Time-Domain Equalization for Underwater Acoustic OFDM Systems with Insufficient Cyclic Prefix

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***Abstract*—Single Input Single Output (SISO) Orthogonal Frequency Division-Multplexing (OFDM) over short to medium range shallow water channels suffers from low bandwidth efficiency. This happens due to large Cyclic Prefix (CP) and relative small number of sub-carriers. To increase the bandwidth efficiency, a time domain channel shortening equalizer (CSE) can be inserted before the Fast Fourier Transform (FFT). The CSE shortens the channel impulse response (CIR) so that a smaller CP is needed. This paper analyses the performance of four time domain CSEs: 1) Minimum Mean Square Error (MMSE) Unit Tap Constraint (UTC) 2) MMSE Unit Energy Constraint (UEC) 3) Maximum Shortening Signal to Noise Ratio (MSSNR) 4) Minimum Inter Symbol Interference (Min ISI) and Frequency Domain Decision Feedback Equalizer (FD-DFE). Analysing simulated and real data, the MMSE UTC equalizer shows the best performance in terms of Bit Error Rate (BER). When bit-loading is applied, the BER of Min ISI approach has comparable performance to UTC.**

I. INTRODUCTION

Long and time-varying impulse responses are the two main characteristics of an underwater acoustic link. Long impulse responses contribute to Inter Symbol Interference (ISI), which is undesirable due to its negative impact on the Bit Error Rate (BER). In recent years, work has been done on implementing Orthogonal Frequency Division-Multplexing (OFDM) for Underwater Acoustic (UWA) communication [1]. When the Cyclic Prefix (CP) is longer than the channel impulse response (CIR), OFDM is an effective method to tackle ISI. However, a long CP reduces the bandwidth efficiency of the system. In addition, high Doppler frequencies induce Inter Carrier Interference (ICI) and hence, a limit on the number of sub-carriers is mandatory for good system performance. Bandwidth efficiency, a measure of the channel throughput, is defined as $\frac{N_c}{N_c+N_p}$ where N_c is the number of sub-carriers and N_p is the CP length. Hence, to keep the bandwidth efficiency high, it is important that the CP is chosen to be as short as possible.

A time domain equalizer can be inserted before the OFDM demodulator to shorten the effective channel so that a smaller N_p is required. A Minimum Mean Square Error (MMSE) channel shortening equalizer (CSE) minimizes the mean square error (MSE) between the equalizer output and the output of the target impulse response (TIR) [2]. The method was first developed to be used with maximum-likelihood sequence estimation (MLSE) to achieve higher data rates on bandlimited noisy channels. The role of the CSE is to reduce the channel impulse response to allow practical use of the high performance Viterbi algorithm. In order to avoid a trivial solution, some constraint like Unit Energy Constraint (UEC) and Unit Tap Constraint (UTC) must be imposed on the TIR. For the Maximum Shortening Signal to Noise Ratio (MSSNR) approach, a Finite Impulse Response (FIR) filter is generated to minimize the energy outside the length of a TIR while setting a unit energy constraint on the desired component of the received signal [3]. However, the method is not optimized to yield the best result in terms of sub-carrier signal to noise ratio (SNR). The Minimum Inter Symbol Interference (Min ISI) is a frequency weighted version of MSSNR [4]. It minimizes the energy outside the length of the TIR according to the sub-carriers SNR. By using a water pouring algorithm, the ISI is placed into spectral regions of low SNR and thus, maximizing the data rate. Both MSSNR and Min ISI have been implemented in Asymmetric Digital Subscriber Line (ADSL) system to increase the bandwidth efficiency. An alternative to time domain equalizers is their frequency domain counterpart. A FD-DFE is developed for OFDM system with insufficient CP [5].

In this paper, a computer generated experiment is conducted to compare the performance between the different methods in a UWA channel. The channel used is estimated from received UWA field trial data. The performance of the methods is also evaluated under two different trial settings.

Notation: Superscripts ^{T}, ^{H} and * stand for transpose, Hermitian transpose, and conjugate, respectively. Column vectors (matrices) are denoted by boldface lowercase (uppercase) letters.

