Robust Equalization of Mobile Underwater Acoustic Channels

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Abstract—Several underwater acoustic channels exhibit impulsive ambient noise. As a consequence, communication receivers implemented on the basis of the Gaussian noise assumption may yield poor performance even at moderate signal-to-noise ratios (SNRs). This paper presents a new channel-estimate-based decision feedback equalizer (CEB–DFE) that deals with high platform mobility, exploits any sparse multipath structure, and maintains robustness under impulsive noise. The key component of this DFE is a linear-complexity sparse channel estimator, which has the ability to detect and reject impulses based on two noise models: contaminated Gaussian and symmetric alpha stable ($S_{\alpha S}$). By processing phase-shift keying (PSK) signals from three mobile shallow-water acoustic links, the gain of the proposed receiver over existing equalizers is demonstrated.

Index Terms—Affine projection sign algorithm (APSA), Doppler compensation, improved-proportionate normalized least mean squares (IPNLMS), interpolation, motion synchronization, normalized least mean squares (NLMS), outliers, recursive least squares (RLS), resampling, sparse equalization.

I. INTRODUCTION

UNDERWATER acoustic channels are severely bandlimited due to low-frequency ship noise and absorption of high-frequency energy. In addition, any transmitted sound signal undergoes both time and frequency spreading [1]. For instance, medium range (1–10 km) acoustic links are typically confined to less than 40 kHz of bandwidth and experience multipath delay spreads that can easily exceed 60 ms.

The bandwidth limitation renders coherent modulation, i.e., minimum mean squared error (MMSE) adaptive equalizers may suffer severe performance degradation and so robust equalizers become an attractive solution. Robust equalization is a mature subject in wireless radio channels and may be useful as a starting point. For example, robust equalizers for the Middleton class-A noise model [11]–[14] and the (Sos) noise model [15]–[18] have been introduced.

DFEs can be divided into two classes. The first class adjusts its filter coefficients based on the channel impulse response, which in turn is estimated from the received signal. The second class adjusts its filter coefficients directly from the received signal. In fast varying channels such as shallow waters where multiple reflections from the moving sea surface are significant, channel-estimate-based DFEs (CEB–DFEs) optimize their coefficients faster than direct adaptation DFEs (DA–DFEs) and consequently show better performance [5]. Additional gains are possible if any available knowledge about the channel structure can be incorporated into the channel estimator. For instance, exploiting channel sparseness (i.e., a big fraction of the energy of the channel impulse response is concentrated in a small fraction of its duration) leads not only to better channel estimation accuracy but also to reduction of receiver computational complexity since only the significant channel coefficients can be retained in the equalization process [7].

Although CEB–DFEs have been thoroughly tested in underwater acoustic channels, by and large, if not all, the results are related to the assumption that the noise probability density function (pdf) is Gaussian. However, a number of underwater acoustic environments have impulsive noise sources, such as ice cracking [8] and snapping shrimp noise [9], [10]. For such environments, minimum mean square error (MMSE) adaptive equalizers may suffer severe performance degradation and so robust equalizers become an attractive solution. Robust equalization is a mature subject in wireless radio channels and may be useful as a starting point. For example, robust equalizers for the Middleton class-A noise model [11]–[14] and the (Sos) noise model [15]–[18] have been introduced.

In this paper, a novel CEB–DFE receiver that can cope with impulsive noise is presented. Robust performance is achieved via the channel estimator, which is able to identify and suppress noise impulses. Based on our recent work in [19], we employ two channel estimation algorithms: the improved-proportionate M-estimate affine projection algorithm (IPMAPA) and the improved-proportionate p-norm affine projection algorithm (IPpNAPA). Both algorithms are robust under impulsive noise, exploit sparse multipath, and require linear computational complexity with respect to the channel parameters. In addition, the receiver can cope with nonconstant platform motion without the need of periodically inserting training data for synchronization.

The performance of the proposed receiver is tested in three mobile shallow-water links by processing phase-shift keying (PSK) signals. Our results firmly conclude that the performance of the CEB–DFE is improved by using IPMAPA/IPpNAPA rather than traditional channel estimators.

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The remainder of the paper is organized as follows. Section II presents the transmitter, and Section III details the design of the new CEB–DFE, including the algorithms IPMAPA and IPpNAPA. Section IV discusses the experimental layout and presents the demodulation results. Section V concludes the paper.

