An Algorithm for Sparse Underwater Acoustic Channel Identification under Symmetric α-Stable Noise

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***Abstract*— A novel adaptive algorithm is derived for sparse channel identification in the presence of Symmetric α-Stable (SαS) noise. The algorithm is based on the minimization of a new cost function, which is the sum of two terms. The first term is the distance between the previous and the current channel estimate. The distance metric is Riemannian, the same as in the improved-proportionate normalized least-mean-square (IPNLMS) algorithm, so that the sparse nature of the filter taps is taken into account. The second term depends on an appropriately defined 1-norm of the a posteriori estimation error and ensures robustness under SαS noise. The resulting algorithm, the so-called sign-IPNLMS (sIPNLMS), has linear computational complexity with respect to its filter coefficients. The superior performance of the sIPNLMS algorithm over the original IPNLMS, the recursive least-squares (RLS), and the normalized least-mean-square (NLMS) is shown by identifying two measured, sparse, underwater acoustic channels under the presence of recorded snapping shrimp ambient noise and simulated SαS noise. In addition, our proposed algorithm shows similar performance with IPNLMS under Gaussian noise and hence it becomes promising for either impulsive or non-impulsive noise environments.**

I. Introduction

Wideband underwater acoustic (UWA) channels are characterized by long and time-varying impulse responses for many link geometries [1]. Moreover, these channels show a sparse structure, namely, a big fraction of the energy of the channel impulse response is concentrated in a small fraction of its duration [2],[3]. Employing standard adaptive filters, such as the normalized least-mean-square (NLMS) and the recursive least-squares (RLS) filter, for channel estimation, poor performance is observed in two aspects: (a) slow convergence of the filter taps to their steady state values since the convergence rate of the algorithm is proportional to the total channel length [4]; (b) high steady-state misadjustment due to the estimation noise that inevitably occurs during the adaptation of the low-energy filter taps. Since both RLS and NLMS do not exploit the a priori knowledge of channel sparseness, designing adaptive algorithms with better performance seems plausible.

In addition to extended time-varying multipath, warm shallow water channels, such as in Singapore waters, are severely contaminated by impulsive noise generated by snapping shrimp [5]. The noise source level could be as high as $190\,dB$ re $1\,\mu$Pa at $1\,$m. Impulsive snapping shrimp noise is well modeled as a Symmetric α-Stable (SαS) random process. Since this kind of noise process exhibits an infinite second moment, any adaptive filter that tries to minimize the mean squared error between the reference signal and its output will suffer poor performance under large noise disturbances. To the best of our knowledge, an adaptive algorithm that deals with both the channel sparsity and the noise impulsiveness does not exist in the UWA communications literature.

Recently, a class of sparse adaptive algorithms based on the natural gradient (NG) have shown better convergence and steady-state misadjustment than RLS over a sparse shallow water channel [9], [10]. The improved performance is due to the fact that the NG-based algorithms exploit the Riemannian structure of the parameter space to adjust the gradient search direction towards the minimum of the chosen cost function [8]. In this work, a new algorithm is derived by reformulating the cost function of the improved proportionate NLMS (IPNLMS) algorithm [6] so that the inherent channel sparsity is exploited and robustness against impulsive noise is enforced. The cost function is the sum of two terms: the first term is the Riemannian distance between the previous and the current channel estimate while the second term depends on the 1-norm of the a-posteriori estimation error. The resulting algorithm has linear complexity with respect to the number of the filter taps.

A computer generated experiment is conducted to compare the performance between the proposed algorithm and IPNLMS, RLS, and NLMS. Recorded m-sequences over two short-range shallow water links are used to extract sparse channel responses. These estimated channel responses are to be identified under recorded snapping shrimp noise and simulated SαS noise. The results prove the superior performance of the new algorithm in terms of initial convergence rate, tracking, and robustness to noise disturbances.

Notation: Superscripts $^{\mathsf{T}}$, †, and * stand for transpose, Hermitian transpose, and conjugate, respectively. Column vectors (matrices) are denoted by boldface lowercase (uppercase) letters. The 1-norm of the L-tap complex vector $\mathbf{x}$ is defined as $\|\mathbf{x}\|_1 = \sum_{i=0}^{L-1} |x_i|$ and the 2-norm of $\mathbf{x}$ is $\|\mathbf{x}\|_2 = \sqrt{\mathbf{x}^\dagger \mathbf{x}}$. Finally, the sign function of a real number t is expressed as:

$$sgn(t) \triangleq \begin{Bmatrix} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{Bmatrix}. \quad (1)$$