II. TIME DOMAIN CHANNEL SHORTENING EQUALIZERS

The MMSE CSE is a class of equalizers that generates a FIR filter that minimizes the error between the output of the equalizer and the output of the TIR in the mean square sense. The TIR is shorter than the original CIR and in an OFDM system, shorter than the CP. Fig. 1 shows the block diagram of MMSE CSE. The received symbol vector is $\boldsymbol{y}[\mathrm{m}]$. The CIR

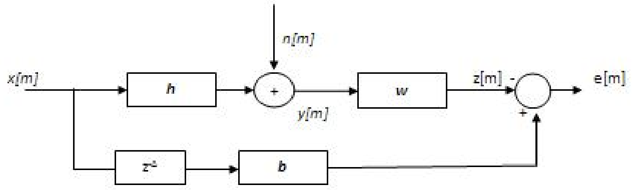

Fig. 1. MMSE Channel Shortening Equalizer.

h is longer than the TIR ***b***. The equalizer is represented by $\mathbf{w}$. The term Δ is the relative delay between CIR and the TIR.

A. UTC

In order to avoid a trivial solution of $\mathbf{b} = \mathbf{w} = 0$, a constraint is placed on $\mathbf{b}$. For MMSE UTC, the MSE is minimized subject to $\mathbf{b}^H\mathbf{e}_i = 1$ where $\mathbf{e}_i$ is the ith unit vector [2]. For a fixed Δ the optimal TIR coefficients are given by

$$\mathbf{b}_{opt} = \frac{\mathbf{R}_\Delta^{-1}\mathbf{e}_{i_{opt}}}{\mathbf{R}_\Delta^{-1}(i_{opt}, i_{opt})} \tag{1}$$

where $\mathbf{R}_\Delta$ is the input-output channel-dependent cross correlation matrix zero padded according to the relative delay and i_{opt} represents the index that yields the minimum MSE. $\mathbf{R}_\Delta^{-1}(i,i)$ is the ith diagonal component of $\mathbf{R}_\Delta^{-1}$. The process is repeated across a range of relative delay Δ. The optimum equalizer coefficients are

$$\mathbf{w}_{opt}^* = \mathbf{b}_{opt}^H\mathbf{R}_{xy}\mathbf{R}_{yy}^{-1} \tag{2}$$

where $\mathbf{R}_{xy}$ and $\mathbf{R}_{yy}$ are the input-output cross-correlation matrix and the output autocorrelation matrix, respectively.

B. UEC

An alternative constraint on $\mathbf{b}$ is the UEC [2]. This constraint has an advantage over UTC because the exhaustive search procedure for the optimal index i is no longer required. Under the constraint $\mathbf{b}^H\mathbf{b} = 1$, $\mathbf{b}_{opt}$ is chosen to be the eigenvector that corresponds to the minimum eigenvalue of $\mathbf{R}_\Delta$. Then, $\mathbf{w}$ is calculated as in (2).

C. MSSNR

Let $\mathbf{h}_{eff}$ be the effective CIR, namely, the convolution of the channel with the filter $\mathbf{w}$. A window of consecutive N_b (TIR length) samples of $\mathbf{h}_{eff}$ is given by $\mathbf{h}_{win}$. The rest of the samples of $\mathbf{h}_{eff}$ is given by $\mathbf{h}_{wall}$. The expressions of the energy inside and outside of the window, respectively, becomes [3]

$$\mathbf{h}_{win}^H\mathbf{h}_{win} = \mathbf{w}^H\mathbf{H}_{win}^H\mathbf{H}_{win}\mathbf{w} = \mathbf{w}^H\mathbf{A}\mathbf{w} \tag{3}$$

$$\mathbf{h}_{wall}^H\mathbf{h}_{wall} = \mathbf{w}^H\mathbf{H}_{wall}^H\mathbf{H}_{wall}\mathbf{w} = \mathbf{w}^H\mathbf{B}\mathbf{w} \tag{4}$$

