Model-Based Signal Detection in Snapping Shrimp Noise

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***Abstract*—In a number of scenarios, detecting the presence or absence of a known signal may be of practical interest. One such example lies in a communication setting, where packet detection is a vital first step to decode transmitted data. The detection problem can be formulated as a binary hypothesis test within the Neyman-Pearson (NP) framework. Our scenario of interest is warm shallow waters, where the sea floor is inhabited by colonies of snapping shrimp. The ambient noise in such a case is *impulsive* and exhibits *memory*. We investigate the performance of optimal detectors corresponding to *four* additive noise models in snapping shrimp noise. In the literature, proposed detectors typically take only the amplitude statistics of the noise process into account. By also considering the memory, we show that there is substantial improvement in detection performance. The detector in the latter case is based on the recently introduced stationary α-sub-Gaussian noise with memory order m (αSGN(m)) model, which effectively characterizes the temporal amplitude statistics of the snapping shrimp noise process.**

I. Introduction

In shallow tropical waters, snapping shrimp noise dominates the acoustic ambient noise spectrum at frequencies greater than 2kHz [1]. As the noise process is *impulsive*, it is modeled well by heavy-tailed (non-Gaussian) distributions [2]–[4]. Moreover, the process also has memory. This results in the clustering of impulses, thus making the noise process *bursty* as well [3], [5]. If not mitigated, snapping shrimp noise can be severely detrimental to communication and signal processing schemes operating in such environments [6], [7].

Signal detection is an integral part of an underwater acoustic communications receiver. In a typical scheme, data is broken down into symbols and is transmitted over several packets [8]. At the receiver, *each* packet needs to be accurately detected *before* one can process the data within. If not, this results in a miss or false alarm, both of which translate to loss of throughput. In the latter case, the receiver starts accepting garbage values and may therefore be unable to detect any transmissions made during this time. To aid detection, a preamble is added to a packet, which is essentially a signal of large time-bandwidth product that is *known* to the receiver. Thus, the problem may be modeled as a *binary hypothesis* test, i.e., detecting the presence or absence of a known signal in noise [9].

In [6], [10], the authors highlight the performance of various detectors in snapping shrimp noise. The Neyman-Pearson (NP) framework was employed and results were compiled for a binary hypothesis test for synthetic data. The authors observed that the *amplitude* statistics of snapping shrimp noise were tracked well by a heavy-tailed symmetric α-stable (SαS) distribution. By modeling the noise process as white SαS noise (WSαSN), detection performance of the maximum-likelihood (ML) estimate of signal strength was shown to *significantly* outperform the linear correlator (LC) in snapping shrimp noise. The latter is the ML estimate of signal strength in white Gaussian noise (WGN) [6], [9]. However, the WSαSN model is still sub-optimal as it assumes independent and identically distributed (IID) samples and does not take into account the memory of the snapping shrimp noise process [3], [5]. To address this, the stationary α-sub-Gaussian noise with memory order m (αSGN(m)) model was introduced in the literature [5]. The latter effectively characterizes both *amplitude* and *temporal* statistics of the snapping shrimp noise process.

The contribution of this work is as follows: we investigate the performance of the log-likelihood ratio (LLR) detector for αSGN(m) in *actual* snapping shrimp noise. Following [6], we synthesize data by immersing a known signal in recorded snapping shrimp noise samples and compile results for binary hypothesis testing within the NP framework. For comparison, we also highlight the performance of the LLR detectors corresponding to WGN, WSαSN and colored Gaussian noise (CGN). The CGN model characterizes only the memory and is unable to track the amplitude statistics of snapping shrimp noise. Our results clearly highlight the relative improvement between detectors that exploit increasingly more information about the noise process. Moreover, we show that the LLR detector for αSGN(m) offers *substantial* improvement over its WSαSN counterpart, thus highlighting the added potential of exploiting memory of the snapping shrimp noise process as well as its amplitude statistics.

This paper is organized as follows: In Section II we summarize key concepts of univariate stable distributions and outline the binary detection problem. We then discuss conventional additive noise models and their LLR detectors in Section III. The αSGN(m) model is presented and its LLR detector derived in Sections IV & V, respectively. Finally, we wrap up our discussion in Section VI by evaluating the performance of all detectors in snapping shrimp noise.