II. SNR Definition in Symmetric α-Stable Noise

Snapping shrimp is a major component of ambient noise at high frequencies (2 kHz - 300 kHz) in warm shallow waters[5]. This kind of noise is highly impulsive and can become a performance limiting factor for systems that are optimized for Gaussian noise. Snapping shrimp noise is well modeled by a SαS distribution. Although there is no closed form for the SαS probability density function (pdf), it can be described by its characteristic function [5]

$$\varphi(\omega) = e^{-\gamma|\omega|^\alpha} \quad (2)$$

where $\alpha \in (0, 2]$ describes the impulsiveness of the noise (smaller α leads to more impulsive noise) and $\gamma > 0$ denotes the dispersion of the distribution. In Singapore waters, typical values of α range between $1.6 - 1.9$ while the parameter γ depends on the operational bandwidth. When $\alpha = 2$, the SαS pdf boils down to the Gaussian pdf and γ equals to half the variance. For $\alpha < 2$, second or higher moments are infinite and therefore, the standard (signal-to-noise ratio) SNR definition cannot be applied.

An SNR definition for baseband (complex) signals is rather involved because the in-phase and quadrature components of the SαS noise are generally dependent. In passband, however, an SNR definition (in dB) can be readily defined as [5]:

$$SNR_\alpha \triangleq 10 \log_{10} \frac{P_s}{2\gamma^{2/\alpha}} \quad (3)$$

where P_s is the signal power and $\gamma^{2/\alpha}$ plays the same role as the variance. When $\alpha = 2$, Eq. (3) becomes the usual SNR definition in Gaussian noise.

III. Algorithm Description

We now tackle the problem of sparse channel identification under impulsive SαS noise. The baseband received signal $y[n]$ is expressed by

$$y[n] = \sum_{k=0}^{K-1} h_k[n]^* x[n-k] + w[n] = \mathbf{h}[n]^\dagger \mathbf{x}[n] + w[n] \quad (4)$$

where $\mathbf{h}[n]$ denotes the K dimensional channel vector at discrete time n, $\mathbf{x}[n]$ stands for the input signal, and $w[n]$ is a SαS distributed random variable. We also present here some important definitions that will be used for the algorithm derivation:

$$e[n] \triangleq y[n] - \hat{\mathbf{h}}[n-1]^\dagger \mathbf{x}[n] \quad (5)$$

$$\hat{\mathbf{h}}[n] \triangleq \hat{\mathbf{h}}[n-1] + \mathbf{r}[n] \quad (6)$$

$$\begin{aligned} \varepsilon[n] &\triangleq y[n] - \hat{\mathbf{h}}[n]^\dagger \mathbf{x}[n] \\ &= e[n] - \mathbf{r}[n]^\dagger \mathbf{x}[n] \end{aligned} \quad (7)$$

where $\hat{\mathbf{h}}[n]$ is the estimated impulse response at time n, $e[n]$ denotes the a priori prediction error of $y[n]$, $\varepsilon[n]$ stands for the a posteriori prediction error of $y[n]$, and $\mathbf{r}[n]$ is the channel update vector.

We follow the general algorithmic framework in [7] that states that an efficient adaptive algorithm must exhibit a good balance between its needs to be conservative (avoid radical changes of $\hat{\mathbf{h}}[n]$ from one iteration to the other) and corrective (ensure better channel estimate if the same input and output were to be observed at two consecutive times). According to this framework, our algorithm is derived by minimizing the cost function:

$$J(n) = \mathbf{r}[n]^\dagger \mathbf{Q}(\hat{\mathbf{h}}[n-1])\mathbf{r}[n] + \mu \, |\varepsilon[n]|_1^p, \;\; \mu > 0, \quad (8)$$

where each of the terms of $J(n)$ is explained below.

The term $\mathbf{r}[n]^\dagger \mathbf{Q}(\hat{\mathbf{h}}[n-1])\mathbf{r}[n]$ in Eq. (8), which ensures the conservativeness of the algorithm, is the Riemannian distance between $\hat{\mathbf{h}}[n-1]$ and $\hat{\mathbf{h}}[n]$. The idea of using a Riemannian distance metric stems from the fact that $\mathbf{h}[n]$ is sparse and hence, the space of $\mathbf{h}[n]$ close to the coordinate axes is highly curved or warped with respect to the Euclidean space[8]. The Riemannian metric tensor $\mathbf{Q}\left(\hat{\mathbf{h}}[n]\right)$ is a $K \times K$ positive definite matrix that describes the local curvature of the parameter space at $\hat{\mathbf{h}}[n]$. For our purposes, we use the diagonal matrix of the IPNLMS algorithm [6], which has proven to be suitable for sparse shallow water channels [9], [10]. The diagonal entries of $\mathbf{Q}\left(\hat{\mathbf{h}}[n]\right)^{-1}$ are given by

$$q_k[n] = \frac{1-\beta}{2K} + \frac{(1+\beta)\left|\hat{h}_k[n]\right|}{2\left\|\hat{\mathbf{h}}[n]\right\|_1 + \epsilon}, 0 \le k \le K-1 \quad (9)$$

where ϵ is a small constant to avoid division by zero when the algorithm is initialized and $\beta \in [-1, 1]$ is the parameter that controls the sparseness (larger β is favorable to more sparse solutions).