where $\mathbf{H}_{win}$ is the truncated Toeplitz matrix of CIR representing the samples in the window and $\mathbf{H}_{wall}$ is the rest of the toeplitz matrix. By minimizing the $\mathbf{h}_{wall}^H\mathbf{h}_{wall}$ while imposing the $\mathbf{h}_{win}^H\mathbf{h}_{win} = 1$ constraint, we can apply Cholesky Decomposition to find the $\mathbf{w}$ that solve for the minimum of $\mathbf{h}_{wall}^H\mathbf{h}_{wall}$. When the length of $\mathbf{w}$ is shorter than N_b, both $\mathbf{A}$ and $\mathbf{B}$ are positive semidefinite. However, when the delay spread is large, a longer filter is needed to effectively shorten the channel. In [6], the optimization becomes the maximization of the window energy by keeping the unity constraint on the wall energy. This allows flexibility in the length of the equalizer filter. These methods, however, are zero forcing equalizers and do not account for the statistics of the noise. To factor in the noise, the new cost function becomes

$$\mathbf{w}_{opt} = \arg\min_{\mathbf{w}}\{\mathbf{w}^H\mathbf{H}_{wall}^H\mathbf{R}_{xx}\mathbf{H}_{wall}\mathbf{w} + \mathbf{w}^H\mathbf{R}_{nn}\mathbf{w}\} \tag{5}$$

where $\mathbf{R}_{xx}$ and $\mathbf{R}_{nn}$ is the input and the noise auto-correlation matrix, respectively. Cholesky Decomposition is used to find the minimum in (5).

D. Min ISI

In [4], a frequency weighted MSSNR algorithm called the Minimum ISI (Min ISI) is introduced. The algorithm factors in the SNR of each subcarrier when performing the optimization of the CSE coefficients. This frequency weighting places ISI into spectral regions of low SNR in effect maximizing the data rate by applying a water-pouring algorithm. The objective function of Min ISI is:

$$\begin{aligned}\mathbf{w}_{opt} = \quad & \arg\min_{\mathbf{w}} \Big(\mathbf{w}^H\mathbf{H}_{wall}^H \textstyle\sum_{i=0}^{N_c}\mathbf{q}_i S_x(\omega_i)\,\mathbf{q}_i^H\mathbf{H}_{wall}\mathbf{w} \\ & + \mathbf{w}^H\mathbf{\Theta}^T \textstyle\sum_{i=0}^{N_c}\mathbf{q}_i S_n(\omega_i)\,\mathbf{q}_i^H\mathbf{\Theta}\mathbf{w}\Big)\end{aligned} \tag{6}$$

where S_x and S_n is the power spectral density of the transmitted symbols and noise, respectively, and $\mathbf{\Theta}$ is the padding matrix for dimension matching. The frequency weighting of the algorithm is based on the statistics of the noise. If the noise is white, (6) reduces to (5).

III. FREQUENCY DOMAIN EQUALIZER

An alternative to the time domain CSE is a Frequency Domain DFE (FD-DFE). The input to the equalizers is the symbol in frequency domain. For an OFDM symbol with insufficient CP, there will be ISI and ICI [5].

$$\begin{aligned}\mathbf{Y}_i &= \mathbf{Y}_S + \mathbf{Y}_{ICI} + \mathbf{Y}_{ISI} + \mathbf{N} && (7)\\ &= \mathbf{C}_1\mathbf{X}_i + \mathbf{C}_2\mathbf{X}_i + \mathbf{S}\mathbf{X}_{i-1} + \mathbf{N} && (8)\end{aligned}$$

where $\mathbf{X}_i$ and $\mathbf{Y}_i$ is the transmitted symbol vector and received symbol vector, respectively. The index i, in this case, is the time index. $\mathbf{Y_{ici}}$ and $\mathbf{Y_{isi}}$ represent the ICI and ISI portion of the symbol whereas $\mathbf{N}$ is the noise vector. The matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ contain the elements of the CIR Toeplitz matrix that represents the ISI and ICI, respectively. The FD-DFE consists of a feedforward filter matrix $\mathbf{W}$ and a feedback filter matrix $\mathbf{B}$. The estimated symbol is given by:

$$\begin{aligned}\hat{X}_i(k) &= \mathbf{W}_k^H\mathbf{Y} - \mathbf{B}_k^H\hat{\mathbf{X}}_{i-1}\\ &= \begin{bmatrix}\mathbf{W}_k\\ -\mathbf{B}_k\end{bmatrix}^H\begin{bmatrix}\mathbf{Y}\\ -\hat{\mathbf{X}}_{i-1}\end{bmatrix}\end{aligned} \tag{9}$$

where k is the sub-carrier index. If the previous decisions are assumed to be correct. The MMSE Wiener-Hopf solution is implemented by minimizing $E[|\hat{X}_i(k) - X_i(k)|^2]$. The solution is

$$\begin{bmatrix} \mathbf{W}_k \\ -\mathbf{B}_k \end{bmatrix} = \mathbf{R}^{-1}\mathbf{P}_k \qquad (10)$$

where

$$\mathbf{R} = E\left[\begin{bmatrix} \mathbf{Y} \\ -\mathbf{X}_{i-1} \end{bmatrix}\begin{bmatrix} \mathbf{Y} \\ -\mathbf{X}_{i-1} \end{bmatrix}^H\right] \qquad (11)$$

$$\mathbf{P}_k = E\left[\begin{bmatrix} \mathbf{Y} \\ -\mathbf{X}_{i-1} \end{bmatrix} X_i^*(k)\right] \qquad (12)$$

IV. PERFORMANCE RESULTS

A. Simulation Results

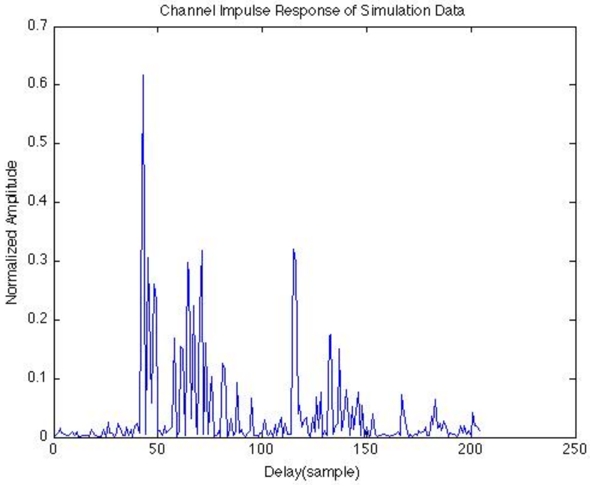

Fig. 2. Channel Impulse Response for simulation.

The performance of the methods is evaluated based on a computer simulation. The CIR shown in Fig. 2 corresponds to a measured impulse response taken from a short range shallow water link experiment. The number of sub-carriers is 512 and the CP length is 100. The symbols in frequency domain are QPSK modulated. The channel impulse response is longer than the CP hence ISI is present. Two OFDM signals with a CP length of 100 and 150 symbols, respectively are used. The equalizers are implemented on the OFDM with shorter CP (Short OFDM). The OFDM signal with long CP (Long OFDM) is used as a benchmark to illustrate the BER performance of an ISI-free scenario. Fig. 3 shows the BER performance of the different methods. For M-ary symbols, the E_b/N_o in dB is given by

$$E_b/N_o = SNR_{symbol} - 10\log_{10}(\log_2 M \times \frac{N_c}{N_c + N_p}) \qquad (13)$$

where SNR_{symbol} is the SNR per channel symbol. Note that although UEC methods yield lower normalized MSE than UTC, UTC outperforms UEC. This is because UEC generates deeper nulls in the TIR than UTC. In the presence of noise, the subcarrier in the deep null region will have high BER. Compared to the long OFDM, UTC system has slightly better BER performance. At BER 10^{-3}, the E_b/N_o gain of UTC over the long OFDM is around 2 dB. When the noise is white, the performances of MSSNR and Min ISI are identical.