II. Problem Definition & Concepts

A. Univariate Stable Distributions

A *univariate* stable distribution can be expressed in terms of four parameters, namely the characteristic exponent $\alpha \in (0, 2]$, the skew parameter $\beta \in [-1, 1]$, the scale $\delta \in \mathbb{R}^+$ and location $\mu \in \mathbb{R}$ [2], [11]. It may therefore be represented

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by the abridged notation $\mathcal{S}(\alpha, \beta, \delta, \mu)$. Stable distributions are generally heavy-tailed. Moreover, the degree of heaviness is exclusively controlled by α. The lower the value of α, the heavier the tails of the distribution [2], [11]. Similarly, as $\alpha \to 2$, the tails become increasingly lighter. In fact, for $\alpha = 2$, the distribution is no longer dependent on β and is equivalent to a Gaussian distribution with mean μ and variance $2\delta^2$, i.e., $\mathcal{S}(2, \beta, \delta, \mu) \stackrel{d}{=} \mathcal{N}(\mu, 2\delta^2)$, where $\stackrel{d}{=}$ implies equality in distribution [2], [11]. The Gaussian distribution is the only member of the stable family to have light (exponential) tails.

A special subclass of the stable family is the SαS distribution. A *univariate* SαS distribution is stable but with $\beta = \mu = 0$ [2], [11]. It may therefore be denoted by $\mathcal{S}(\alpha, \delta)$. As highlighted by its name, the probability density function (PDF) of a SαS random variable is symmetric in its argument. Moreover, from the discussion on stable random variables, we have $\mathcal{S}(2, \delta) \stackrel{d}{=} \mathcal{N}(0, 2\delta^2)$. Thus, the zero-mean Gaussian distribution is a member of the SαS family. One disadvantage of working with SαS distributions (and stable distributions in general) is the lack of closed-form expressions for their PDFs. The only exceptions to this are the Gaussian ($\alpha = 2$) and the Cauchy ($\alpha = 1$) cases. Therefore, one must revert to numerical routines when dealing with most SαS PDFs [2], [11].

The concepts and notation presented in this section are sufficient to understand the noise models considered in this text. The binary detection problem is discussed next.

B. The Binary Detection Problem

Within the NP framework, the binary detection problem can be formulated by selecting one of two possible hypotheses,

$$\begin{aligned} \mathcal{H}_0 &: \quad x_n = w_n \\ \mathcal{H}_1 &: \quad x_n = \theta s_n + w_n \end{aligned} \tag{1}$$

$\forall\ n \in \{1, 2, \dots, N\}$, where x_n, s_n and w_n represent time samples of the received signal, transmitted signal and noise process, respectively [6], [9]. The parameter $\theta \in \mathbb{R}^+$ is a measure of signal strength and is known [6]. It is sometimes convenient to express (1) in vector form, i.e.,

$$\begin{aligned} \mathcal{H}_0 &: \quad \mathbf{x} = \mathbf{w} \\ \mathcal{H}_1 &: \quad \mathbf{x} = \theta\mathbf{s} + \mathbf{w} \end{aligned} \tag{2}$$

where $x_n,\ s_n$, and w_n are the n^{th} elements of $\mathbf{x},\ \mathbf{s}$ and $\mathbf{w}$, respectively. We consider the signal to have finite energy, i.e., $\mathbf{s}^\mathsf{T}\mathbf{s} = \mathcal{E}$ for some $\mathcal{E} \in \mathbb{R}^+$.