The term $\mu \, |\varepsilon[n]|_1^p$ in Eq. (8), which promotes the corrective-ness of the algorithm, uses the 1-norm of the complex number $\varepsilon[n]$, defined as

$$|\varepsilon[n]|_1 \triangleq |\mathrm{Re}\{\varepsilon[n]\}| + |\mathrm{Im}\{\varepsilon[n]\}|. \quad (10)$$

The power $p \in (0, \alpha)$ ensures robust convergence and tracking performance of the algorithm since any moment of the form $E\left[|\varepsilon[n]|_1^p\right], \; p \ge \alpha$, becomes infinite under SαS noise. In addition, when $\mu \to \infty$, the optimization problem essentially becomes the minimization of $\mathbf{r}[n]^\dagger \mathbf{Q}(\hat{\mathbf{h}}[n-1])\mathbf{r}[n]$ subject to $\varepsilon[n] = 0$. Since we expect the noise to be impulsive, μ should be chosen close to zero so that the algorithm focuses on minimizing the distance between $\hat{\mathbf{h}}[n-1]$ and $\hat{\mathbf{h}}[n]$ and therefore, radical channel updates that compromise robustness are prohibited.

Taking the gradient of $J(n)$ with respect to $\mathbf{r}[n]^*$, applying the chain rule, and setting the resulting vector to zero we have[1]

$$\mathbf{Q}(\hat{\mathbf{h}}[n-1])\mathbf{r}[n] + \mu \frac{\partial\, |\varepsilon[n]|_1^p}{\partial\, |\varepsilon[n]|_1} \frac{\partial\, |\varepsilon[n]|_1}{\partial \varepsilon[n]} \frac{\partial \varepsilon[n]}{\partial \mathbf{r}[n]^*} = 0. \qquad (11)$$

Using the following identities:

$$\frac{\partial\, |\varepsilon[n]|_1^p}{\partial\, |\varepsilon[n]|_1} = p\, |\varepsilon[n]|_1^{p-1} \qquad (12)$$

$$\frac{\partial\, |\varepsilon[n]|_1}{\partial \varepsilon[n]} = \frac{1}{2} csgn(\varepsilon[n])^* \qquad (13)$$

$$\frac{\partial \varepsilon[n]}{\partial \mathbf{r}[n]^*} = -\mathbf{x}[n] \qquad (14)$$

we finally obtain

$$\hat{\mathbf{h}}[n] = \hat{\mathbf{h}}[n-1] + \tilde{\mu}\, |\varepsilon[n]|_1^{p-1}\, csgn(\varepsilon[n])^* \mathbf{Q}(\hat{\mathbf{h}}[n-1])^{-1}\mathbf{x}[n], \qquad (15)$$

where $\tilde{\mu} = \mu \cdot p/2$ and

$$csgn(\varepsilon[n]) \triangleq sgn(\mathrm{Re}\,\{\varepsilon[n]\}) + j \cdot sgn(\mathrm{Im}\,\{\varepsilon[n]\}). \qquad (16)$$

Eq. (16) is essentially a functional form of the sign of a complex number and thus, the name $csgn(\cdot)$. Note that Eq. (13) is impractical to implement because $\varepsilon[n]$ is not known at time step n but, rather, needs be determined. In steady-state, however, it is plausible to assume that $\hat{\mathbf{h}}[n]$ is very close to $\hat{\mathbf{h}}[n-1]$ and so, $e[n] \simeq \varepsilon[n]$. Hence, the sign-IPNLMS (sIPNLMS) algorithm is deduced as:

$$\hat{\mathbf{h}}[n] = \hat{\mathbf{h}}[n-1] + \tilde{\mu}\, |e[n]|_1^{p-1}\, csgn(e[n])^* \mathbf{Q}(\hat{\mathbf{h}}[n-1])^{-1}\mathbf{x}[n] \qquad (17)$$

where the entries of $\mathbf{Q}(\hat{\mathbf{h}}[n-1])^{-1}$ are given by Eq. (9). The algorithm is initialized with $h_k[0] = 0,\ 0 \leq k \leq K-1$.