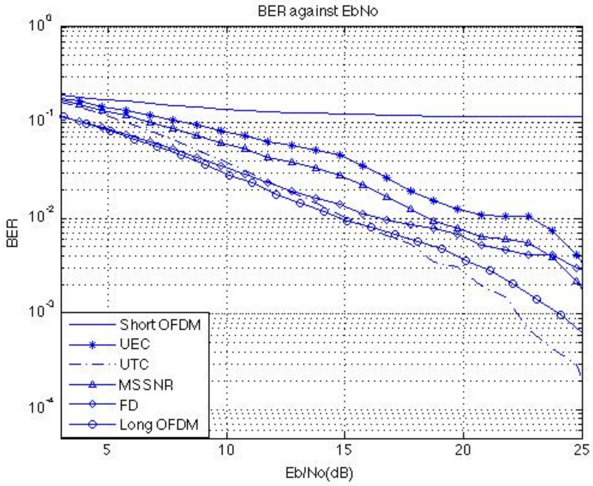

Fig. 3. BER of performance of employed methods based on simulated data.

To investigate the performance of the equalizers under colored noise, a new noise sequence is generated. The power spectral density (PSD) of the noise is shown in Fig. 4. Note that out of 512 sub-carriers, 100 sub-carriers have lower SNR. The results are shown in Fig. 5. Min ISI performed better than MSSNR. This is because Min ISI is a frequency weighted solution of the CSE problem. For sub-carriers of high SNR, the BER performance depends on ISI. Hence by giving higher priority to subcarrier with high SNR in terms of shortening the channel the system achieves a better overall performance in BER.

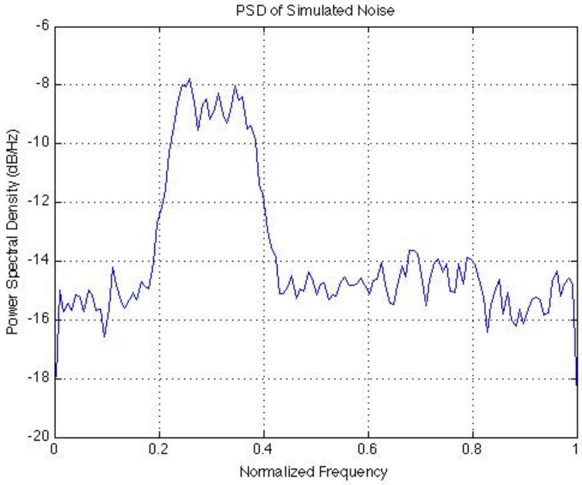

Fig. 4. PSD of colored noise.

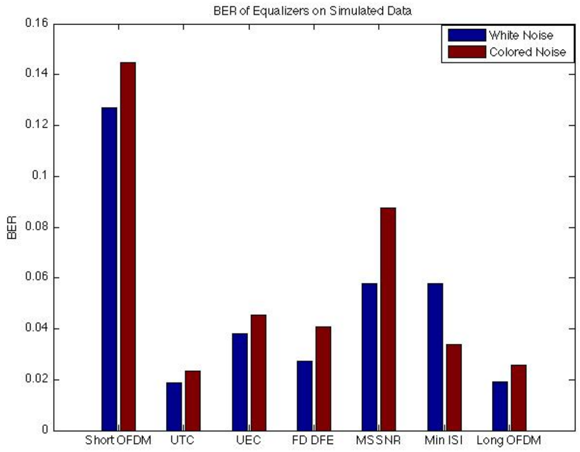

Fig. 5. BER of different methods.

