Improving Broadband Acoustic Doppler Current Profiler with Orthogonal Coprime Pulse Pairs and Robust Chinese Remainder Theorem

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***Abstract*—Broadband acoustic Doppler current profilers (ADCPs) are instruments that are widely used in underwater observation. However, conventional broadband ADCPs face the problem of being limited in accuracy due to velocity ambiguity. This paper proposes a method based on using orthogonal coprime pulse pairs and the robust Chinese remainder theorem to mitigate this limitation. The proposed method breaks through the limit of velocity ambiguity of conventional broadband ADCP and improves the performance significantly. The simulations show that our proposed method can decrease the standard deviation of current velocity measurement by nearly three time, when compared to the conventional method.**

Keywords- Broadband acoustic Dopper current profiler, current measurement, robust Chinese remainder theorem, orthogonal coprime pulses, velocity ambiguity

I. INTRODUCTION

Acoustic Doppler current profilers (ADCPs) are widely used in measuring ocean currents [1]-[5]. Modern ADCPs usually operate in broadband due to their lower measurement variance as compared to narrowband ADCPs [1], [2].

The transmitted signal of broadband ADCPs usually consists of a series of coded pulses represented in Fig. 1(a), which correlate well around a chosen measurement lag, namely the length of the coded pulse, and have little correlation at other lags [2]. For broadband ADCPs, the pulse-pair (or "covariance") method [1], [6], [7] is generally used to estimate Doppler shift by extracting the corresponding phase at the chosen measurement lag. Unfortunately, for conventional broadband ADCPs, using the pulse-pair method entails a trade-off between accuracy and maximum measurable velocity (ambiguity velocity) described below, which limits the performance.

For the pulse-pair method, the extracted phase may only fall within $-\pi$ to π range. Any other observed value (which can arise due to a high true velocity of current being observed) wraps around and falls within the $(-\pi, \pi)$ limit, leads to a phase ambiguity. This phase ambiguity in turn leads to an ambiguity in the estimation of Doppler shift. Thus, the maximum measurable velocity of conventional broadband ADCPs is the maximum velocity that can be determined unambiguously, called ambiguity velocity. As analyzed in Section II, the ambiguity velocity is determined by the length of the coded pulse, namely the chosen measurement lag. However, having a higher ambiguity velocity requires the coded pulse to be shorter, which results in worsening accuracy [2]. We also highlight this problem in our simulations in Section V. In summary, the ambiguity velocity limits the performance of conventional broadband ADCPs.

Conventional broadband ADCPs requires the users to configure the ambiguity velocity before system deployment. Because the ambiguity velocity must accommodate all possible velocities that we wish to measure [2], this results in the selection of coded pulse lengths that unfavorably limits the achievable measurement accuracy. In this paper, we focus on finding a solution to improve broadband ADCPs by breaking the conventional limit of the ambiguity velocity for a given signal length.

To improve the performance of broadband ADCPs, we propose to transmit two orthogonal coprime pulse pairs simultaneously as shown in Fig. 1(b) and employ the robust Chinese remainder theorem (CRT) [8]-[12] to resolve velocity ambiguity. The simulations show that our proposed method can decrease the measurement standard deviation by nearly three times.

This paper is organized as follows. In Section II, we briefly describe the basic theory of conventional broadband ADCPs. Section III will introduce the robust CRT. Our proposed method is shown in Section IV. Section V presents the simulation study comparing the proposed method to the conventional method. Finally, conclusions are given in Section VI.

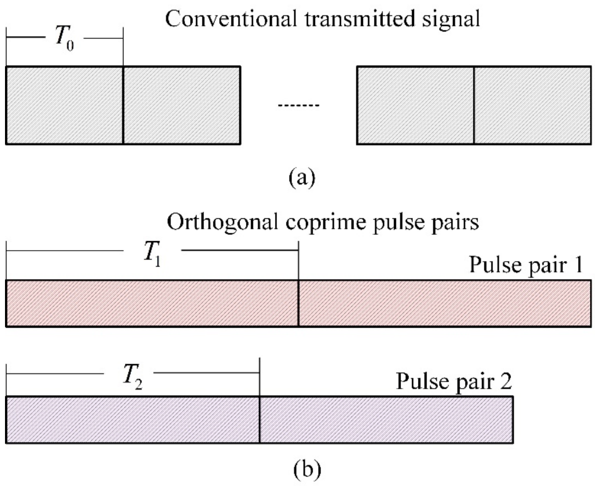

Fig. 1. Schematic of conventional transmitted signal (a) and our proposed two orthogonal coprime pulse pairs (b).