Note that the sIPNLMS has $O(K)$ complexity, which is highly desirable for hardware implementation. The main burden comes from computing $\left\|\hat{\mathbf{h}}[n]\right\|_1$ and $\mathbf{Q}(\hat{\mathbf{h}}[n-1])^{-1}\mathbf{x}[n]$ at each iteration. For complex data, $\left\|\hat{\mathbf{h}}[n]\right\|_1$ requires $4K$ floating point operations while $\mathbf{Q}(\hat{\mathbf{h}}[n-1])^{-1}\mathbf{x}[n]$ requires $4K-2$ floating point operations.

IV. Performance Results

In this section, the sIPNLMS algorithm is compared with two widely used stochastic gradient descent algorithms (NLMS and RLS) and one NG-based algorithm (IPNLMS) by using a computer generated experiment. The experiment considers the identification of two sparse time-varying channels $\mathbf{h}[n]$ in the presence of snapping shrimp ambient noise and simulated SαS noise. We stress that the channels are not simulated but are measured from different field experiments. Hence, the complexities one would encounter in a real experiment are not neglected. As a performance measure, the normalized misadjustment (in dB),

$$20\log_{10}\left(\frac{\left\|\mathbf{h}[n]-\hat{\mathbf{h}}[n]\right\|_2}{\|\mathbf{h}[n]\|_2}\right), \qquad (18)$$

is employed.

[1]The concept of the gradient of a cost function with respect to complex variables is discussed in Appendix B of [4].

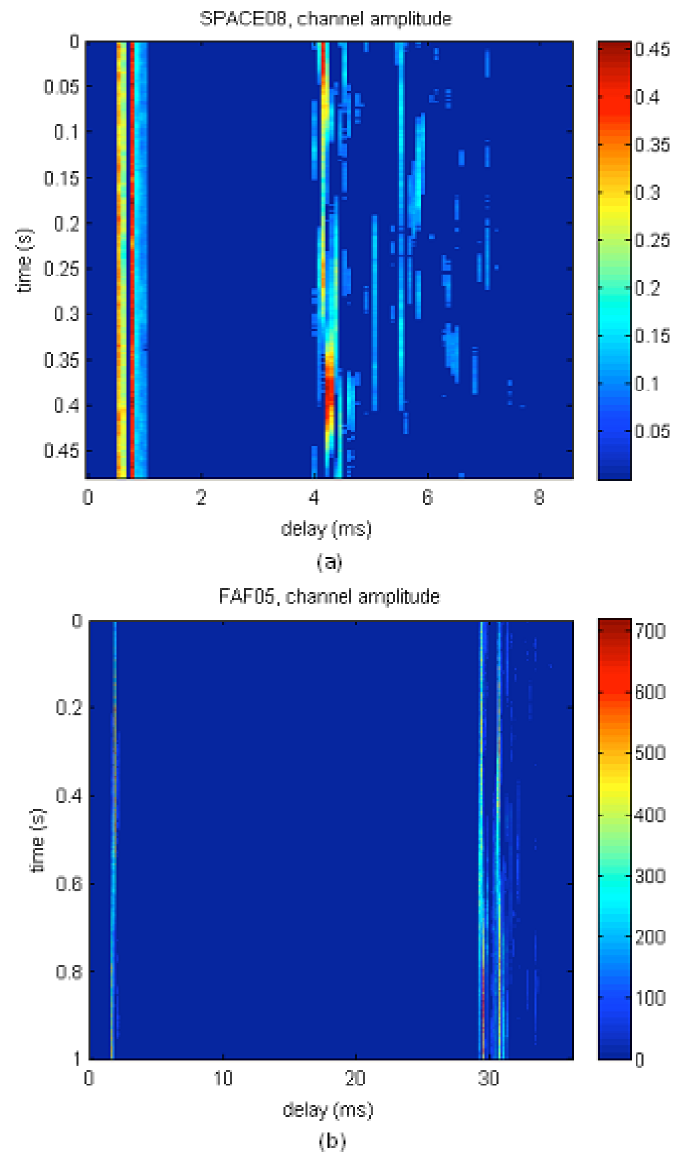


Fig. 1. Snapshots of the estimated baseband channel impulse response. The L_0-IPNLMS algorithm [10] was employed for adaptive channel estimation. The horizontal axis represents delay, the vertical axis represents absolute time and the colorbar represents the amplitude. (a) SPACE08 experiment. The snapshots are computed at a rate of 6510.4 Hz. (b) FAF05 experiment. The snapshots are computed at a rate of 6250 Hz.