According to the NP Lemma, the LLR detector is optimal in the sense that it maximizes the probability of detection (P_D) for a given probability of false alarm (P_{FA}) [9]. Mathematically, the LLR detector is given by

$$L(\mathbf{x}) = \log \frac{f_{\vec{W}}(\mathbf{x} - \theta\mathbf{s})}{f_{\vec{W}}(\mathbf{x})}, \tag{3}$$

where $f_{\vec{W}}(\mathbf{w})$ is the PDF of the random vector $\vec{W} \in \mathbb{R}^N$ with outcome $\mathbf{w}$. For some $\gamma \in \mathbb{R}$, the detector decides in favor of $\mathcal{H}_1$ if $L(\mathbf{x}) > \gamma$ and $\mathcal{H}_0$ otherwise [9]. The threshold γ is determined from a given P_{FA}, which in turn is expressed as $P_{FA} = P(L(\mathbf{x}) > \gamma; \mathcal{H}_0)$. The notation $P(L(\mathbf{x}) > \gamma; \mathcal{H}_0)$ is read as the probability of $L(\mathbf{x}) > \gamma$ given $\mathcal{H}_0$ is true. The subsequent detection performance may then be evaluated from $P_D = P(L(\mathbf{x}) > \gamma; \mathcal{H}_1)$. As θ is assumed known, the resulting detector is essentially a *clairvoyant* detector and offers the best possible performance in a practical setting [9].

It is clear from (3), that the performance of an LLR detector in snapping shrimp noise depends greatly on the adopted noise model. These are summarized next.

III. Conventional Noise Models & LLR Detectors

In the literature, LLR detection in WGN and CGN is a well-investigated subject [9]. If $\vec{W}$ are samples of WGN, then the joint-densities in (3) can be broken down in a product of univariate zero-mean Gaussian PDFs. Given that $W_n \sim \mathcal{N}(0, 2\delta^2)$, the WGN LLR detector with known s_n and θ is

$$L(\mathbf{x}) = (2\theta\mathbf{x}^\mathsf{T}\mathbf{s} - \theta^2\mathcal{E})/4\delta^2,$$

or equivalently [9],

$$L'(\mathbf{x}) = \mathbf{x}^\mathsf{T}\mathbf{s}. \tag{4}$$

In CGN, $\vec{W}$ is a zero-mean Gaussian random vector with an arbitrary covariance matrix $\mathbf{\Sigma} \in \mathbb{R}^{N\times N}$, i.e., $\vec{W} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$. The simplified LLR statistic is thus [9]

$$L'(\mathbf{x}) = \mathbf{x}^\mathsf{T}\mathbf{\Sigma}^{-1}\mathbf{s} = \mathbf{x}'^\mathsf{T}\mathbf{s}', \tag{5}$$

where $\mathbf{x}' = \mathbf{\Sigma}^{-1/2}\mathbf{x}$ and $\mathbf{s}' = \mathbf{\Sigma}^{-1/2}\mathbf{s}$. The matrix $\mathbf{\Sigma}^{-1/2}$ stems from the Cholesky decomposition of $\mathbf{\Sigma}$, i.e., $\mathbf{\Sigma}^{-1} = (\mathbf{\Sigma}^{-1/2})^\mathsf{T}\mathbf{\Sigma}^{-1/2}$. With this choice of variables, we observe that (5) results from a left multiplication operation of (2) by the *whitening* matrix $\mathbf{\Sigma}^{-1/2}$ and subsequently invoking (4).

For WSαSN, the noise samples are IID SαS *random variables*. Therefore, if $\vec{W}$ consists of samples of WSαSN, then $W_n \sim \mathcal{S}(\alpha, \delta)\ \forall\ n \in \{1, 2, \dots, N\}$. The corresponding LLR detector can then be written in the general form

$$L(\mathbf{x}) = \log \frac{\prod_{n=1}^{N} f_W(x_n - \theta s_n)}{\prod_{n=1}^{N} f_W(x_n)}, \tag{6}$$

where $f_W(\cdot)$ is a univariate PDF corresponding to $\mathcal{S}(\alpha, \delta)$ [6]. Empirical amplitude distributions of snapping shrimp noise are tracked well within $1.5 \leq \alpha < 2$ [3], [6]. Therefore, (6) cannot be further simplified due to $f_W(\cdot)$ being unavailable in closed-form [2], [12].

In retrospect, we see that the CGN and WSαSN models are able to track the memory and amplitude statistics, respectively, of the snapping shrimp noise process. However, the WGN model is unable to do so.