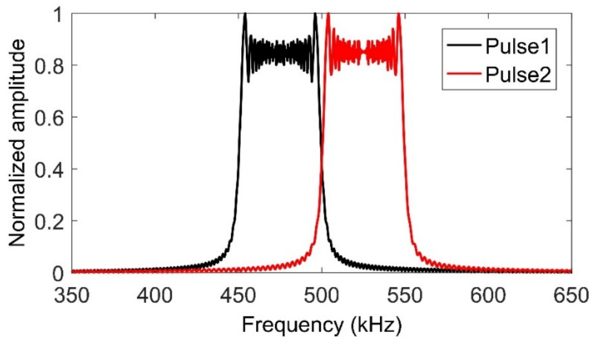

Fig. 2. Spectrum of the orthogonal coprime LFM pulses.

II. CONSTRAINTS OF CONVENTIONAL BROADBAND ADCP

Fig. 1(a) illustrates a schematic of the transmitted signal of conventional broadband ADCPs, which consists of repeated coded pulses. The length of each coded pulse T_0 determines the ambiguity velocity of the conventional broadband ADCP. Generally, the total length of the transmitted signal, which consists of multiple coded pulses, determines the depth cell size of broadband ADCPs [2].

For conventional broadband ADCPs, the measurement velocity obtained by using the pulse-pair method [5], [6], [9] is expressed as

$$v = \frac{c}{4\pi f_0 T_0}\Delta\phi, \tag{1}$$

where c is the sound speed in water, f_0 is the central frequency of the coded pulse, T_0 is the pulse length, $\Delta\phi$ is the phase change at the measurement lag T_0 . For v to be meaningful, the range of the phase change should be

$$-\pi \le \Delta\phi \le \pi. \tag{2}$$

If $\Delta\phi$ is larger than π or lower than -π, it leads to a phase ambiguity, which translates to velocity ambiguity. Therefore, the maximum measurable velocity (referred to as ambiguity velocity) of conventional broadband ADCPs can be expressed as

$$v_{\max 0} = \frac{c}{4 f_0 T_0}. \tag{3}$$

For a chosen f_0 , to increase $v_{\max 0}$, T_0 needs to be decreased. However, the decrease of T_0 will increase the measurement deviation. This is re-demonstrated in Section V.

III. ROBUST CRT

The traditional CRT demonstrates that given L integers, where M_1, M_2, …, M_L are coprime, and a positive integer N < lcm{M_1, M_2, …, M_L } where lcm represents the least common multiple, N can be uniquely reconstructed from its remainders modulo the L positive integers [8], [9]. However, the traditional CRT is not robust, which means a small error from any remainders may result in a large reconstruction error [8]-[12].

To mitigate the effect of remainder errors, several robust methods have been proposed recently, which are referred to as robust CRT [8]-[12]. The robust CRT can be generalized to be applied to real numbers. Our paper proposes to use the robust CRT to reconstruct the real velocities from the measured, and possibly erroneous velocities. The closed-form robust CRT [9] for real numbers is introduced briefly in the following.

Let R be a positive real number, which can be written as

$$R = n_i M\Gamma_i + r_i, 1 \le i \le L \tag{4}$$

where n_i is an unknown folding integer with $1 \le i \le L$, and all Γ_i are known positive integers and coprime, M is a known real-valued normalization factor decided by the system design and r_i is the real-value remainder with $0 \le r_i < M\Gamma_i$. The goal of robust CRT is to robustly reconstruct R from given noisy real-valued remainders $\hat{r}_i$, which are the erroneous version of r_i .

The closed-form robust CRT algorithm is explained in the following:

Step 1: calculate the differences $\hat{q}_{i,1}$:

