AUV positioning based on Interactive Multiple Model

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***Abstract*— The research topic of autonomous underwater vehicles (AUVs) has attracted much attention over years since they provide marine researchers easy ways to access the ocean for surveying and site investigation, etc. To accomplish these applications, an AUV has to know its position accurately. Therefore, AUV localization is very important problem. In this paper, we propose an interactive multiple model (IMM)-based method for AUV localization because this method is capable of tackling complex behaviors of vehicles with different dynamic models. Several filtering techniques, namely, Kalman filter (KF), particle filter (PF) and modified PF (MPF), are investigated to estimate the position of the AUV. In development of the MPF, an ℓ_1-norm is used to compute particle's cost instead of their weight to allow us to operate the filter without the use of measurement information. Those filters are running in parallel and the estimates are integrated by the IMM-based algorithm to obtain the position of the AUV. The sensor unit onboard consists of global positioning system (GPS), Doppler velocity log (DVL), inertial measurement unit (IMU) and a digital compass. Different dynamic models are studied to demonstrate the performance of the IMM-based methods, namely, IMM-KF, IMM-PF and IMM-MPF. Field trials using the STARFISH AUV show the capability of the algorithm.**

I. Introduction

The research topic of autonomous underwater vehicles (AUVs) has attracted much attention over years since they provide marine researchers easy ways to access the ocean for surveying and site investigation, etc [1]. To accomplish those applications, AUV has to know its position all the time. Therefore, AUV localization is very important issue. Several techniques have been developed using Doppler velocity log (DVL), inertial measurement unit (IMU), global position system (GPS), acoustic or optical sensors to estimate the position of the AUV. In [2], an acoustic-based localization technique is developed by using freely floating acoustic buoys equipped with GPS device. The problems is that the floating buoys must be deployed in the area of interest in advance to be able to make this work. This limitation is evident for a lot of applications since sometimes we do know much about the working area and we have to carry enough buoys every time. Other acoustic-based localization techniques include the long baseline (LBL) [3] and short baseline (SBL) [4] systems. For the LBL system, number of seafloor transponders is deployed underwater. Then, AUV can localize itself using distance information between the AUV and transponders. For the SBL system, using high-frequency directional emitter, a support ship can estimate the AUV position with respect to the mother ship. The usage of additional devices such as, transponders and support ship, can really limit the applications of such systems. To make AUV localization more independent, several techniques involving using onboard DVL, IMU, and GPS as well, are developed. In [5], an integrated DVL/IMU navigation system is presented based on extended KF, but ranging aid is still used in their system. In [6], a particle filtering approach is developed to improve positioning accuracy based on GPS/DVL/IMU measurements.

In this paper, we are interested in AUV positioning using GPS/IMU/DVL data. Since the movement of the AUV is complicated, the interactive multiple model (IMM) method is presented to combine state estimates from individual filters. The IMM-based methods have been used to improve the positioning accuracy in navigation systems. In our work, IMM-based approaches combined with several filtering techniques such as, Kalman filter (KF), particle filter (PF) and modified PF (MPF) are investigated. The KF is a optimal state estimate of linear and Gaussian state space models. KF and its variants [7] are most commonly adopted filtering methods to estimate the state. However, the performance of the KFs degrades as the system becomes nonlinear or non-Gaussian [7]. In such situation, PF is a more suitable choice since it does not requires linearity in system or Gaussianity in noise. PF uses random samples, called particles, to approximate the posterior function at every time step. The approximated posterior function approaches the true one when the number of particles goes to infinity [7], [8]. One problem in implementation of PF is that the noise distribution must be needed to compute the importance weight of particles. However, in reality, this information is either not available or not accurate. Therefore, a more robust approach, called MPF, is developed without noise distribution. In development of the MPF, ℓ_1-norm is used to calculate the particles' cost instead of their importance weights. To cooperate with the IMM, in this paper, three dynamic models, namely, constant velocity (CV), constant acceleration (CA) and constant turn (CT) are studied to demonstrate the performance of the proposed methods.