IV. The αSGN(m) Model

In [5], it was observed that scatter plots of closely-spaced samples of snapping shrimp noise followed *near-elliptical* geometries. The αSGN(m) model was then shown to not only track the amplitude statistics of the noise process, but also the dependency between adjacent samples. The model is based on the multivariate α-sub-Gaussian (αSG) distribution, which is a member of the SαS family ($\alpha \neq 2$) with the added characteristic of being *elliptic* as well [13].

More precisely, the αSGN(m) model is based on a sliding-window framework and constrains *any* immediately adjacent $m+1$ samples to be αSG [5]. Moreover, as the marginals of a multivariate SαS distribution are SαS [11], [12], a sample of αSGN(m) is essentially a *heavy-tailed* SαS random variable. Thus the amplitude distribution of an αSGN(m) process is heavy-tailed SαS. Let $W_n \sim \mathcal{S}(\alpha, \delta)$ (for $\alpha \neq 2$) be samples of αSGN(m) and $\vec{W}_{n,m} = [W_{n-m}, W_{n-m+1}, \ldots, W_n]^\mathsf{T}$ be a *random vector* in $\mathbb{R}^{m+1}$ whose elements are the current sample (at index n) and m immediately previous samples. Then

$$\vec{W}_{n,m} \stackrel{d}{=} A_n^{1/2} \vec{G}_{n,m}, \tag{7}$$

where $A_n \sim \mathcal{S}(\frac{\alpha}{2}, 1, 2(\cos(\frac{\pi\alpha}{4}))^{2/\alpha}, 0)$ is a right-skewed stable random variable, $\vec{G}_{n,m} = [G_{n-m}, G_{n-m+1}, \ldots, G_n]^\mathsf{T}$ and $\vec{G}_{n,m} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_m)\ \forall\ n \in \mathbb{Z}$ [5], [13]. We note that $\mathbf{R}_m \in \mathbb{R}^{(m+1)\times(m+1)}$. Moreover, both A_n and $\vec{G}_{n,m}$ are independent of each other and their statistics *do not* vary with time [13]. Consequently, αSGN(m) is a *stationary* process. Due to the sliding window framework, the distribution of W_n depends on the immediately previous m samples, i.e., $\vec{W}_{n-1,m-1}$. This implies that the αSGN(m) process is Markov of order m. From the sliding window framework and stationarity of αSGN(m), $\mathbf{R}_m = [r_{ij}]$ is essentially a *symmetric Toeplitz matrix* [5]. Therefore, to satisfy $W_n \sim \mathcal{S}(\alpha, \delta)$, the main-diagonal elements of $\mathbf{R}_m$ need to be equal to δ^2, i.e., $r_{ii} = \delta^2\ \forall\ i \in \{1, 2, \ldots, m+1\}$ [13]. One may also express $\mathbf{R}_m$ in the block form

$$\mathbf{R}_m = \begin{bmatrix} \mathbf{R}_{m-1} & \mathbf{r}_m \\ \mathbf{r}_m^\mathsf{T} & r_{(m+1)(m+1)} \end{bmatrix}, \tag{8}$$

where $\mathbf{r}_m = [r_{1(m+1)}, r_{2(m+1)}, \ldots, r_{m(m+1)}]^\mathsf{T}$ [5].

The αSGN(m) model offers a more general framework compared to its WSαSN counterpart. In fact, αSGN(0) and heavy-tailed WSαSN are equivalent processes. In general, for a given m, the αSGN(m) process depends on $m+2$ parameters, i.e., α and the $m+1$ elements that construct $\mathbf{R}_m$. On the other hand WSαSN only depends on the two parameters α and δ.

V. The LLR Detector for αSGN(m)

We exploit the Markovity and stationarity properties of αSGN(m) to achieve a simpler form of the LLR detector in (3). This corresponds to expressing $f_{\vec{W}}(\cdot)$ in a suitable form. In probability theory, the chain rule allows breaking a joint PDF into a product of univariate conditional densities [14]. Let us define the random vector $\vec{W}_n = [W_1, W_2, \ldots, W_n]^\mathsf{T}$ and its outcome $\mathbf{w}_n = [w_1, w_2, \ldots, w_n]^\mathsf{T}$. From the chain rule, we have

$$\begin{aligned} f_{\vec{W}}(\mathbf{w}) &= f_{\vec{W}_m}(\mathbf{w}_m) f_{\vec{W}|\vec{W}_m}(\mathbf{w}|\mathbf{w}_m) \\ &= f_{\vec{W}_m}(\mathbf{w}_m) \prod_{n=m+1}^{N} f_{W_n|\vec{W}_{n-1}}(w_n|\mathbf{w}_{n-1}). \end{aligned}$$