Notation: Superscripts $\top$, $\dagger$, and $*$ stand for transpose, Hermitian transpose, and conjugate, respectively. Column vectors (matrices) are denoted by boldface lowercase (uppercase) letters. Let $z \in \mathbb{C}$ and $p \geq 1$. The $L_p$-norm of $z$ is defined as $|z|_p \triangleq (|\Re\{z\}|^p + |\Im\{z\}|^p)^{1/p}$. Let $z \in \mathbb{C}^N$. The $L_p$-norm of $z$ is defined as $|z|_p \triangleq (\sum_{i=0}^{N-1} |z_i|^p)^{1/p}$. The complex gradient of a scalar function $f(z)$ with respect to $z$ is denoted as $\nabla z f(z)$ and is defined in [20].

II. TRANSMITTER

Our goal is to achieve high rate communications relying on coherent modulation. To this end, the transmitter uses linear and memoryless modulation methods based on $M$-ary constellations [21]. The information-bearing symbol stream is pulse shaped via a raised cosine (RC) filter with symbol interval $T$ and rolloff factor $\gamma$. The baseband signal is given by

$$u(t) = \sum_n d(n) g(t - nT);$$

(1)

where $\{d(n)\}$ represents the information-bearing sequence of $M$-ary symbols, and $g(t)$ is the RC response. The signal $u(t)$ is modulated onto a carrier $f_c$ and transmitted through the ocean. The occupied frequency range is $f_c \pm (1 + \gamma)/2T$.

Remark 1: Note that error-correction coding could improve system performance, however, it would impede understanding of how efficiently the receiver mitigates the ISI. For this reason, channel coding is omitted in this work.

III. RECEIVER

From the communications perspective, time-varying multipath propagation dominates the characterization of any underwater acoustic link. Thus, the UWA channel is typically modeled as a linear time-varying system, which is described by the (lowpass equivalent) input delay-spread function $h(\tau, t)$. The variable $\tau$ corresponds to the time variations of the impulse response due to physical processes (e.g., moving surface waves, tides, currents, and internal waves) while the variable $t$ represents the channel multipath delay for a fixed value of $\tau$. The lowpass equivalent (baseband) output $r(t)$ is related to the input $u(t)$ via the formula [6]

$$r(t) = \int_{-\infty}^{+\infty} h^*(\tau, t) u(t - \tau) d\tau + w(t).$$

(2)

Equation (2) may also be interpreted as a system with impulse response $h(\tau, t')$ at time $t$ when an impulse is applied at time $t - \tau$. Here $w(t)$ models the additive impulsive ambient and thermal noise, which is independent from $u(t)$.

Transmitter–receiver motion induces a different time scale at the received signal. Since the signal bandwidth is usually comparable to the center frequency, time scaling is not well represented by just a Doppler shift. Hence, the baseband received signal (with respect to the center frequency $f_c$) is expressed as

$$r'(t) = r(t + \Delta t - \tau_0) e^{j2\pi f_c(\Delta T - \tau_0)} + w(t)$$

(3)

where $\Delta$ stands for the (time-varying) dilation/compression factor and $\tau_0$ is the arrival time of the beginning edge of the signal. From noise perspective, platform motion is immaterial because the noise bandwidth is much larger than the signal bandwidth for all practical purposes.

The proposed CEB–DFE receiver can be seen in Fig. 1. The processing line of the received signal includes three stages: motion compensation, adaptive channel estimation, and decision feedback equalization.

A. Adaptive Resampling

The standard method for motion synchronization is to compute the time difference between two known pulse (e.g., chirp) transmissions [22]. If there is a deviation from the expected time difference, it is directly translated into a simple scaling factor. This method may be well suited for constant velocity platforms, however, it is not efficient for rapid platform acceleration (e.g., autonomous underwater vehicles). Furthermore, it suffers an overhead, i.e., a significant amount of time is not devoted for communications. Motion compensation via adaptive resampling [23], [24] offers the possibility to transmit very long communication signals with no extra overhead for synchronization. Here, the novelty is that adaptive resampling is performed in conjunction with adaptive channel estimation at the symbol rate. As a result, fast platform motion is decoupled from slow environmental fluctuations leading to improved channel estimates.
Let us denote $r(n') = r(n'T/4)$ the baseband signal sampled at four samples/symbol. Motion-induced time scaling is compensated by resampling the signal via linear interpolation. The output of the linear interpolator, denoted as $y(n)$, is downsampled to two samples per symbol and is given by

