Near-optimal detection in snapping-shrimp dominated ambient noise

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Abstract

Detection of a known signal in presence of noise is a common requirement in many applications including sonar, ranging, environmental sensing and communications. The optimal detection of signals in noise requires detailed knowledge of the noise statistics. The linear correlator, commonly used in the form of a matched filter, is known to be optimal in the presence of Gaussian noise. However, the performance of the linear correlator is poor in warm shallow waters where snapping shrimp dominate acoustic noise in the range 2-300 kHz. Since snapping shrimp noise consists of a large number of individual transients, its statistics are highly non-Gaussian. In this paper, we show that the ambient noise statistics can be described accurately by the symmetric alpha-stable family of probability distributions. The knowledge of the probability distribution allows us to design maximum-likelihood and locally optimal detectors, which perform well in such noise. Surprisingly, we found that a simple non-parametric sign correlation detector also performs well in presence of snapping shrimp noise. Although the performance of the sign correlator is slightly inferior to that of the maximum-likelihood detector, it is very simple to implement and does not require detailed knowledge of the noise statistics. This makes it an attractive detector for use in warm shallow waters.

Introduction

Snapping shrimp (families *Alpheus* and *Synalpheus*) produce loud snapping sounds by extremely rapid closure of their snapper claw. The closure produces a high velocity water jet leading to the formation of a cavitation bubble, which collapses rapidly, causing a loud broadband snapping sound [1]. The shrimp are usually found in such large numbers that there is a permanent crackling background noise in warm shallow waters throughout the world. Source levels as high as 190 dB (peak-to-peak) re 1 μPa @ 1 m have been reported [2]. Due to the impulsive nature of the snaps, the sound is broadband. At low frequencies, noise from shipping is significant; from about 2 kHz to well above 200 kHz, snapping shrimp noise is clearly detectable [3]. As ambient snapping shrimp noise is composed of impulsive noise sources, the resulting noise statistics at these frequencies are non-Gaussian [4][5].

The problem of detecting a known signal with unknown amplitude in noise is commonly encountered in areas such as communications, target detection, ranging and environmental sensing. If the noise statistics are known, an optimal detector can be designed based on the maximum-likelihood (ML) criterion. If the noise is Gaussian, the ML detector simplifies to a linear correlator (LC) [6]. Unlike a general ML detector, the LC does not require knowledge of the standard deviation of the Gaussian distribution. In the presence of non-Gaussian noise, the LC is no longer optimal. In spite of this, many signal processing algorithms still use the LC (or equivalently, a matched filter) for signal detection in non-Gaussian noise due to its simple implementation and the lack of detailed statistical information about the noise.

Since the LC is not optimal in snapping shrimp dominated ambient noise, a significant potential exists for enhancing the detection performance of signal processing algorithms in these waters.

Ambient Noise Statistics

Ambient Noise Data

Ambient noise recordings have been made in warm shallow waters in and around Singapore throughout the year. We present analysis of the ambient noise statistics from 4 different data sets to demonstrate the ubiquity of the non-Gaussian noise statistics.

Data Set 1 was collected using a high frequency data acquisition system (HiDAQ) capable of omni-directional acoustic recordings from 1-180 kHz [7]. The data was collected in Singapore waters with muddy/sandy sea bed, water depths of 15-20 m and water temperature in the range of 25-30°C through out the year. The ambient noise was digitally sampled at 500 kSa/s and pre-whitened using a 64-order finite impulse response (FIR) filter. It was further bandpass filtered with a FIR filter to contain frequencies between 10 and 180 kHz.

Data Set 2 was collected using a pop-up ambient noise data acquisition system (PANDA) capable of omni-directional acoustic recordings from 11 Hz – 8.3 kHz [8]. The data was collected in Singapore waters with muddy/sandy sea bed close to a coral reef with water depths of 15-20 m and water temperature in the range of 25-30°C through out the year. The ambient noise was digitally sampled at 20 kSa/s and then bandpass filtered using a digital FIR filter to contain frequencies between 2 and 8 kHz.