On invoking the Markovity and stationary properties of αSGN(m), the conditional density can be expressed as

$$\begin{aligned} f_{W_n|\vec{W}_{n-1}}(w_n|\mathbf{w}_{n-1}) &= f_{W_n|\vec{W}_{n-1,m-1}}(w_n|\mathbf{w}_{n-1,m-1}), \\ &= f_{W_{m+1}|\vec{W}_m}(w_n|\mathbf{w}_{n-1,m-1}). \end{aligned}$$

where $\mathbf{w}_{n,m} = [w_{n-m}, w_{n-m+1}, \ldots, w_n]^\mathsf{T}$. This results in

$$\begin{aligned} f_{\vec{W}}(\mathbf{w}) &= f_{\vec{W}_m}(\mathbf{w}_m) \prod_{n=m+1}^{N} f_{W_{m+1}|\vec{W}_m}(w_n|\mathbf{w}_{n-1,m-1}) \\ &= f_{\vec{W}_m}(\mathbf{w}_m) \prod_{n=m+1}^{N} \frac{f_{\vec{W}_{m+1}}(\mathbf{w}_{n,m})}{f_{\vec{W}_m}(\mathbf{w}_{n-1,m-1})}. \end{aligned} \tag{9}$$

On substituting (9) in (3), the LLR detector takes on the form

$$\begin{aligned} L(\mathbf{x}) = &\log \frac{f_{\vec{W}_m}(\mathbf{x}_m - \theta\mathbf{s}_m)}{f_{\vec{W}_m}(\mathbf{x}_m)} \\ &+ \log \frac{\prod_{n=m+1}^{N} \frac{f_{\vec{W}_{m+1}}(\mathbf{x}_{n,m} - \theta\mathbf{s}_{n,m})}{f_{\vec{W}_m}(\mathbf{x}_{n-1,m-1} - \theta\mathbf{s}_{n-1,m-1})}}{\prod_{n=m+1}^{N} \frac{f_{\vec{W}_{m+1}}(\mathbf{x}_{n,m})}{f_{\vec{W}_m}(\mathbf{x}_{n-1,m-1})}}. \end{aligned} \tag{10}$$

We note that $\vec{W}_{m+1}$ is an $(m+1)$-dimensional αSG random vector as it consists of $m+1$ immediately adjacent samples of αSGN(m). Similarly, as $\vec{W}_m$ is a marginal of $\vec{W}_{m+1}$, it is αSG as well. Moreover, from (8), the underlying covariance matrix of $\vec{W}_m$ is $\mathbf{R}_{m-1}$. Like its univariate counterpart, a multivariate αSG PDF needs to be numerically evaluated. One notes that (10) requires $4(N-m)+2$ calls to multivariate αSG distributions.

VI. Results & Discussion

A. Simulation Setup

From (7), we note that the αSGN(m) model depends on the underlying samples $G_n\ \forall\ n \in \mathbb{Z}$. This in turn is essentially a stationary autoregressive process of order m (AR(m)) with Gaussian innovations [15]. Consequently, any immediate N samples of G_n is a Gaussian random vector and will thus be elliptically distributed [16]. Recalling that the dependence between adjacent snapping shrimp noise samples is near-elliptic, the Gaussian AR(m) process is able to track this. Therefore, we consider this for our CGN model. Moreover, $\mathbf{\Sigma} = [\sigma_{ij}]$ also has a symmetric Toeplitz structure, with its first row determined by $\sigma_{1k} = 2r_{1k}$ for $1 \leq k \leq m+1$ and

$$\sigma_{1k} = 2\mathbf{r}_m^\mathsf{T}\mathbf{R}_{m-1}^{-1}[\sigma_{1(k-1)}, \sigma_{1(k-2)}, \ldots, \sigma_{1(k-m)}]^\mathsf{T},$$

for $m+2 \leq k \leq N$ [16, pg. 59]. The remaining matrix can be constructed using properties of a symmetric Toeplitz matrix. Note that the main-diagonal elements of $\mathbf{\Sigma}$ are equal to $2\delta^2$. Thus, this implies $W_n \sim \mathcal{N}(0, 2\delta^2)$, as is in the WGN case.