$$y(n) = (I(n)r(n') + (I(n) - 1)r(n' + 1)) e^{-j\phi(n)}$$  \hspace{1cm} (4)

$$\phi(n) = \phi(n - 1) + 2\pi(I(n) - 1) f_c T/2$$  \hspace{1cm} (5)

$$I(n) = I(n - 1) + K_1\theta(n - 1)$$  \hspace{1cm} (6)

$$\theta(n - 1) = \text{Im}\left\{d[n-1]d^*(n-1)\right\}$$  \hspace{1cm} (7)

where $n' = \{1, 3, \ldots\}, n = \{1, 2, \ldots\}, d[n]$ is the soft decision when the DFE operates in decision-directed mode [or when the DFE operates in training mode], $\theta(n)$ is the phase error measurement of the transmitted symbol $a(n)$, $I(n)$ is the one-tap linear interpolator ($I[0] = 1$), $K_1$ is a first-order phase-locked loop (PLL) tracking parameter, and $\phi(n)$ is the carrier-phase estimate ($\phi(0) = 1$). Typical values for $K_1$ range between $10^{-5} - 10^{-4}$ depending on the platform speed. We have noticed that once a value is chosen, it can remain fixed for the entire duration of the signal regardless the motion fluctuation.

### B. Robust Adaptive Channel Estimation

After motion compensation, the received signal (at time $nT$) can be expressed in a vector form as

$$y(n) = h(n)u(n) + \nu(n)$$  \hspace{1cm} (8)

where

$$u(n) = \begin{bmatrix} u(nT) \\ \vdots \\ u(nT + T_2/2) \end{bmatrix}$$  \hspace{1cm} (9)

and

$$h(n) = \begin{bmatrix} h(nT - (N_c + 1)T/2) \\ \vdots \\ h(nT + N_a T/2) \end{bmatrix}$$  \hspace{1cm} (10)

are the samples of the transmitted signal and the channel impulse response (including transmit and receive filters), respectively. $z(n)$ denotes the noise. Parameters $N_c$ and $N_a$ denote, respectively, the causal and acausal taps with respect to the channel tap $h(0, nT)$. Note that $u(n)$ assumes values based on either known (past and future) symbols (training mode) or decided (past) symbols (decision-directed mode). The goal here is to reliably estimate $h(n)$ in the presence of impulsive noise. Let us call this estimate as $\hat{h}(n)$.

In impulsive noise environments, it is well known that $L_2$-norm-based channel estimation algorithms are not appropriate even in moderate signal-to-noise ratios (SNRs). Recently, the authors have introduced a framework that systematically generates sparse robust adaptive algorithms [19]. For the purposes of this work, we use two algorithms from that framework: the IPMAA and the IPPNAPA. Both algorithms are generated by minimizing the following cost function:

$$J(n) = \sum_{i = n - L - 1}^{n} f(\tilde{e}(i)) + \delta r(n)^H P(n - 1) r(n)$$  \hspace{1cm} (11)

$$\tilde{e}(i) = y(i) - h(n)^H u(i), \quad i = (n - L + 1)T, \ldots, nT$$  \hspace{1cm} (12)

$$r(n) = \hat{h}(n) - \hat{h}(n - 1)$$  \hspace{1cm} (13)

where $f()$ is a scalar loss function whose purpose is to downweight noise impulses [IPMAA and IPPNAPA depend on the choice of $f()$], $\tilde{e}(i)$ is the posterior error considered over a window of length $L$ symbol intervals (typically $L < 10$ depending on channel coherence time), $\delta \geq 0$ is a regularization parameter, and $P(n)$ is Hermitian positive–definite matrix whose entries depend on $h(n)$. The term $r(n)^H P(n - 1) r(n)$ denotes the Riemannian distance between $\hat{h}(n)$ and $\hat{h}(n - 1)$ and its purpose is to exploit channel sparseness [19]. By setting $\nabla_r J(n) = 0$ (algebra and definitions are described in Appendix A), IPMAA/IPPNAPA can be summarized by the following equations:

$$\hat{h}(n) = \hat{h}(n - 1) + \mu A(n) B(n) e(n)^*$$  \hspace{1cm} (14)

$$A(n) = G(n - 1) U(n)$$  \hspace{1cm} (15)

$$B(n) = (U(n)^H A(n) + \delta Q(n)^{-1})^{-1}$$  \hspace{1cm} (16)

where $U$ is the $(N_c + N_a) \times L$ matrix of input samples, $G$ is a diagonal matrix of size $N_c + N_a$, and $Q$ is a diagonal matrix of size $L$. Initialization starts with $\hat{h}(0) = 0$. As already mentioned, the choice of the loss function and subsequently the matrix $Q$ will generate either IPPNAPA or IPMAA.