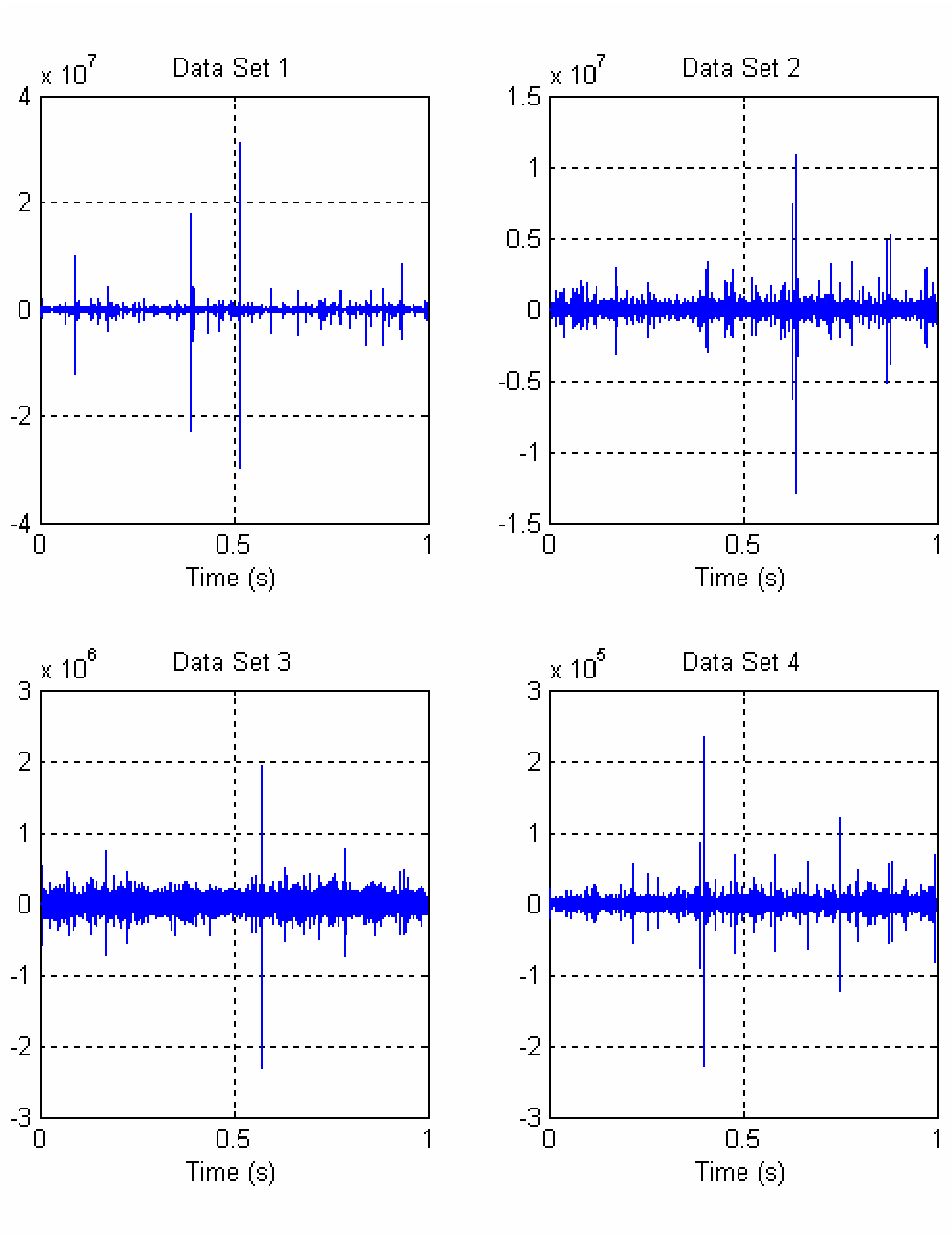

Figure 1. Sample time series from the ambient noise data sets

Data Set 3 was recorded using the PANDA in South China Sea in water depths of about 40 m and water temperature in the range of 25-30°C through out the year. The ambient noise was digitally sampled at 20 kSa/s and then bandpass filtered using a digital FIR filter to contain frequencies between 2 and 8 kHz.