In our simulations, we process a synthetic linear frequency modulated (LFM) signal immersed in snapping shrimp noise. The noise data sets are sampled at 180kHz and were recorded off the coast of Singapore by the Acoustic Research Laboratory of the National University of Singapore. For the SαS models, α and δ are estimated via the ML method [17]. We consider $m = 4$ for the αSGN(m) model, as this is adequate to capture the temporal statistics in snapping shrimp noise sampled at 180 kHz [5]. Thereafter, $\mathbf{R}_4$ is estimated from the noise samples by the sample-covariation method discussed in [5], [18]. This is then used to construct $\mathbf{\Sigma}$ for the CGN LLR detector.

In our simulations we plot detection performance against the measure $\mathcal{E}\theta^2/(2\delta^2)$. We note that $2\delta^2$ is the per-sample variance in WGN and CGN. Therefore, $\mathcal{E}\theta^2/(2\delta^2)$ is essentially the energy-to-noise ratio (ENR) in these cases [9]. On the other hand, second-order moments are nonexistent for heavy-tailed stable distributions. However, by expressing the variance in terms of δ, the ENR can be used for heavy-tailed SαS models. Similar measures are adopted in [2], [4], [6].

B. Results

We consider two *different* snapping shrimp data sets, namely $\mathcal{D}_1$ and $\mathcal{D}_2$. For $\mathcal{D}_1$, the estimated parameters (with obvious notation) are $\hat{\alpha} = 1.56$, $\hat{\delta} = 12.524$ and

$$\hat{\mathbf{R}}_4 = \hat{\delta}^2 \begin{bmatrix} 1.0000 & 0.5804 & 0.2140 & 0.1444 & -0.0135 \\ 0.5804 & 1.0000 & 0.5804 & 0.2140 & 0.1444 \\ 0.2140 & 0.5804 & 1.0000 & 0.5804 & 0.2140 \\ 0.1444 & 0.2140 & 0.5804 & 1.0000 & 0.5804 \\ -0.0135 & 0.1444 & 0.2140 & 0.5804 & 1.0000 \end{bmatrix}.$$

The LLR detection performance is plotted in Fig. 1. For $\mathcal{D}_2$, $\hat{\alpha} = 1.54$, $\hat{\delta} = 12.062$

$$\hat{\mathbf{R}}_4 = \hat{\delta}^2 \begin{bmatrix} 1.0000 & 0.6369 & 0.2704 & 0.1624 & 0.0396 \\ 0.6369 & 1.0000 & 0.6369 & 0.2704 & 0.1624 \\ 0.2704 & 0.6369 & 1.0000 & 0.6369 & 0.2704 \\ 0.1624 & 0.2704 & 0.6369 & 1.0000 & 0.6369 \\ 0.0396 & 0.1624 & 0.2704 & 0.6369 & 1.0000 \end{bmatrix},$$

and results are presented in Fig. 2. The parameters were estimated from the first 33 seconds of $\mathcal{D}_1$ and $\mathcal{D}_2$. Detection performance was then evaluated for at least the next 1500 seconds. Results are compiled for $P_{FA} = 10^{-4}$ and $N = 1000$. From the sampling rate, this implies a preamble of length ~ 5.6ms. Moreover, a LFM signal that sweeps between $18-36$kHz is employed for s_n. The signal is thus centered at 27kHz and is of bandwidth 18kHz.