The channel impulse response of Fig. 1(a) was obtained during the Surface Processes and Acoustic Communications Experiment (SPACE08) off the coast of Martha's Vineyard in northeast America in October 26th, 2008. Both the transmitter and the receiver were mounted on rigid tripods and were

TABLE I

CHOSEN PARAMETERS OF ALL ALGORITHMS

	SPACE08	FAF05
NLMS	$\eta = 0.4,\ \delta = 10$	$\eta = 0.6,\ \delta = 10$
RLS	$\lambda = 0.996,\ \delta = 100$	$\lambda = 0.997,\ \delta = 100$
IPNLMS	$\mu = 0.3,\ \beta = 0$	$\mu = 0.2,\ \beta = 0.5$
	$\delta = 0.067$	$\delta = 0.011$
sIPNLMS	$\mu = 0.2, \beta = 0, p = 1.5$	$\mu = 7, \beta = 0.5, p = 1.5$

located $4\,\mathrm{m}$ and $3.25\,\mathrm{m}$, respectively, above the sea floor. The sea depth was $15\,\mathrm{m}$ and the horizontal range of the link was $60\,\mathrm{m}$. The transmitted signal was a 6510.4 bps-rate, BPSK modulated m-sequence with $12.5\,\mathrm{kHz}$ carrier frequency. Note that the impulse response has a sparse structure and is highly time-varying due to sound scattering off the moving sea surface. For this link, the channel delay spread is $8.6\,\mathrm{ms}$ corresponding to a total channel length of 112 taps, when the received signal is sampled at 2 samples/bit.

The channel impulse response of Fig. 1(b) was obtained during the Focused Acoustic Fields (FAF05) experiment off the coast of Pianosa, Italy in July 22nd, 2005. The transmitter was attached on the hull of the research vessel Leonardo, $4.5\,\mathrm{m}$ below the sea level. The receiver was mounted on a moving autonomous undersea vehicle (AUV). The range of the link was approximately $700\,\mathrm{m}$ and the ocean depth at the receiver side was $85\,\mathrm{m}$. The transmitted signal was a 6250 bps-rate, QPSK modulated m-sequence with $12\,\mathrm{kHz}$ carrier frequency. Note again that the estimated impulse response is highly time-varying due to the motion of the AUV and exhibits a long delay spread of $36\,\mathrm{ms}$. When the received signal is sampled at 2 samples/symbol, the corresponding channel length is 454 taps.

In our simulations, a BPSK modulated m-sequence[2] is applied to the each of the above channels, and hence the output signal is expressed as a linear time-varying convolution sum. Then, each channel output is added to a noise series, initially recorded in warm shallow waters in Singapore at $500\,\mathrm{kHz}$. Prior to adding the noise to the output, the noise is bandpass filtered for α and γ estimation and then shifted to baseband, downsampled to the symbol rate of the corresponding channel, and scaled such that the average SNR_α is $7\ dB$. Since the channels have similar carrier frequencies and symbol rates, the SαS parameters α and γ are approximately the same for both channels. Specifically, after running a Kolmogorov-Smirnov goodness of fit test, α and $\gamma^{1/\alpha}$ are found to be 1.84 and $9.004 \times 10^{-4}\,\mathrm{Pa}$, respectively.

Choosing the right algorithm parameters is not a straightforward procedure but rather, channel dependent. Moreover, the system requirements of fast initial convergence rate, small steady-state misadjustment, fast tracking of channel changes, and robustness to noise disturbances are competing to each other for any adaptive algorithm. To ensure a fair comparison between sIPNLMS and IPNLMS (both have $O(K)$ complexity), we chose the parameters μ and β such that both algorithms show the same initial convergence rate for each channel. The parameters of NLMS and RLS were chosen for best steady-state performance. Table I summarizes the chosen algorithm parameters used to generate the results below. The parameters η and λ denote the step-size and the forgetting factor of the NLMS and RLS, respectively [4]. The parameter δ is the respective regularization constant for NLMS, RLS, and IPNLMS.