Data Set 4 was recorded using a directional acoustic recording system with a beam-width of about 3° at about 40 kHz. The ambient noise was digitally sampled at 200 kSa/s and bandpass filtered using a digital FIR filter to contain frequencies between 35 and 42 kHz. The recording was made with the directional receiver placed in the horizontal orientation in about 10 m water depth and a muddy sea bed.

A one second sample time-series from each of the data set is shown in Figure 1. The impulsive nature of the data is visible in the time-series. A Lilliefors test [9] was applied to 10,000 samples randomly chosen from each data set to test the hypothesis that the data has a Gaussian distribution. The hypothesis was rejected for all data sets at a 1% level of significance. This suggests that all 4 ambient noise data sets exhibit non-Gaussian statistics.

Symmetric α-Stable Distribution

The probability density function (PDF) for impulsive noise decays less rapidly than the Gaussian PDF, leading to heavy tails. The family of *stable* distributions provides a useful theoretical tool for such signals [10]. Stable distributions are a direct generalization of the Gaussian distribution and include the Gaussian as a limiting case. The defining feature of stable distributions is the *stability property*, which states that the sum of two independent stable random variables with the same characteristic exponent is stable with the same characteristic exponent [10].

The stable distribution is described by four parameters: the characteristic exponent (α), the scale parameter (γ), the location parameter (a) and the symmetry parameter (β). An important subclass of the α-Stable distributions, known as the Symmetric α-Stable (SαS) distribution is characterized by $a = 0$ and $\beta = 0$. The SαS distribution can be most conveniently described by its characteristic function [11].

$$\varphi_\alpha(\theta) = \exp\left(-\gamma|\theta|^\alpha\right) \qquad \ldots(1)$$

In the above expression, α is the characteristic exponent controlling the heaviness of the tails. The scale parameter (γ), also known as *dispersion*, determines the spread of the distribution in a similar way to the variance in a Gaussian distribution. When $\alpha = 2$, the SαS distribution reduces to a Gaussian distribution and γ equals half the variance. For all other values of α, the variance of the stable distribution is infinite. A related parameter often used with stable distributions is c (defined as $\gamma^{1/\alpha}$), which plays the same role as the standard deviation for Gaussian random variables.

Unfortunately, no closed form expression exists for the general SαS PDF and cumulative distribution function (CDF), except for the Gaussian ($\alpha = 2$) and Cauchy ($\alpha = 1$) cases. However, there are efficient numerical methods for computing the PDF [12].

For an acoustic signal, the mean noise pressure must be zero; hence the location parameter for the distribution must also be zero. The other parameters of the distribution have to be estimated. Several estimators for the parameters of the SαS have been developed [10]. Of these, fractile-based estimators are easy to use and are known to be robust. To estimate the noise parameters, we use a fractile estimator for α developed by McCulloch [13] and a fractile estimator for c developed by Fama and Roll [14]. We use these estimators to fit SαS distribution to the ambient noise data.

Figure 2 shows an amplitude probability plot of the ambient noise data and the best SαS fit. The model describes the data quite well. This is confirmed via a Kolmogorov-Smirnov goodness of fit test [15] applied to 10,000 random data samples. The hypothesis that the SαS model describes the data could not be rejected at a 1% level of significance.