The rest of paper is organized as follows. In Section II, the algorithm development including introduction to Bayes filter, KF, PF, as well as IMM method and three different dynamic models is presented. Section III demonstrates the performance of the proposed approaches using field trial data. In Section IV, the conclusions are drawn.

II. Algorithm Development

A. Bayes Filter-Conceptual Solution

To define a tracking system, consider the following dynamic state space model:

$$\begin{aligned} \mathbf{x}_t &= f_t(\mathbf{x}_{t-1}, \mathbf{v}_{t-1}) \\ \mathbf{z}_t &= h_t(\mathbf{x}_t, \mathbf{n}_t) \end{aligned} \tag{1}$$

where $f_t(\cdot)$ is a transition function, which describes the evolution of the state with time t, $h_t(\cdot)$ is a measurement function, which defines the relationship between noisy observations and the state, $\mathbf{v}_{t-1}$ is a independent and identically distributed (i.i.d.) process noise, $\mathbf{n}_t$ is a i.i.d. measurement noise, $\mathbf{v}_t$ and $\mathbf{n}_t$ are mutually independent. In order to track a target under Bayesian framework, we need to calculate the posterior probability distribution function (PDF) of the state, i.e., $\pi(\mathbf{x}_t|\mathbf{z}_{1:t})$, where $\mathbf{z}_{1:t} = \{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_t\}$ denotes all the observations up to the current time t. Let the initial density of the state vector be $\pi(\mathbf{x}_0) = \pi(\mathbf{x}_0|\mathbf{z}_0)$, where $\mathbf{z}_0$ means no measurements. The PDF $\pi(\mathbf{x}_t|\mathbf{z}_{1:t})$ is obtained recursively in two stages, namely, prediction and update. Assuming that at time $(t-1)$ the required PDF $\pi(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1})$ is available, the prediction density of the state at time t is obtained by the following equation [8], [9]

$$\pi(\mathbf{x}_t|\mathbf{z}_{1:t-1}) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1})\pi(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1})d\mathbf{x}_{t-1} \tag{2}$$

At time t the observation $\mathbf{z}_t$ becomes available, the update stage is performed. Via the Bayes' rule, an update of the prediction density is given as

$$\pi(\mathbf{x}_t|\mathbf{z}_{1:t}) \propto p(\mathbf{z}_t|\mathbf{x}_t)\pi(\mathbf{x}_t|\mathbf{z}_{1:t-1}) \tag{3}$$

where $p(\mathbf{x}_t|\mathbf{x}_{t-1})$ is transition distribution defined by state model, $p(\mathbf{z}_t|\mathbf{x}_t)$ is the likelihood function defined by measurement model. The recursive propagation of the posterior density, using (2) and (3), is only a conceptual solution in the sense that in general it cannot be determined analytically. In what follows, two famous filters, namely, KF and PF, to solve the above dynamic system are introduced in details.

B. Kalman Filter

The KF is an optimal solution to the system (1) where it becomes linear and Gaussian. For such system, (1) can be rewritten as:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{F}_t\mathbf{x}_{t-1} + \mathbf{v}_{t-1} \\ \mathbf{z}_t &= \mathbf{H}_t\mathbf{x}_t + \mathbf{n}_t \end{aligned} \tag{4}$$

where $\mathbf{F}_t$ and $\mathbf{H}_t$ are known linear function, $\mathbf{v}_{t-1}$ and $\mathbf{n}_t$ are Gaussian noise defined by covariance matrix $\mathbf{Q}_{t-1}$ and $\mathbf{R}_t$, respectively. Another assumption to use KF is that the initial density $\pi(\mathbf{x}_0)$ is Gaussian distributed. The KF algorithm is given as follows: [10]