Fig. 2(a) shows the normalized misadjustment of each algorithm based on the impulse response of Fig. 1(a) and the noise series of Fig. 2(b). Clearly, IPNLMS and sIPNLMS outperform RLS and NLMS in terms of both convergence and tracking. For instance, both IPNLMS and sIPNLMS need $45\,\mathrm{ms}$ to converge to $-9.7\,dB$, about $3.2\,dB$ and $4.2\,dB$

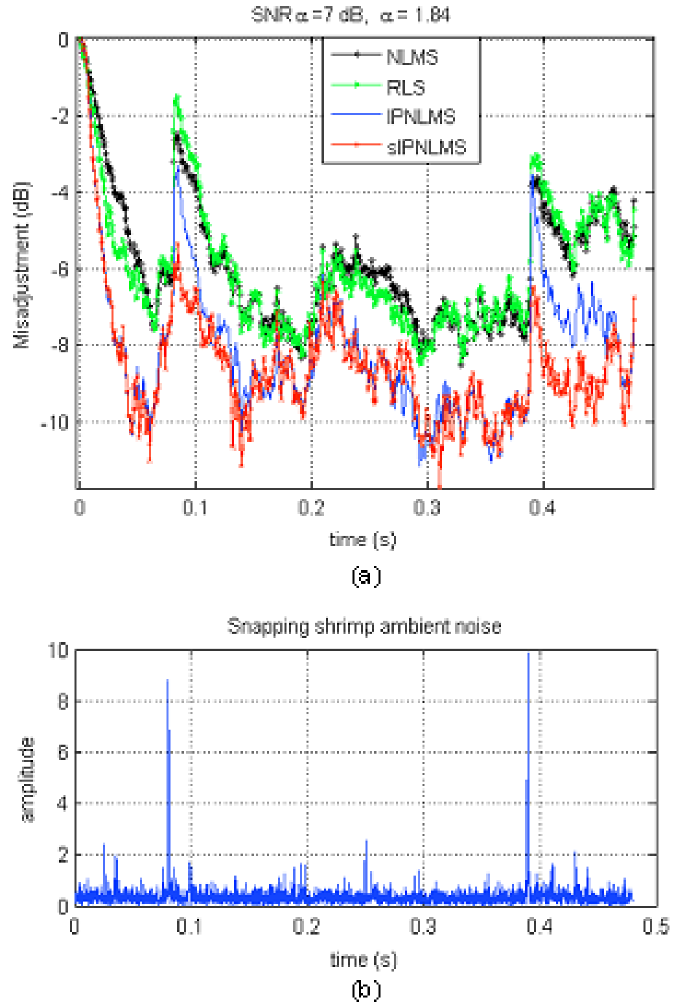
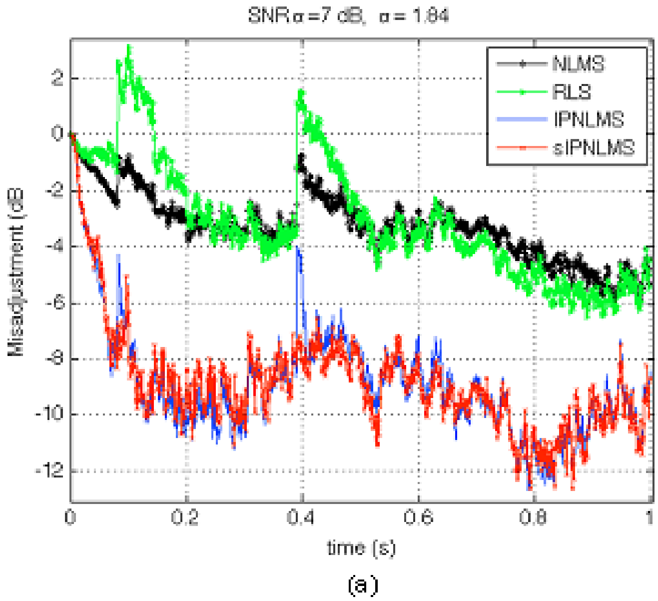


Fig. 2. Results for the SPACE08 channel: (a) Normalized misadjustment for all algorithms; (b) amplitude of the corresponding baseband ambient noise.

[2]We chose not to pulse shape the m-sequence because the convergence rate and the misadjustment of RLS and NLMS depend on the condition number of the input covariance matrix. The higher the condition number, the poorer the performance. Since the m-sequence exhibits an almost white autocorrelation function, a well-conditioned input covariance matrix is guaranteed.