1) The IPPNAPA: The key assumption is that the passband noise is modeled by the SαS pdf. The SαS distribution, denoted as $S(\alpha, \delta)$, is defined by means of its characteristic function $\varphi(\omega) = e^{-i \delta \omega}$. The characteristic exponent $\alpha \in [0, 2]$ controls the heaviness of the pdf tails and the scale parameter $\delta > 0$ controls the spread of the pdf around zero. When $\alpha = 2$, the SαS pdf boils down to the Gaussian pdf $N(0, 2\delta^2)$. Parameters $\alpha$ and $\delta$ can be estimated from the ambient noise using fractile-based estimators [26].

In many practical situations, the real and imaginary parts of the baseband (complex) noise $z(n)$ follow the same $S(\alpha, \delta)$ but are generally dependent. The scale parameter $\delta$ is equal to $\epsilon \cdot \delta$, $\epsilon$ being a constant that depends on the passband-to-baseband filtering [27]. In addition, it is known that SαS distributions lack
moments of order \( p \geq \alpha \), but all moments of order \( p < \alpha \) do exist [28]. This motivates the usage of the \( L_p \)-norm, \( p \in [1, \alpha] \) as a loss function. As a result, the diagonal elements \( \{q(e(n-k))\}_{k=0}^{L-1} \) of the \( Q \) matrix are given by [19]

\[
q(e) = \begin{cases} 
1, & 0 \leq \|e\|_2 < \xi \\
\frac{p}{2e^p} \left[ \text{Re}(e) \right]^{p-1} \text{sgn}(\text{Re}(e)) \\
-j \left[ \text{Im}(e) \right]^{p-1} \text{sgn}(\text{Im}(e)) , & \xi \leq \|e\|_2 < \Delta \\
0, & \Delta \leq \|e\|_2.
\end{cases}
\]  

(17)

The threshold parameters \( \xi \) and \( \Delta \) are responsible for detecting and downweighting impulses in the intervals \([\xi, \Delta]\) and rejecting any impulse stronger than \( \Delta \). These thresholds are proportional to \( \delta \) but their exact value depends on the received SNR. In contrast, the choice \( p = \alpha - 0.15 \) is fairly robust [19].

2) The IPMAP A: The key assumption is that the noise \( z(n) = y(n) - h(n)^t u(n) \) is modeled as complex Gaussian noise, but “contaminated” with impulses (or outliers). Using an M-estimator function \( f(\cdot) \) is the typical approach to achieve robustness against impulses, however, redescending M-estimators is a more effective solution because they can differentiate between gross and moderate impulses. A typical redescending M-estimator is Hampel’s loss function [25]. Based on this function, the diagonal elements \( \{q(e(n-k))\}_{k=0}^{L-1} \) of matrix \( Q \) are given by [19]

\[
q(e) = \begin{cases} 
1, & 0 \leq \|e\|_2 < \xi \\
\frac{\xi}{\|e\|_2}, & \|e\|_2 \leq \Delta \\
\frac{\|e\|_2 - \frac{\Delta}{T}}{\Delta - \frac{T}{T}} , & \Delta \leq \|e\|_2 < T \\
0, & T \leq \|e\|_2.
\end{cases}
\]  

(19)

The threshold parameters \( \xi, \Delta, \) and \( T \) are responsible for detecting and downweighting impulses in the intervals \([\xi, \Delta] \) and rejecting any impulse with amplitude greater than \( T \). These thresholds are computed based on the assumption that the impulse-free signal \( z(n) \) is Rayleigh distributed with scale parameter (or mode) \( \sigma \). We choose the thresholds as: \( \xi = 2.45\sigma, \Delta - 2.72\sigma, \) and \( T = 3.03\sigma \) [19]. A robust recursive estimate of \( \sigma \) (denoted as \( \hat{\sigma} \)) can be obtained through the prior error signal \( e(n) = y(n) - \hat{h}(n-1)^t u(n) \) and the median operator [25] as follows:

\[
\hat{\sigma}(n) = \sqrt{\frac{(\sigma^2 + \sigma^2)(n)}{2}}
\]  