- Prediction:

$$\begin{aligned} \mathbf{x}_{t|t-1} &= \mathbf{F}_t\mathbf{x}_{t-1|t-1} \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}_{t-1} + \mathbf{F}_t\mathbf{P}_{t-1|t-1}\mathbf{F}_t^H \end{aligned} \tag{5}$$

- Update:

$$\begin{aligned} \mathbf{x}_{t|t} &= \mathbf{x}_{t|t-1} + \mathbf{K}_t(\mathbf{z}_t - \mathbf{H}_t\mathbf{x}_{t|t-1}) \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} - \mathbf{K}_t\mathbf{H}_t\mathbf{P}_{t|t-1} \\ \mathbf{K}_t &= \mathbf{P}_{t|t-1}\mathbf{H}_t^H(\mathbf{H}_t\mathbf{P}_{t|t-1}\mathbf{H}_t^H + \mathbf{R}_t)^{-1} \end{aligned} \tag{6}$$

where $\mathbf{K}_t$ is called Kalman gain. Like we mention earlier, KF is only applicable to linear and Gaussian system. For nonlinear system, Taylor expansion is exploited to obtain so-called extended KF (EKF). However, the EKF still suffers from large performance loss when the system is severely nonlinear. In what follows, a technique called PF is introduced to handle the nonlinearity and non-Gaussian problems.

C. Particle Filter

PF uses smartly generated random sample to approximate the posterior function, which is an efficient way to solve nonlinear and/or non-Gaussian problems [7]. The key idea behind PF is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights. According to the law of large numbers, this Monte Carlo method becomes an equivalent representation of the usual functional description, and the sequential importance sampling approaches the optimal Bayesian estimator. Given the large set of N particles $\{\mathbf{x}_{t-1}^{(i)}\}_{i=1}^N$ and their associated weights $\{w_{t-1}^{(i)}\}_{i=1}^N$. The posterior density at time $(t-1)$ is approximated as

$$\pi(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}) \approx \sum_{i=1}^{N} w_{t-1}^{(i)}\delta(\mathbf{x}_{t-1} - \mathbf{x}_{t-1}^{(i)}) \tag{7}$$

where $\delta()$ is the Dirac delta function. Moreover, the new particles $\{\mathbf{x}_t^{(i)}\}_{i=1}^N$ are generated from the properly designed proposal function:

$$\mathbf{x}_t^{(i)} \sim q(\mathbf{x}_t|\mathbf{x}_{t-1}^{(i)}, \mathbf{z}_{1:t}), \qquad i = 1, \cdots, N \tag{8}$$

While the importance weight $w_t^{(i)}$ is recursively updated as

$$w_t^{(i)} \propto w_{t-1}^{(i)} \times \frac{p(\mathbf{z}_t|\mathbf{x}_t^{(i)})p(\mathbf{x}_t^{(i)}|\mathbf{x}_{t-1}^{(i)})}{q(\mathbf{x}_t|\mathbf{x}_{t-1}^{(i)}, \mathbf{z}_{1:t})} \tag{9}$$

Based on the new particles and their associated weights, the minimum mean square error estimate is [7]

$$\hat{\mathbf{x}}_t = \mathbb{E}[\mathbf{x}_t|\mathbf{z}_{1:t}] = \int \mathbf{x}_t\pi(\mathbf{x}_t|\mathbf{z}_{1:t})d\mathbf{x}_t \approx \sum_{i=1}^{N} w_t^{(i)}\mathbf{x}_t^{(i)} \tag{10}$$

where $\mathbb{E}$ denotes the expectation operator. In PFs after a certain number of recursive steps, all but one particle will have negligible weights, leading to the degeneracy phenomenon. In order to avoid this problem, the resampling step must be taken. Resampling eliminates samples with low importance weights and multiplies samples with high importance weights, and the details can be found in [7]. One problem about using PF is that one has to know the likelihood function in (9). In practice, it is very likely either we cannot have this information or the likelihood function is inaccurate. Therefore, to overcome this problem, we propose a MPF in which the measurement noise information is unnecessary. In the importance weight calculation step, we use ℓ_1-norm to compute the particle cost, i.e.,

$$\begin{aligned} c_t^{(i)} &= \|\mathbf{z}_t - h_t(\mathbf{x}_t^i)\|_1 \\ w_t^{(i)} &= 1/c_t^{(i)} \end{aligned} \tag{11}$$

Therefore, this step replaces step (9) for particle importance weight update in PF.