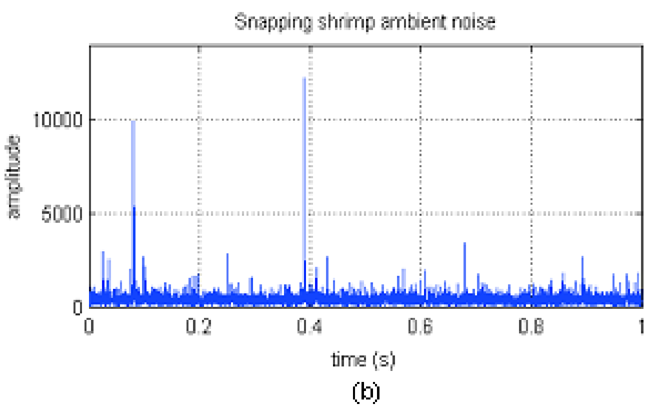


Fig. 3. Results for the FAF05 channel: (a) Normalized misadjustment for all algorithms; (b) amplitude of the corresponding baseband ambient noise.

better than RLS and NLMS, respectively. Moreover, sIPNLMS demonstrates better tracking performance under impulsive noise than the rest of the algorithms. Specifically, at $0.39\,\mathrm{s}$, the received signal is distorted by a loud snap, as seen in Fig. 2(b). At that time also, the channel impulse response shows a significant change due to a strong surface reflected multipath component arriving $4\,\mathrm{ms}$ after the direct arrival. Note that the sIPNLMS demonstrates about $3.4\,dB$, $3.1\,dB$, and $3.6\,dB$ better misadjustment than IPNLMS, NLMS, and RLS, respectively.

In Fig. 3(a), we now use the impulse response of Fig. 1(b) and the noise series of Fig. 3(b) to compare all the algorithms. IPNLMS and sIPNLMS still outperform RLS and NLMS in terms of both convergence and tracking. For instance, after tracking the channel for $0.8\,\mathrm{s}$, both IPNLMS and sIPNLMS show $-10.8\,dB$ misadjustment, about $5.9\,dB$ and $6.5\,dB$ better than RLS and NLMS, respectively. Furthermore, sIPNLMS is still more robust under impulsive noise than all other algorithms. For instance, when a loud snap occurs at about $0.39\,\mathrm{s}$, sIPNLMS shows about $3.6\,dB$, $6.4\,dB$, and $8.8\,dB$ better misadjustment than IPNLMS, NLMS, and RLS, respectively.

The above results provide insight into the mechanism of the new algorithm but they are generated by using a single noise realization. Since we did not have enough noise recordings to average out the corresponding misadjustments, we used simulated noise for the following results. Figs. 4(a) and (b) show the algorithm performances for SPACE08 and FAF05, respectively, when the simulated passband noise is SαS with $\alpha = 1.65$. The SαS noise simulator can be found in [11]. The simulated noise went through exactly the same processing as the recorded noise above so that SNR_α remains $7\ dB$. Clearly, sIPNLMS outperforms the rest of the algorithms in terms of both initial convergence rate and tracking validating the use of the 1-norm of the a posterior error. Figs. 5(a) and (b) illustrate the algorithm performances for SPACE08 and FAF05, respectively, when the simulated baseband noise is a circular symmetric Gaussian random process. The SNR is chosen $7\ dB$ for consistency with the previous results. Clearly, both sIPNLMS and IPNLMS outperform NLMS and RLS, e.g., in Fig. 5(b), RLS and NLMS show between $-4\,dB$ and $-5\,dB$ worse tracking performance than the sparse algorithms. This result confirms that RLS and NLMS are not suitable for sparse UWA channel estimation. In addition, note that sIPNLMS and IPNLMS have similar performances for both channels rendering sIPNLMS suitable for either impulsive or non-impulsive noise environments.

V. Conclusion

A new algorithm, the sIPNLMS, was proposed for sparse channel estimation under SαS noise. The algorithm was derived by minimization of a cost function, which is the sum of two terms: the first term is the Riemannian distance between the previous and the current channel estimate and the second term is the p-th power ($p < \alpha$) of the 1-norm of the a-posteriori estimation error. The use of the Riemannian distance modifies the gradient search direction for faster adaptation while the 1-norm of the a-posteriori estimation error ensures robustness to noise impulsiveness. A comparison between sIPNLMS and IPNLMS, RLS, and NLMS was carried out by estimating two measured sparse underwater acoustic links under recorded snapping shrimp ambient noise and simulated SαS noise. These results confirmed the superior performance of sIPNLMS algorithm to all other algorithms. In addition, simulation results using white Gaussian noise showed that sIPNLMS and IPNLMS had similar performances rendering sIPNLMS a promising algorithm for either impulsive or non-impulsive noise channels.