(20)

\[
\hat{\sigma}^2(n) = \lambda_\sigma \hat{\sigma}^2(n-1) + \sigma^2(1 - \lambda_\sigma) \text{med}(\hat{e}_\sigma(n))
\]  

(21)

\[
\hat{\sigma}_i^2(n) = \lambda_{\sigma i} \hat{\sigma}_i^2(n-1) + \sigma^2(1 - \lambda_{\sigma i}) \text{med}(\hat{e}_i(n))
\]  

(22)

\[
\hat{e}_\sigma(n) = [\hat{e}_1^2(n), \ldots, \hat{e}_k^2(n) - N_w + 1]^t
\]  

(23)

\[
\hat{e}_i(n) = [\hat{e}_1^2(n), \ldots, \hat{e}_i^2(n) - N_w + 1]^t
\]  

(24)

\[
e = -1.483 \left(1 + \frac{5}{N_w} \right)
\]  

(25)

where \( N_w \) is the observation window of the error signal, \( \hat{\sigma}^2(n) \) and \( \hat{\sigma}^2(n) \) denote, respectively, the estimated variance of the real \( (e_r(n)) \) and imaginary part \( (e_i(n)) \) of \( e(n) \), \( c \) is a finite sample correction factor, and \( \lambda_\sigma \) is a forgetting factor. Note that \( O(N_w \log_2(N_w)) \) operations are required for the computation of \( \hat{\sigma}^2(n) \). Furthermore, \( \hat{e}_\sigma(n) \) always contains signal-dependent noise from channel estimation errors and thus, parameter \( N_w \) should be chosen to be smaller than the channel coherence time.

C. Decision Feedback Equalization

Let \( L_c \) and \( L_a \) denote, respectively, the causal and acausal taps of the linear equalizer and let

\[
y(n) = \left[ y(nT - (L_c + 1)\frac{T}{2}) \right]
\]  

\[
\vdots
\]  

\[
y(nT) 
\]  

\[
\vdots
\]  

\[
y(nT + L_c\frac{T}{2})
\]  

(26)

be the vector of received samples that corresponds to the \( n \)th symbol interval. The received signal can be written as

\[
y(n) = H(n)^t u(n) + z(n)
\]  

(27)

where

\[
u(n) = \left[ u(nT - (N_c + L_c - 2)\frac{T}{2}) \right]
\]  

\[
\vdots
\]  

\[
u(nT) 
\]  

\[
\vdots
\]  

\[
u(nT + (N_a + L_a)\frac{T}{2})
\]  

(28)

\[
\hat{H}(n) = \left[ \begin{array}{cccc}
0 & \cdots & 0 & 0 \\
0 & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots
\end{array} \right]
\]  

(29)

is the \((L_c + L_a + N_c + N_a - 1) \times (L_c + L_a)\) convolution matrix. Let us now partition \( \hat{H}(n) \) into causal and acausal parts as follows:

\[
\hat{H}(n) = \left[ \begin{array}{cc}
H_c(n) & H_{ac}(n)
\end{array} \right]
\]  

(30)

where \( H_c(n) \) includes the rows of \( \hat{H}(n) \) that correspond to the causal input signal

\[
u_c(n) = \left[ u(nT - (N_c + L_c - 2)\frac{T}{2}) \\
\vdots \\
u(nT - T) \right]
\]  

(31)

and \( H_{ac}(n) \) includes the rows of \( \hat{H}(n) \) that correspond to the acausal input signal

\[
u_{ac}(n) = \left[ u(nT - \frac{T}{2}) \\
\vdots \\
u(nT + (N_a + L_a)\frac{T}{2}) \right]
\]  

(32)
The ISI-free signal at the input of the linear equalizer can be constructed as follows:

\[ \tilde{y}(n) = y(n) - \hat{H}_e(n)^T \hat{u}_e(n). \]  

The soft estimate \( \hat{d}(n) \) of the transmitted symbol is obtained by

\[ \hat{d}(n) = p(n)^T \tilde{y}(n) \]  

where \( p(n) \) denotes the equalizer filter. Adaptation of \( p(n) \) at every data symbol arrival is achieved via the exponentially weighted recursive least squares (RLS) algorithm \[35\] \[36\] \[37\] where \( \delta' \) is a regularization parameter and \( \lambda \) is the exponential weighting factor. The RLS computational complexity is \( O((L_c + L_a)^2) \), however, as indicated in [7] and verified in our experimental results, \( p(n) \) can be designed shorter than the total delay spread of the channel and still deliver good performance.