B. *GLINT 2008*

A comparison of the aforementioned methods is investigated based on real data recorded during the GLINT08 experiment off the coast of Pianosa Island, Italy. The analysis is performed on the data collected from three hydrophones (depth 17.6m) on a moored vertical array approximately 1.6km away from the source. The SNR was around 14dB. Fig. 6 shows snapshots of the estimated channel impulse response of the data. Table I shows the OFDM parameters of the symbol. In total, 30 OFDM symbols (15360 bits) were transmitted. Channel estimation is performed by using sparse adaptive algorithm [7]. For an OFDM signal with short CP (0.8ms), the BER of the system is high because the CP is not long enough for ISI free demodulation. From Fig. 6, we can see that a significant portion of the channel energy lies outside the CP (0.8ms). Since the channel impulse response is time-varying, the channel has to be continuously tracked in order to have the correct equalizer filter coefficients. We use the previously decoded OFDM symbol as the pilot signal to the channel estimator. To reduce the BER, a rate 1/2 convolution encoding is performed at the transmitter. The first column of Table II assumes perfect decoding, i.e., the actual transmitted signal was used as a pilot. The second column is based on pilot generated from the decoder output (decision directed). In both cases, the BER performance of OFDM is worse than the rest. UTC performs better than UEC because of the presence of deep nulls in the TIR of the UEC method. In the case of no bit loading, the BER performance of Min ISI is the same as MSSNR. In the imperfect feedback scenario, the decoding errors render the pilot signal inaccurate and thus leads to poorer channel estimation in the next OFDM period and degradation in the BER performance. Imperfect feedback rate 1/2 has lower BER than perfect feedback at the expense of reduced bit-rate.

TABLE I
OFDM PARAMETERS OF GLINT 08

No. of Subcarrier, N_c	256
No. of CP, N_p	16
No. of suffix, N_s	2
Sampling Frequency, F_s	96000 Hz
Bandwidth, F_d	20000 Hz
Carrier Frequency, F_c	30000 Hz
Symbol Length	30

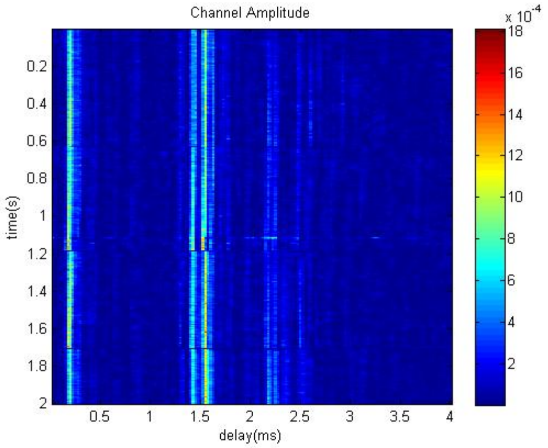

Fig. 6. Snapshots of the estimated time-varying channel impulse response for GLINT 08. The horizontal axis represents delay, the vertical axis represents absolute time and the colorbar represents the amplitude. The intensity ranges linearly.

C. *Singapore waters 2010*

The experimental data were recorded in the area of Selat Pauh in Singapore waters on April 21st, 2010. Both the transmitter and the receiver were mounted on rigid tripods, 4m above the sea floor. The sea depth was 15m and the horizontal range of the link was 350m. The sound speed profile was isovelocity 1540m/s and the sea surface was calm during the experiment. The average received SNR was 11.5dB. Table III shows the OFDM parameters of the transmitted signal. Of the 256 subcarriers, 129 subcarriers carried data. The rest are reserved for Peak-to-Average Power Ratio (PAPR) or are null carriers. The CP of 2ms is clearly inadequate for the long CIR.

TABLE II
BER PERFORMANCE IN GLINT 2008

Method	Perfect Feedback	Imperfect Feedback	Imperfect Feedback (Code 1/2 rate)
OFDM	0.1473	0.1548	0.1231
UTC	0.0932	0.1104	0.0733
UEC	0.1273	0.1431	0.1045
MSSNR	0.1224	0.1337	0.1008
MIN ISI	0.1235	0.1357	0.0997
FD DFE	0.1123	0.1245	0.0905
Bit Rate (kbits/s)	37.4	37.4	18.7

TABLE III
OFDM PARAMETERS OF SINGAPORE'10

No. of Subcarrier, N_c	256
No. of CP, N_p	10
Sampling Frequency, F_s	200000 Hz
Bandwidth, F_d	5000 Hz
Carrier Frequency, F_c	27500 Hz
Symbol Length	30