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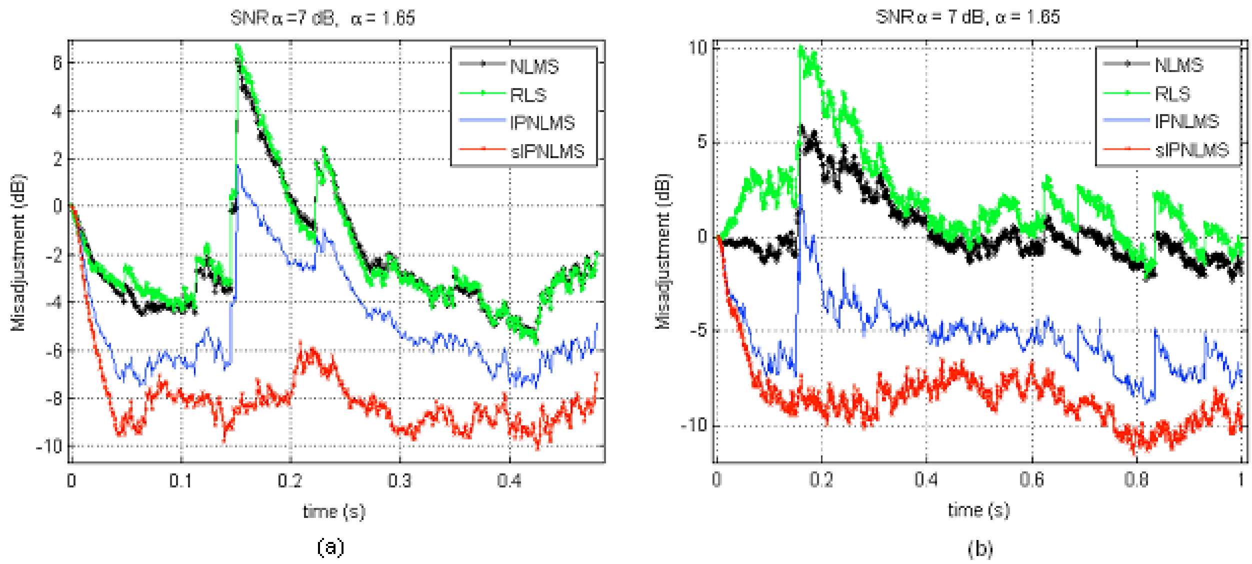

Fig. 4. Normalized misadjustment for all algorithms averaged out over 200 SaS noise realizations: (a) SPACE08 channel; (b) FAF05 channel.

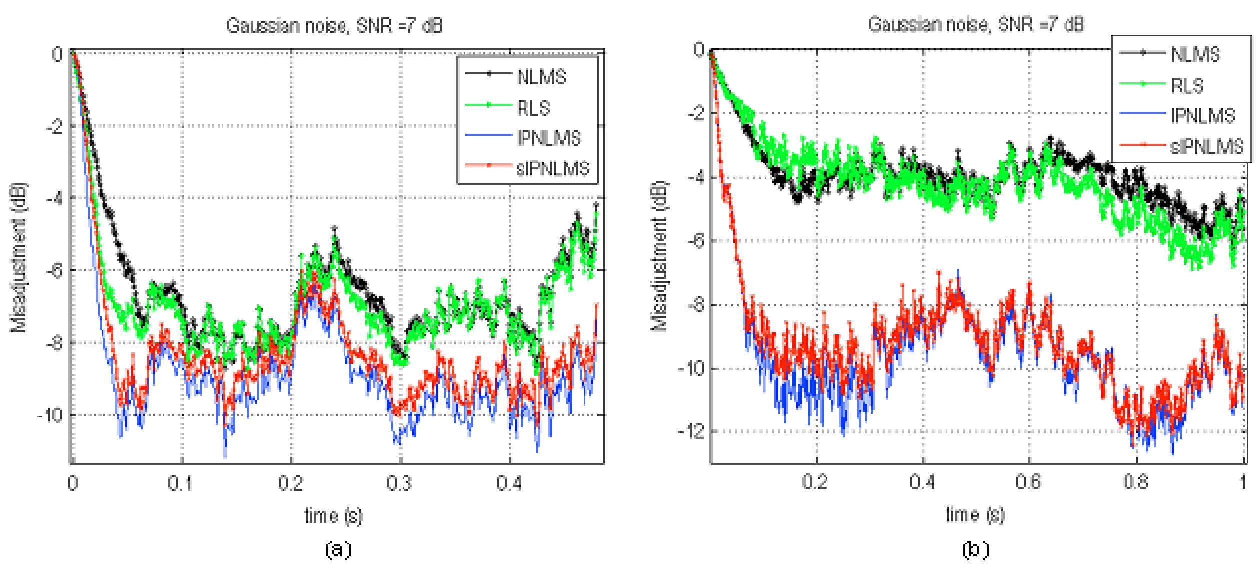

Fig. 5. Normalized misadjustment for all algorithms averaged out over 200 Gaussian noise realizations: (a) SPACE08 channel; (b) FAF05 channel.

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