Remark 2: This DFE implementation is robustified via the channel estimator. If a strong impulse occurs at time \( n \), it means that the prior error signal \( e(n) \); high probability is larger than \( \Delta \) (for IPpNAPA) or \( T \) (for IPMAPA). In such and only event, the receiver modifies (6) to \( I(n+1) = I(n) \) and (35) to \( p(n+1) = p(n) \). This robustification strategy is sufficient to deal with a small fraction of impulses in the data as the experimental results confirm.

IV. SEA EXPERIMENT AND RESULTS

The proposed receiver was tested in experimental data obtained in a shallow-water environment. To assess the receiver performance based on the IPMAPA and the IPpNAPA, we use linear complexity algorithms from the adaptive filter literature. These are: improved-proportionate normalized least mean squares (IPNLMS) [30], normalized least mean squares (NLMS) [29], and affine projection sign algorithm (APSA) [31]. We stress that IPNLMS is a sparse-aware algorithm (all sparse adaptive filters used in this paper employ the same \( G \) matrix; see Appendix A), but not robust under impulsive noise. The NLMS algorithm is neither sparse aware nor robust in the presence of impulses. The APSA uses the \( L_1 \)-norm of the error signal and therefore is robust under impulsive noise, but cannot exploit channel sparseness.

A. Experimental Layout

The experiment was conducted in the sea of Selat Pauh, Singapore, on October 23, 2013. The projector (transmitter) was deployed off a vessel and submerged about 3 m below the sea surface. The received signals were recorded at a different vessel 3 m below the sea surface. The sea depth was about 15–20 m and the sound-speed profile was isovelocity (1540 m/s). The sea surface was calm but often the links encountered ship wakes.
Fig. 2(a) illustrates the experimental layout. Two signal formats are considered, i.e., 4- and 8-PSK signals modulated by a pseudonoise sequence. The baud rate was 3000 symbols/s, the carrier frequency was 17 kHz, and the rolloff factor of the RC pulse was 0.7 resulting in a bandwidth of 14 450–19 550 Hz. Several transmissions at different ranges and vessel velocities were repeated. In this paper, we report results based on two ranges: 1.2 km (files 125350 and 130304) and 2.9 km (file 160250). The links 125350 and 130304 have almost the same range (note that the vessels were slightly drifting due to sea currents), yet link 125350 was utilized 10 min before link 130304.

Fig. 2(b) shows the transmit/receive positions. The sampling rate at the receiver was 250 kHz. Before we present the demodulation results, it is instructive to gain insight into the ambient noise and channel characteristics.

B. Ambient Noise

Here, we present an ambient noise data set (14 450–19 550 Hz) recorded during the experiment. Fig. 3(a) clearly shows that the noise series includes instantaneous (impulse-like) sharp sounds. The source of these impulses is due to snapping shrimp [9]. Studies have shown that the $\kappa S$ distribution efficiently
models snapping shrimp dominated ambient noise [9], [10]. Fig. 3(b) verifies this result by plotting the fit along with the Gaussian and the empirical fit of the noise samples of Fig. 3(a).

C. Channel Characterization

Fig. 4(a)–(c) shows the time evolution of the amplitude of the baseband impulse response of all links. Wideband Doppler distortion due to motion is compensated. The channel estimates are generated by using the IPNLMS receiver in training mode. Due to high received SNR, these channel estimates are very reliable since the receiver can achieve error-free communications. Also note that a reduced filter size equalizer that does not span the total delay spread of the channel is used. Table I summarizes the receiver parameters for each link. To gain further insight into how fast each link fluctuates, Fig. 4(d)–(f) shows the time-varying energy of each link by computing $|\hat{h}(n)|^2$.

Several important features can be observed from these responses. The first is that channels 130304 and 160250 exhibit very sparse and long multipath spread as compared to channel 125350. For further validation, Fig. 5(c) shows the spectrogram of the beginning edge of the received signal for the 130304 link. A chirp pulse (0.2 s) and its delayed replica due to multipath can be clearly seen. A similar spectrogram can be obtained for channel 160250 (omitted for brevity). It is yet unclear the origin of the distant reflector for channels 130304 and 160250. We believe this reflector is either an island or a reef close to the area of operations [see Fig. 2(b)].