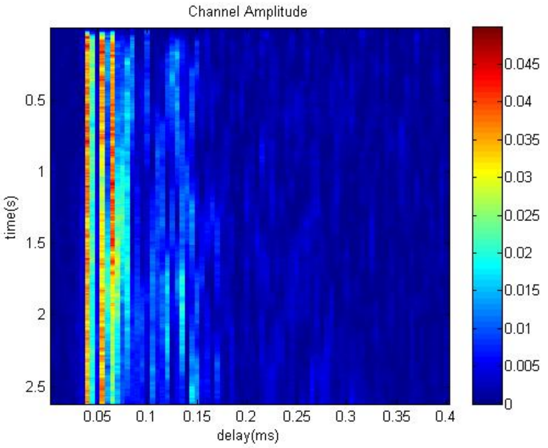

Fig. 7. Snapshots of the estimated time-varying channel impulse response for Singapore '10. The horizontal axis represents delay, the vertical axis represents absolute time and the colorbar represents the amplitude. The intensity ranges linearly.

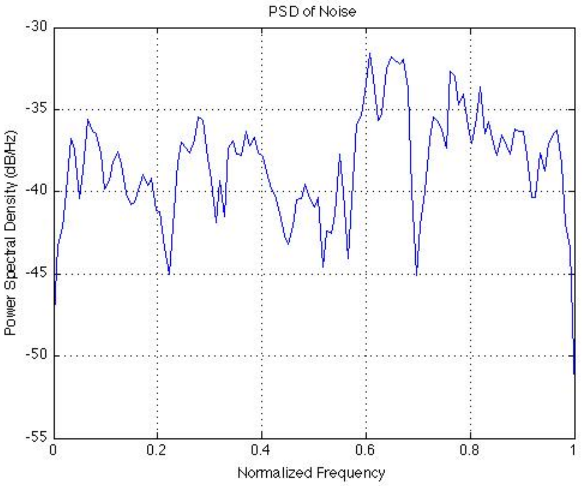

Fig. 8. PSD of noise for Singapore'10.

Table IV shows the BER result of the equalizers. The UTC method performs better than the other methods when no bit-loading is performed. This is due to smaller number of spectral nulls in the frequency response of UTC TIR. The noise is colored as seen in Fig. 8. Like in GLINT 08, the decoded symbol is used as the pilot for channel estimation. The results in the first three columns of Table IV are documented under the same setting as the first three columns of Table II. In order to showcase the performance of Min ISI, bit loading is performed on the transmit signal. There are 30 sub-carriers with low SNR which are encoded with a convolution code rate of $\frac{1}{3}$. The rest of the sub-carriers are convolution $\frac{1}{2}$ encoded. The BER result of this setting is tabulated in the last column of Table IV. In the case of bit-loading with non-white noise, Min ISI has the lowest BER. This is because frequency weighting is done and the priority in channel shortening is given to sub-carriers with higher SNR, which carries more information bits.

TABLE IV
BER PERFORMANCE IN SINGAPORE'10

Method	Perfect Feedback	Imperfect Feedback	Imperfect Feedback (Code 1/2 rate)	Bit Loading
OFDM	0.1383	0.1893	0.0983	0.0693
UTC	0.0903	0.1563	0.0692	0.0231
UEC	0.1203	0.1832	0.1102	0.0754
MSSNR	0.1128	0.1692	0.0927	0.0532
Min ISI	0.1128	0.1692	0.0816	0.0183
FD DFE	0.1093	0.1602	0.0826	0.0442
Bit Rate (kbits/s)	4.85	4.85	2.42	2.24

V. CONCLUSION

Five OFDM systems with improved bandwidth efficiency were investigated for UWA communications. There are: MMSE UTC CSE, MMSE UEC CSE, MSSNR CSE, Min ISI CSE, and FD-DFE. After analyzing simulated and experiment data, we found that the UTC CSE approach performs better than the rest. This is due to the small number of spectral nulls in the frequency response of UTC TIR. Moreover, the other methods perform worse than a typical OFDM system with sufficient CP. When the noise is non-white, the Min ISI approach performs comparably to UTC when bit-loading is exploited.

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