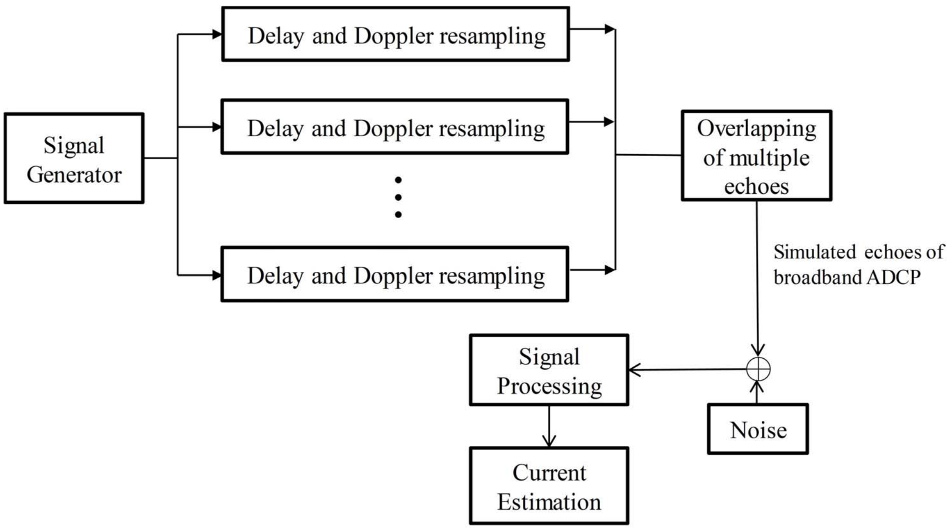

Fig.3. Schematic describing our approach for simulating the received echoes and subsequent processing.

$$\hat{q}_{i,1} = \left[\frac{\hat{r}_i - \hat{r}_1}{M} \right],\ 2 \le i \le L \tag{5}$$

where $[\bullet]$ stands for the closest integer rounded off to.

Step 2: calculate the remainder of $\hat{q}_{i,1}\bar{\Gamma}_{i,1}$ modulo Γ_i :

$$\hat{\xi}_{i,1} = \hat{q}_{i,1}\bar{\Gamma}_{i,1} \bmod \Gamma_i, \tag{6}$$

where $\bar{\Gamma}_{i,1}$ is the modular multiplicative inverse of Γ_1 modulo Γ_i, and can be calculated in advance.

Step 3: calculate $\hat{n}_1$:

$$\hat{n}_1 = \sum_{i=2}^{L} \hat{\xi}_{i,1} b_{i,1} \frac{\gamma_1}{\Gamma_i} \bmod \gamma_1, \tag{7}$$

where $b_{i,1}$ is the modular multiplicative inverse of γ_1 / Γ_i modular Γ_i, γ_1 is

$$\gamma_1 \triangleq \Gamma_1 \cdots \Gamma_L / \Gamma_1 = \Gamma_2 \cdots \Gamma_L. \tag{8}$$

Step 4: Calculate $\hat{n}_i$ for $2 \le i \le L$;

$$\hat{n}_i = \frac{\hat{n}_1\Gamma_1 - \hat{q}_{i,1}}{\Gamma_i}. \tag{9}$$

IV. PROPOSED METHOD

For conventional broadband ADCPs, operating with a high ambiguity velocity results in high uncertainty in measurements. Choosing probe signal with a low ambiguity velocity can help increase the measurement accuracy, but when the setting is used to measure a large velocity without resolving the ambiguity robustly, it can lead to serious errors.

We propose to transmit two orthogonal coprime pulse pairs simultaneously. The aim of choosing orthogonal signals is to guarantee two independent velocity values can be measured at the same time. The ambiguity velocities of the two orthogonal pulse pairs are designed to be coprime so that any phase ambiguity can be resolved by using the robust CRT. When measuring a current velocity that is larger than the two ambiguity velocities of the two pulse pairs respectively, the true current velocity can be reconstructed from the two velocities (erroneous remainders) measured by using the two orthogonal coprime pulse pairs, respectively. The proposed orthogonal coprime pulse pairs are illustrated in Fig. 1 (b). The design of orthogonal signals has been studied by the research community. Many existing orthogonal signals in the open literature can be good candidate for this work. In this paper, we choose the orthogonal linear frequency modulated signals, as shown in Fig. 2.

The coprime property of the pulse pairs will be explained in the following. Let the pulse lengths of the two pairs shown in Fig. 1 (b) be T_1 and T_2. Denote the central frequencies of the two pairs as f_1 and f_2. According to (1) and (2), the ambiguity velocity of the pulse pair 1 can be written as

$$v_{\max 1} = \frac{c}{4 f_1 T_1}. \tag{10}$$

The ambiguity velocity of the pulse pair 2 can be written as

$$v_{\max 2} = \frac{c}{4 f_2 T_2}. \tag{11}$$

Let us denote $v_{\max 1} / v_{\max 2}$ as Γ_1 / Γ_2. The two pulse pair signals are coprime if Γ_1 and Γ_2 are coprime. Given that they are coprime, the true current velocity can be reconstructed from the velocities measured by using the robust CRT. The reconstructing process is described briefly in the following.