The second feature is the channel short-time variability due to constructive/destructive multipath interference. Fig. 4(d) indicates that the energy of channel 125350 fluctuates about 2 dB over a period of 3.5 s. The mean Doppler shift (not shown for brevity) due to drifting oscillates between $\pm 8$ Hz. Fig. 4(e) shows that the energy of channel 130304 fluctuates about 7 dB over a period of 3.2 s. The mean Doppler shift oscillates in a similar fashion as in channel 125350. Note the rapid 5-dB energy increase at 0.9 s. This large fluctuation is challenging to be tracked by adaptive receivers. For instance, it makes a standard DA–DFE running on RLS to fail regardless the high SNR. Fig. 4(f) illustrates that the energy of channel 160250 fluctuates about 6 dB over a period of 2.5 s. The energy lows observed at 1.2 and 1.8 s are due to a prominent Lloyd’s Mirror effect [32].

Fig. 5(a) shows the time-varying Doppler shift of the received signal for the channel 160250. The positive Doppler validates that the transmitter was propelling toward the receiver at a speed of about 2 kn. Fig. 5(b) illustrates the effect of motion on the channel response when it is left uncompensated. The plot focuses on the multipath arrivals close to 0 ms for visualization purposes. Clearly, the channel response varies very rapidly and eventually makes the DFE to fail.

D. Demodulation Results

The performance metric is the symbol error probability (SER) as a function of the received SNR. Since the data were originally acquired in very high SNR, the following plots are computed by scaling and adding extra ambient noise to the original data. At every SNR, the SER is computed after averaging 15 independent ambient noise recordings. Hence, the reported plots illustrate the average performance of each receiver given the link realization. We emphasize that only the first 900 transmitted symbols were used as a training set for the channel estimator and the RLS equalizer. The fixed receiver parameters for all links are listed in Table I. However, the choice for some channel estimation parameters is not straightforward as it depends on the channel coherence time and received SNR. To ensure a fair comparison between all algorithms, their respective parameters are optimized among some representative values such that the
The lowest SER is obtained for a given received SNR. We consider the following parameters and their representative values:

- $\mu \in \{0.1, 0.2, \ldots, 0.5\}$ for NLMS, IPNLMS, IPMAPA, IPpNAPA;
- $N_{\text{sc}} \in \{4, 8, 12\}$ for IPMAPA;
- $\xi \in \{2\delta, 4\delta, 7\delta\}$, $\Delta \in \{8\delta, 13\delta, 18\delta\}$ for IPpNAPA;
- $\mu \in \{0.25\delta, 0.5\delta, \delta, 2\delta, 3\delta\}$ for APSA [31].

Note that the IPMAPA and the IPpNAPA are optimized, respectively, over 15 and 45 different combinations. Given that both algorithms obtain linear computational complexity, optimization based on an exhaustive search of all combinations during the training period is possible. After the training period, the optimized parameters remain fixed for the rest of the signal duration.

Fig. 6(a)–(c) shows the SER performance of all algorithms for the links 125350, 130304, and 160250, respectively. We observe the following.

- At relative low SNRs, the proposed algorithms outperform all other algorithms in all links. At higher SNRs, noise impulsiveness is not the main challenge and so the sparse algorithms deliver similar performances.
- For the 8-PSK links (125350 and 130304), the proposed algorithms require more than 20-dB SNR to deliver good performance. For the 4-PSK link (160250), however, the proposed algorithms require 5 dB less power to deliver a similar performance. This 5-dB difference is explained by noting that 4-PSK constellations are less sensitive to motion-induced Doppler and impulsive noise than 8-PSK constellations.
- With respect to IPNLMS, the proposed algorithms demonstrate 2-dB power savings for a SER of $10^{-2}$ for the link 125350. Moreover, IPMAPA shows an advantage of 1 dB at SER = $10^{-2}$ for the link 130304. On the other hand, the proposed algorithms show no power gain at SER = $10^{-2}$ for channel 160250. This can be explained by noting that link 125350 does not suffer from deep fades, link 130304 suffers from minor fades, while link 160250 suffers two deep fades at 1.2 and 1.8 s [see Fig. 4(e)]. Hence, it seems that in the absence of strong fading, more gain is achieved with the proposed methods.
- The commonly used NLMS and APSA can deliver good performance only for link 125350. Unfortunately, both algorithms fail in the other two (extended-multipath) links due to their inability to exploit channel sparseness.
- Since the ambient noise is well modeled as a SinS, one would expect IPpNAPA to deliver better performance than IPMAPA. However, our plots show that the algorithms have similar performance. These results are consistent with our findings in [19] because when $\alpha$ is larger than about 1.5, then the performance of IPMAPA is close to IPpNAPA.