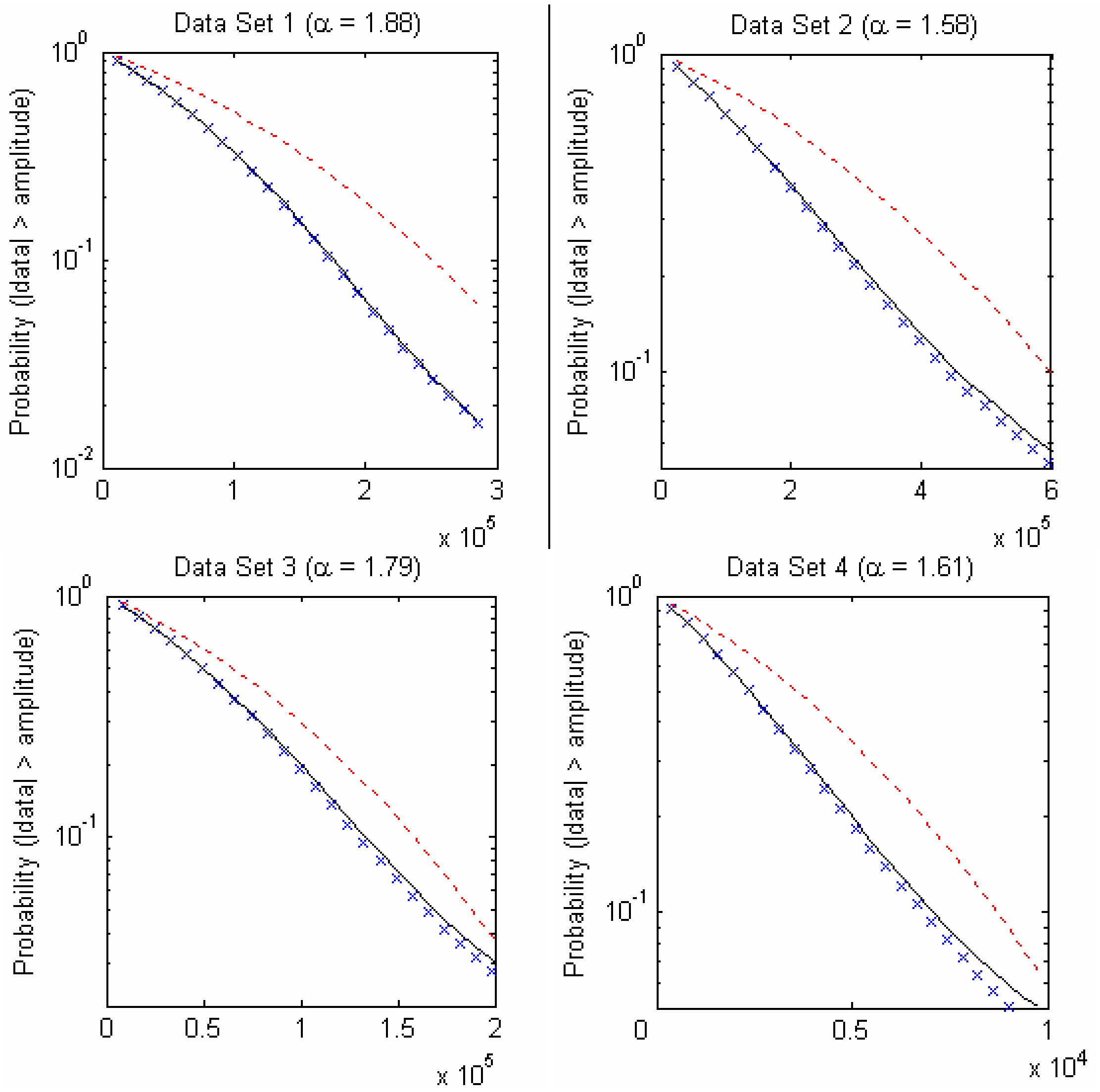

Figure 2. Amplitude probability plot of ambient noise data (crosses), SαS fit (solid line) and Gaussian fit (dashed line)

To compare with the SαS fit, we also fit a Gaussian distribution to the ambient noise data. The mean noise pressure must be zero; hence the mean of the Gaussian distribution is assumed to be zero. The standard-deviation of the Gaussian distribution is estimated using the root-mean-square (RMS) pressure of the ambient noise sample. The Gaussian fit is also shown in Figure 2; it is seen that the fit is much worse than the SαS fit. Both, Lilliefors and Kolmogorov-Smirnov goodness of fit tests reject the Gaussian noise hypothesis at a 1% level of significance.