Without considering the errors of the measured velocities, the real estimated velocity can be formulated as

$$\begin{cases} \hat{v} = \hat{v}_1 + n_1(2v_{\max 1}) \\ \hat{v} = \hat{v}_2 + n_2(2v_{\max 2}) \end{cases}, \tag{12}$$

where $\hat{v}$ is the real velocity, $\hat{v}_1$ and $\hat{v}_2$ are the velocities measured by the two pairs, n_1 and n_2 are folding integers corresponding to each pulse pair. The normalization factor M can be written as

$$M = \frac{2v_{\max 1}}{\Gamma_1} = \frac{2v_{\max 2}}{\Gamma_2}. \tag{13}$$

When considering the errors of the measured velocities, assume that e_1 and e_2 are the errors of $\hat{v}_1$ and $\hat{v}_2$. The robust CRT [8]-[12] can guarantee that if

$$\begin{cases} e_1 < \dfrac{M}{4} \\ e_2 < \dfrac{M}{4} \end{cases}, \tag{14}$$

n_1 and n_2 can be uniquely determined. In our simulation, we find that the error bound of robust Chinese remainder theorem can be realistically fulfilled.

V. Simulations and Results

We undertake a simulation study to compare our method with the conventional method of estimating current velocities. Simulation conditions are summarized as follows. Depth cell size is set to 4 m. The transmitted signals are set to a time length, corresponding to the depth cell size. The central frequency is set to 500 kHz. The system bandwidth is 100 kHz. For the conventional pulse pair, the ambiguity velocity is set to 5 m/s. We choose the linear frequency modulated (LFM) signals as the transmitted pulses. In this paper, orthogonality of the two LFM pulses is realized by making sure they lie in two different sub-bands, as shown in Fig. 2. The ambiguity velocities of the pulse pair 1 and 2 shown in Fig. 1 (b) are set to 0.50 and 0.45 m/s, which are an order smaller than the ambiguity velocity setting in the conventional method.

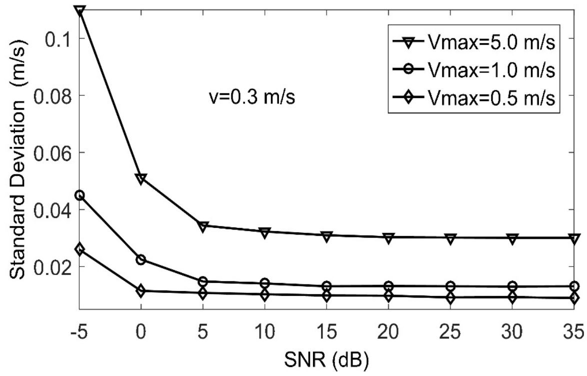

Fig. 4. Standard deviations of current velocity estimates using signals with ambiguity velocities (V_{max}) under the same depth cell (transmitted signal length).

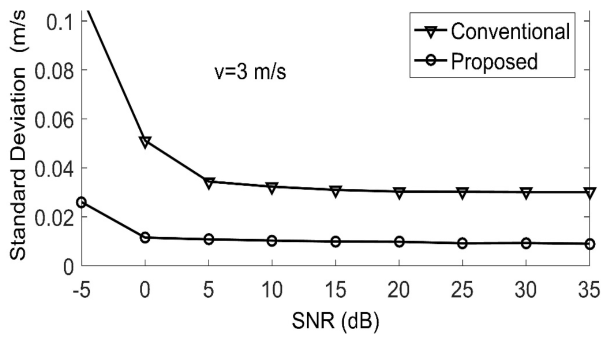

Fig. 5. Standard deviations of current velocity estimates using the conventional method and the proposed method.

We simulate the acoustic data received by the broadband ADCP, as consisting of echoes from the current and ambient noise. The schematic describing our methodology of simulating the echoes, which has also been used in [4] is illustrated in Fig. 3. The echoes are taken as the superposition of Doppler-resampled backscattered signals of a mass of scatterers.

We first verify that for the conventional broadband ADCP, the ambiguity velocity limits the accuracy. Fig. 4 shows that the numerical simulation captures the effect of ambiguity velocities (V_{max}) that limits the accuracies of the current estimation in the conventional method.

We then replace the conventional method with our proposed method in the simulation but keep the remaining setup unchanged. Fig. 5 shows the performance enhancement of the proposed approach compared to conventional approach. It can be found that the proposed method outperforms the conventional method, reducing the standard deviation by nearly 3 time at a SNR of 10 dB. Also, the SNR limit at which performance breaks down, is reduced in the proposed method.

In summary, the simulation results demonstrate the significant improvement of estimation accuracy of our proposed method.

VI. CONCLUSION

The velocity ambiguity in conventional DVLs limits the improvement of measurement accuracy. To reduce the measurement deviation, we propose a method based on designing orthogonal coprime pulse pairs, and use the robust CRT to resolve velocity ambiguity. Simulations verify that, the measurement standard deviation of the proposed method is decreased by nearly 3 times, compared with the conventional method.

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