D. **Interactive Multiple Model**

Single model is not sufficient enough to capture the movement of the AUV, since its trajectory might be very complex underwater. Therefore, the IMM method is used to improve the positioning accuracy. In the IMM approach, the state estimates from each filters are combined according to a Markovian model for the transition between different models. The IMM method is described in following four steps:

• *Interaction*:

The mixing probabilities $\mu_{t-1|t-1}^{j|i}$ for each model M^i and M^j are calculated as

$$\begin{aligned} \bar{c}_j &= \sum_{i=1}^{n} p_{ij}\mu_{t-1}^i \\ \mu_{t-1|t-1}^{j|i} &= \frac{1}{\bar{c}_j} p_{ij}\mu_{t-1}^i \end{aligned} \tag{12}$$

where μ_{t-1}^i is the probability of model M^i in the time step $(t-1)$, $\bar{c}_j$ a normalization factor and p_{ij} is the transition probability from model M_i to M_j. Now we can compute the mixed inputs for each filter as

$$\begin{aligned} \bar{\mathbf{x}}_{t-1|t-1}^i &= \sum_j \mu_{t-1|t-1}^{j|i}\hat{\mathbf{x}}_{t-1|t-1}^j \\ \bar{\mathbf{P}}_{t-1|t-1}^i &= \sum_j \mu_{t-1|t-1}^{j|i} \times \{\mathbf{P}_{t-1|t-1}^j + \\ &(\bar{\mathbf{x}}_{t-1|t-1}^i - \hat{\mathbf{x}}_{t-1|t-1}^j) \times (\bar{\mathbf{x}}_{t-1|t-1}^i - \hat{\mathbf{x}}_{t-1|t-1}^j)^T\} \end{aligned} \tag{13}$$

• *Filtering*:

Now, for each model the filtering is done by using KF/PF/MPF to compute the state estimate $\hat{\mathbf{x}}_{t|t}^i$ and covariance $\mathbf{P}_{t|t}^i$ according to the mixed inputs calculated in interaction step.

• *Model Probability Update*:

The probabilities of each model M^i at time t are calculated as

$$\begin{aligned} c &= \sum_i \Lambda_t^i \bar{c}_i \\ \mu_{t|t}^i &= \frac{1}{c}\Lambda_t^i \bar{c}_i \end{aligned} \tag{14}$$

where Λ_t^i is calculated as

$$\Lambda_t^i = \mathbb{N}(\mathbf{v}_t^i; \mathbf{0}, \mathbf{S}_t^i) \tag{15}$$

where $\mathbb{N}(\cdot)$ represents Gaussian distribution, $\mathbf{v}_t^i$ is the measurement residual and is $\mathbf{S}_t^i$ it's covariance.

• *Combination*:

The combined state estimate $\hat{\mathbf{x}}_{t|t}$ and its covariance $\mathbf{P}_{t|t}^i$ are now calculated as

$$\begin{aligned} \hat{\mathbf{x}}_{t|t} &= \sum_i \mu_{t|t}^i \hat{\mathbf{x}}_{t|t}^i \\ \mathbf{P}_{t|t} &= \sum_i \mu_{t|t}^i \times \{\mathbf{P}_{t|t}^i + (\hat{\mathbf{x}}_{t|t} - \hat{\mathbf{x}}_{t-1|t-1}^i) \times (\hat{\mathbf{x}}_{t|t} - \hat{\mathbf{x}}_{t|t}^i)^T\} \end{aligned} \tag{16}$$

E. **Dynamic Model Set**

To capture the movement of the AUV underwater more accurately, three different dynamic models are studied. The CV, CA and CT, which are described as follows, respectively.