In Fig. 1, the LLR detectors for WGN, CGN, WSαSN and αSGN(4) get increasingly better (in that order) in $\mathcal{D}_1$. This is due to the ability of the respective models to effectively characterize increasingly more information of the snapping shrimp noise process. We note that the LLR detector for CGN performs almost at par with its WGN counterpart. On the other hand, the LLR detector for WSαSN clearly outperforms that of CGN. This implies that exploiting the amplitude statistics of snapping shrimp noise is *significantly* more advantageous than just considering the noise memory. However, if both of these physical attributes are jointly exploited, then the performance increases further by $\sim$ 1.8dB as highlighted in Fig. 1. The trends seen for $\mathcal{D}_1$ are in consensus with those observed in Fig. 2 for $\mathcal{D}_2$. In the latter case, the LLR detector for αSGN(4) offers $\sim$ 2dB gain over its WSαSN counterpart.

Lastly, for $\mathcal{D}_1$, we present another set of performance curves in Fig. 3. All parameters are the same as before, but the signal now sweeps between $63-81$kHz, i.e., s_n still has a bandwidth of 18kHz, but is centered at 72kHz. Note that all detectors actually perform better than their respective counterparts in Fig. 1. This is understood by noting that the

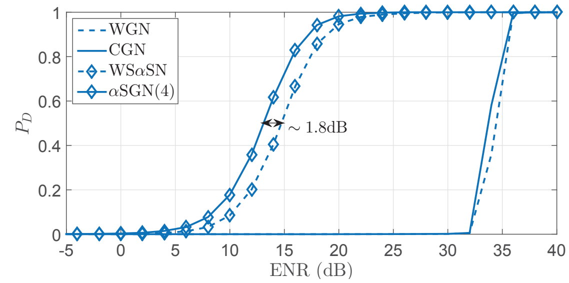

Fig. 1. LLR detection performance in $\mathcal{D}_1$ for the signal centered at 27kHz.

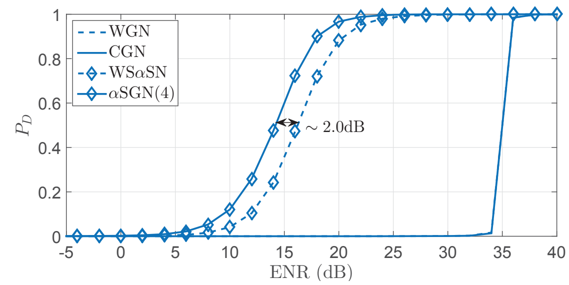

Fig. 2. LLR detection performance in $\mathcal{D}_2$ for the signal centered at 27kHz.

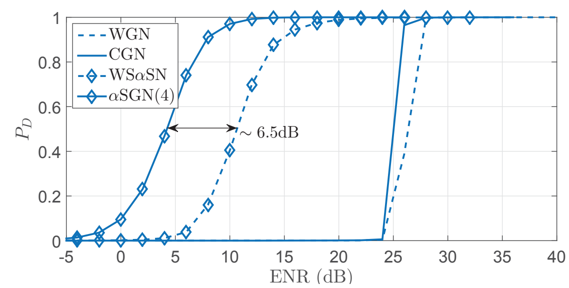

Fig. 3. LLR detection performance in $\mathcal{D}_1$ for the signal centered at 72kHz.

power spectral density (PSD) of snapping shrimp noise is influenced by its memory. The PSD is strong at the lower end of the spectrum but starts tapering off at higher frequencies [1]. In comparison to 27kHz, the *in-band* noise power is actually lower at 72kHz [1] and therefore results in the overall improved performance in Fig. 3. More striking though is that the gain of the LLR detector for αSGN(4) has now increased to $\sim$ 6.5dB over that of WSαSN. This is attributed to the fact that the WSαSN model assumes a flat spectrum [4], while the power of snapping shrimp noise is concentrated towards lower frequencies. Though not illustrated here, the PSD of $\mathcal{D}_1$ at 27kHz is approximately at par with that of the correspondingly tuned WSαSN process. However, at 72kHz, the former decreases while the latter remains the same. On the other hand, the tuned αSGN(4) model is able to track the PSD of $\mathcal{D}_1$ effectively over its entire spectrum. Consequently, the performance of the LLR detector for αSGN(4) *further* outperforms its WSαSN counterpart if the signal's spectrum is placed *away* from the frequency bands where the models' spectra coincide.

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