Detection in Snapping Shrimp Dominated Ambient Noise

Having modeled the snapping shrimp dominated ambient noise using an SαS distribution, optimal or near-optimal detectors can be designed. As the PDF of the SαS distribution does not have a closed form solution, the detectors make use of the numerical approximations of the PDF.

Maximum-Likelihood Detector

A maximum-likelihood (ML) detector can be developed for signals of arbitrary strength in SαS noise. Let *s*(*t*) be the signal, *A* the signal strength and *n*(*t*) the noise. The observed data *x*(*t*) can be written as

$$x(t) = As(t) + n(t) \qquad \text{…(2)}$$

Given the noise PDF $f_n(n)$ of *n*(*t*), a likelihood ratio function *L* can be written as a function of the estimated signal strength *A*

$$L = \frac{\prod_t f_n[x(t) - As(t)]}{\prod_t f_n[x(t)]} \qquad \text{…(3)}$$

Maximizing the likelihood ratio *L*, or equivalently, minimizing the negative log-likelihood ratio then gives us the best estimate of signal strength. The estimated signal strength is expected to be close to zero when no signal is present. For SαS noise, the minimization of the negative log-likelihood ratio does not yield a closed-form solution. Numerical minimization leads to an estimate of signal strength, but typically is too computationally intensive to be used in practice.

Linear Correlation Detector

For the special case of α = 2, the SαS distribution reduces to a Gaussian distribution and the minimization of the negative log-likelihood ratio in (3) results in the familiar linear correlation (LC) detector:

$$\hat{A} = \frac{\sum_t x(t)s(t)}{\sum_t [s(t)]^2} \qquad \text{…(4)}$$

The LC detector is non-parametric i.e. it does not require knowledge of the parameters (standard deviation) of the noise distribution (assumed to be Gaussian). Although the LC detector is not expected to be optimal in SαS noise, it provides a performance benchmark as it is probably one of the most common detectors used in signal processing.

Locally Optimal Detector

The development of locally optimal (LO) detectors in SαS noise has been investigated by several researchers [6][10][16]. LO receivers can be designed for the detection of weak signals by introducing a non-linear transfer function before a standard LC detector. The non-linear transfer function *g*(*x*), can be determined from the noise PDF $f_\alpha(x)$:

$$g(x) = \frac{-1}{f_\alpha(x)} \frac{\partial f_\alpha(x)}{\partial x} \qquad \text{…(5)}$$

Since the SαS PDF is not available in closed form, we resort to numerical methods to compute the transfer function. Once the ambient noise parameters are known, the non-linear transfer function can be numerically computed and stored for later use in the detector. However, detailed parametric knowledge of the noise PDF is required for the computation of the transfer function. Often this knowledge is not available a-priori and hence the LO detector cannot be used.

Sign Correlation Detector

The sign correlation (SC) detector is obtained by replacing the non-linear transfer function in (5) with the *signum* function. The function takes a value of +1 for all positive inputs, -1 for all negative inputs and 0 if the input is 0. The SC detector is known to be locally optimal in double exponential density noise but is also known to exhibit robust performance in many other types of non-Gaussian noise [17]. As the detector does not require knowledge of the parameters of the noise distribution, it is non-parametric. It is mathematically simple yet performs well in snapping shrimp noise as we shall demonstrate. Hence it is an attractive detector for use in signal processing in snapping shrimp dominated ambient noise.