• CV: The CV model is a nearly-constant-velocity one, since accelerations along x and y directions are modeled as small white noise. The state is defined as $\mathbf{x}_t = [x_t, y_t, \dot{x}_t, \dot{y}_t]^T$ and the transition model is

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

• CA: To account the accelerations in the movement, the CA model is introduced. The state is defined as $\mathbf{x}_t = [x_t, y_t, \dot{x}_t, \dot{y}_t, \ddot{x}_t, \ddot{y}_t]^T$ and the transition model is

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & T & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & T & 0 & T^2/2 \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where T is the time step.

• CT: Sometimes, the AUV may make turn, in order to capture this behavior, the CT model is investigated. The state in CT model is the same as in CV model. The corresponding transition model is

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \sin(\omega T)/\omega & (\cos(\omega T) - 1)/\omega \\ 0 & 1 & (1 - \cos(\omega T))/\omega & \sin(\omega T) \\ 0 & 0 & \cos(\omega T) & -\sin(\omega T)/\omega \\ 0 & 0 & \sin(\omega T) & \cos(\omega T) \end{bmatrix}$$

where ω is the turn rate.

Note that to use IMU measurements, we have to consider the bias introduced by the IMU. The bias is modeled as random walk for each direction. Therefore, two additional states $[bx_t, by_t]^T$ for IMU bias in acceleration are introduced and the transition model for the bias is

$$\begin{aligned} bx_t &= bx_{t-1} + w_{bx} \\ by_t &= by_{t-1} + w_{by} \end{aligned} \tag{17}$$

III. Experimental Results

In this section, field underwater trials are conducted to evaluate the performance of the proposed methods from the STARFISH AUV [11]. In the test, the number of particles are 100, the turn rate for CT model is set to be 0.1g, where g is the acceleration of gravity.

A. GPS/DVL performance

The first trial was conducted in Nov. 2008 in Pandan reservoir in Singapore. In this test, the GPS and DVL measurements were collected for AUV positioning. Two dynamic models, namely, CV and CT models, are used for the positioning algorithm. The AUV surfaced three times during trial to obtain GPS data for ground truth, as shown in Figure 1. In our system, GPS device directly outputs the coordinates of the AUV. We model those measurement equation in GPS as noisy observation of true coordinates, i.e.,

$$\begin{aligned} z_{x,t} &= x_t + n_{x,t} \\ z_{y,t} &= y_t + n_{y,t} \end{aligned} \tag{18}$$

where $n_{x,t}$ and $n_{y,t}$ are assumed to be Gaussian. The DVL gives the velocity of the AUV. We formulate those measurements in DVL as noisy observations of true velocity same as in GPS, therefore, we have:

$$\begin{aligned} z_{v_{x,t}} &= v_{x,t} + n_{v_{x,t}} \\ z_{v_{y,t}} &= v_{x,t} + n_{v_{y,t}} \end{aligned} \tag{19}$$

The transition matrix in IMM for this test is

$$\prod = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}$$

Figure 1 shows the tracking result obtained by the proposed methods. It is seen that all filters have similar performance, which demonstrates the effectiveness of the MPF approach. The detailed figure 1 is presented in Figure 2. In Figure 3, the distance errors are plotted when the GPS data is available. From the figure, it can be seen that IMM-PF performs the best and IMM-KF has gains over KF. In Figure 4, the estimated model transition probability is shown using KF.