V. CONCLUSION

A new channel-estimate-based decision feedback equalizer receiver for underwater acoustic communications was presented. By using data from three shallow-water links, the proposed receiver managed to: 1) tolerate high transmitter/receiver mobility; 2) adapt to rapid channel fluctuations by exploiting channel sparseness; and 3) achieve robustness under impulsive noise. This result was demonstrated by testing different channel estimation algorithms and comparing the respective SER performances. The algorithms tested were IPMAPA [19], IPpNAPA [19], IPNLMS [30], NLMS [29], and APSA [31]. The best performance was achieved by IPMAPA/IPpNAPA as these algorithms are designed to suppress impulsive noise while exploiting channel sparseness.

APPENDIX A

DERIVATION OF (14)

Let us define

$$\bar{\epsilon}(n) = y(n) - \hat{h}(n)^T u(n)$$  \hspace{1cm} (38)

$$\epsilon(n) = y(n) - \hat{h}(n - 1)^T u(n)$$  \hspace{1cm} (39)

$$r(n) = \hat{h}(n) - \hat{h}(n - 1)$$  \hspace{1cm} (40)
where \( \tilde{e}(n) \) and \( e(n) \) are the a posteriori and a priori error at symbol time \( nT \), respectively, and \( r(n) \) denotes the channel update vector. Note that we can express \( \tilde{e}(n) \) in terms of \( e(n) \) and \( r(n) \) as
\[
\tilde{e}(n)^* - e(n)^* = u(n)^T r(n). \tag{41}
\]
Via the loss function \( f(z), z \in \mathbb{C} \), we also define
\[
\psi(z) = \frac{\partial f(z)}{\partial z}, \quad q(z) = \psi(z)^* z^*. \tag{42}
\]
We now compute \( \nabla_{r(n)} J(n) \), where \( J(n) \) is given by (11). Computing each term individually, we write
\[
\nabla_{r(n)}^* J(n) = -\sum_{i=-n}^{L+1} q(\bar{e}(i)) e(i)^* u(i) \tag{43}
\]
Using a step size parameter \( \mu \in (0, 1) \), we manage to manipulate the change of the tap values from one iteration to the next. Finally, the channel update equation is deduced as follows:
\[
\hat{h}(n) = \hat{h}(n) + \mu A(n) B(n) e(n)^* \tag{56}
\]
\section{A. The \( G \) Matrix}
As already mentioned above, the purpose of the \( P = G^{-1} \) matrix is to exploit channel sparseness. A good choice for \( G \) is the diagonal matrix of \([30] \). Effectively, \( G \) assigns a variable step size parameter to each channel filter tap. This parameter is a function of the tap's previously estimated magnitude. As a result, active filter taps (i.e., taps with significant values) converge fast, which makes the overall algorithm to have fast convergence in sparse channels. We stress that no prior knowledge of the significant tap position is required. The diagonal elements \( \{g(h_k(n))\}_{k=0}^{N_c+N_a-1} \) of \( G(n) \) are given by \([19], [30] \)
\[
g(h_k(n)) = \frac{1 - \beta}{2(N_c + N_a)} + \frac{(1 + \beta) |\hat{h}_k(n)|_1}{2 |\hat{h}(n)|_1} + \varepsilon \tag{57}
\]
where \( \varepsilon \) denotes a small positive constant to avoid division by zero during initialization of the algorithm. Parameter \( \beta \) controls the sparseness of the solution. Typically, one needs to consider four values: \( \beta = -1 \) (diffuse channel), \( \beta = -0.5 \) (low sparseness), \( \beta = 0 \) (sparse), and \( \beta = 0.5 \) (very sparse). Once \( \beta \) is chosen, parameter \( \delta \) is given by \([30] \)
\[
\delta = \frac{1 - \beta}{2(N_c + N_a)} \delta_{NLMS} \tag{58}
\]
where \( \delta_{NLMS} \) is chosen as in the NLMS algorithm \([29] \). Note that the channel update equation (56) requires \( O(N_c + N_a) \) computational complexity because \( G(n) \) is diagonal and \( N_c + N_a > L \) for typical underwater acoustic channels.

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\section{References}