Detector Performance

We tested the performance of ML, LC, LO and SC detectors for detecting a signal in additive ambient noise using Monte-Carlo simulations. A 2 ms signal centered at 50 kHz and with a bandwidth of 15 kHz was randomly added to a recorded ambient noise sample from Data Set 1. The detection performance of the detectors at a fixed probability of false alarm was then computed based on their ability to correctly determine the presence or absence of the signal in 50,000 iterations. The simulations were repeated for varying values of signal strength to test performance as a function of signal-to-noise ratio (SNR). Since SαS noise has theoretically infinite variance and hence infinite power, the usual definition of SNR is not meaningful for our analysis. We adopted the ratio of signal power to noise dispersion as a measure of SNR [10] since dispersion plays a similar role in SαS noise as variance does in Gaussian noise.

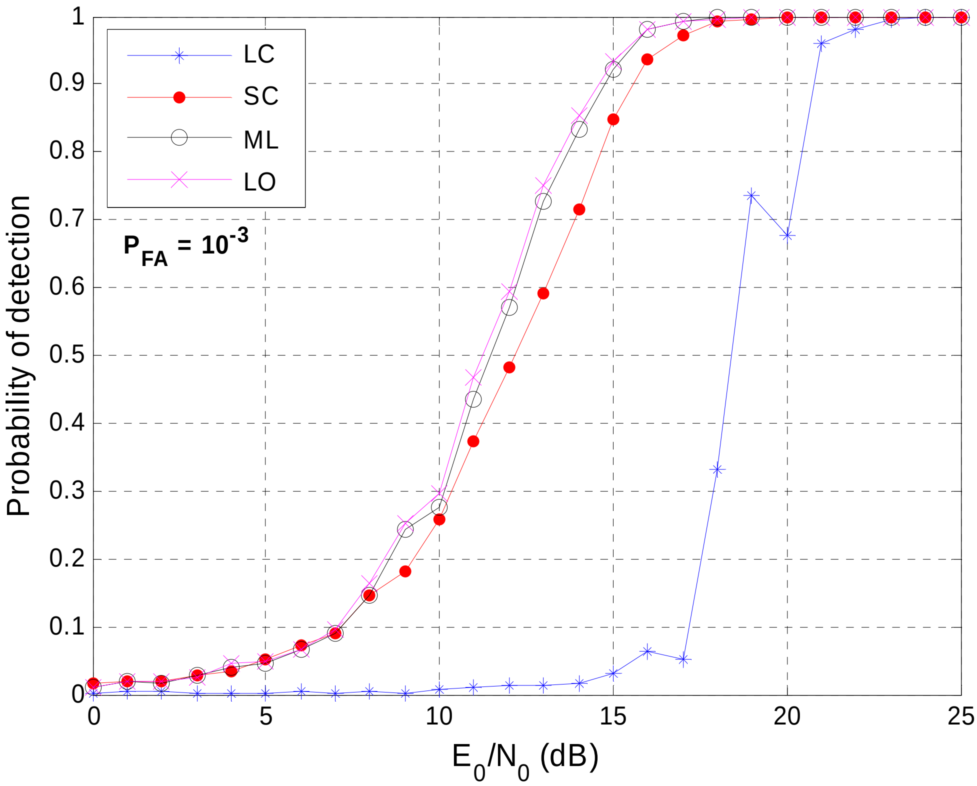

Figure 3. Performance of various detectors in snapping-shrimp dominated ambient noise

Figure 3 shows the detection performance of the four detectors as a function of SNR. The ML and LO detectors demonstrate the best performance as expected. The LC detector is inferior to the ML detector by 10-15 dB. The SC detector performance is only slightly inferior (within 2 dB) to the ML detector.

Conclusions

We have shown that snapping shrimp dominated ambient noise can be represented accurately by the SαS probability distribution. The parameters of the SαS distribution can be determined using fractile-based estimators. The knowledge of the noise probability distribution enables us to develop optimal detectors. The performance of these detectors is better than the more conventional LC detector. However, the optimal detectors require detailed parametric knowledge of the noise distribution. When this knowledge is unavailable, a simple non-parametric SC detector can provide near-optimal performance.

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