B. GPS/IMU performance

The second trial was conducted in Jul. 2009 in the same location. In this test, the GPS and IMU measurements were collected for AUV positioning. All three dynamic models are used for the positioning algorithm. The same measurement equation for GPS is used as in test A. For IMU measurement, the accelerations along x and y directions are formulated as follows:

$$z_{a_t} = C^b_{n,t}[a_{x,t}\ a_{y,t}]^T + b_{a,t} + n_{a_t} \tag{20}$$

where $C^b_{n,t}$ is the direction cosine matrix, $b_{a,t}$ is the bias term introduced by the IMU. The transition matrix in IMM for this test is

$$\prod = \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.90 & 0.05 \\ 0.05 & 0.05 & 0.90 \end{bmatrix}$$

The tracking trajectory is shown in Figure 5 and distance errors when GPS data is available are presented in Figure 6.

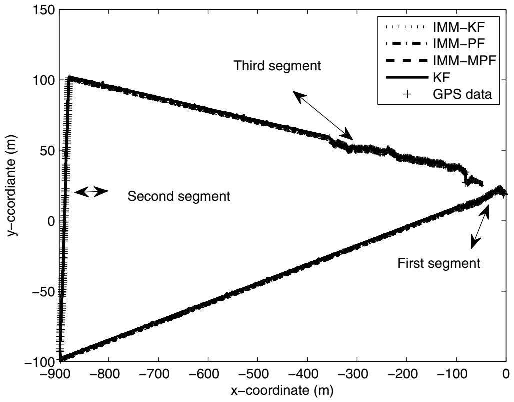

Fig. 1. Trajectory tracking result.

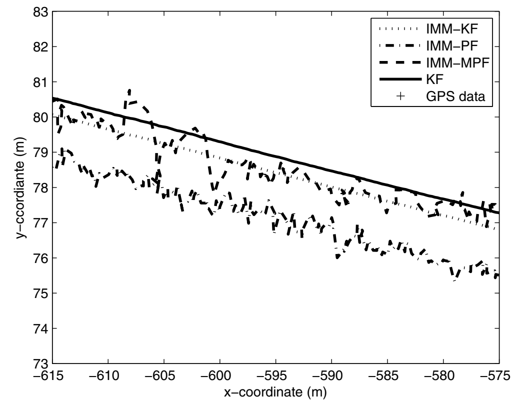

Fig. 2. Trajectory tracking result in detail.

Their detailed information are presented in Figure 7 and 8, respectively. From the figures, it is observed that the IMM-PF gives the best performance again. In Figure 9, the estimated model transition probability is shown using KF.

IV. Conclusion

In this paper, positioning algorithms for use in an AUV are investigated. To improve the accuracy, the IMM-based positioning methods using KF, PF and MPF are presented. Several dynamic models, namely, CV, CA and CT, are also studied. From the field trial results, it is seen that the positioning algorithms demonstrate their effectiveness.

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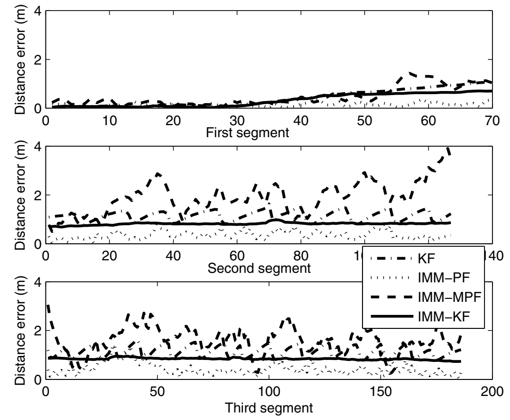

Fig. 3. Distance estimation error.

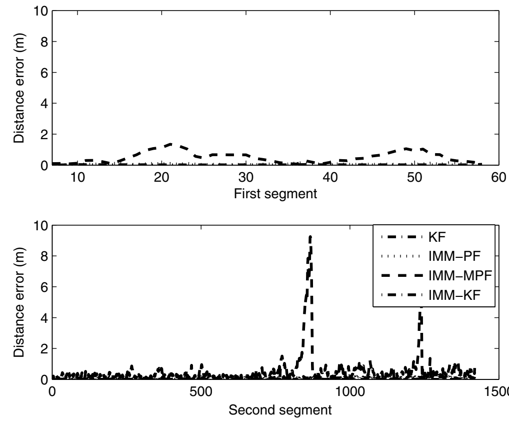

Fig. 6. Distance estimation error.

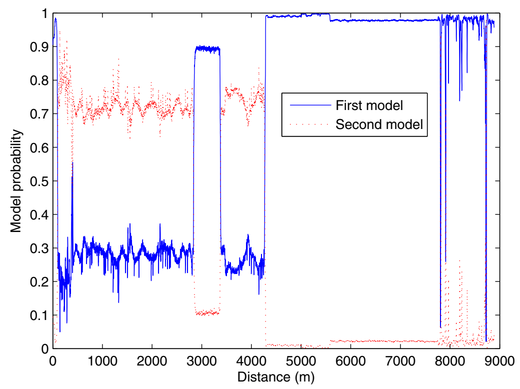

Fig. 4. Model probability transition using KF.

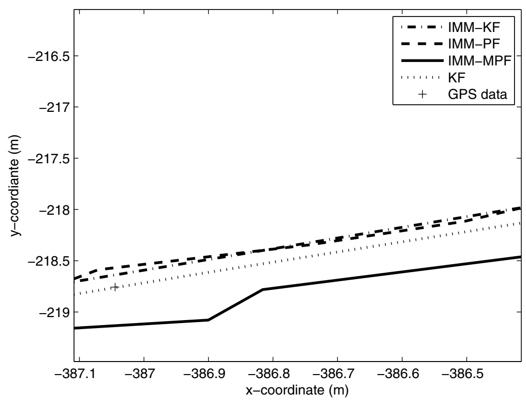

Fig. 7. Trajectory tracking result in detail.

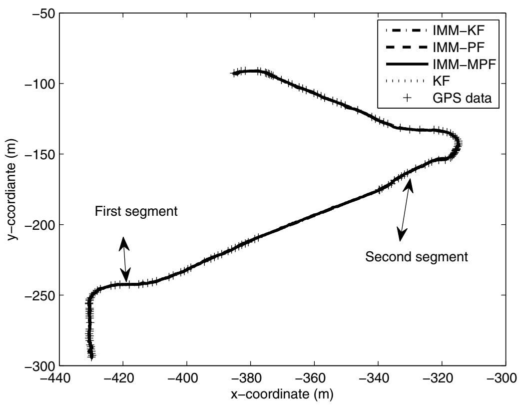

Fig. 5. Trajectory tracking result.

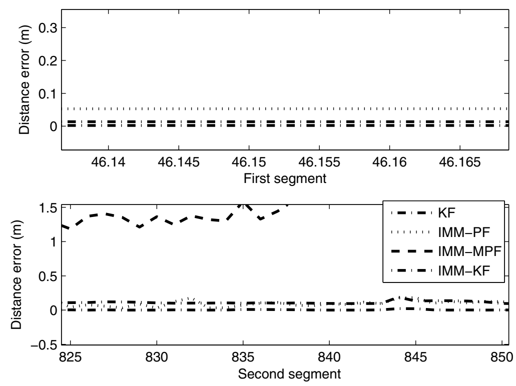

Fig. 8. Distance estimation error in detail.

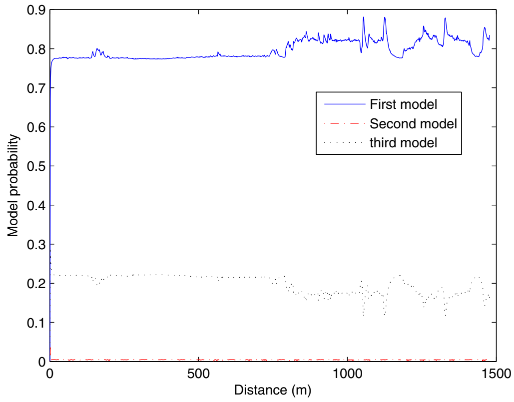

Fig. 9. Model probability transition using KF.

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