Unslotted Transmission Schedules for Practical Underwater Acoustic Multihop Grid Networks with Large Propagation Delays

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***Abstract*—In a recent work, a regular grid-based network topology with multihop relaying was investigated. A transmission strategy which maximizes the throughput while exploiting the large propagation delay was presented, and the upper bound on throughput established. However, deployments of communication nodes in the ocean inevitably result in slight positional deviations from the expected locations of the nodes, a departure from the perfectly aligned regular grid networks assumed. The irregularity in the grid network due to deployment errors degrades the network throughput significantly. We consider this practical problem and propose an algorithm to compute unslotted transmission schedules. We formulate the scheduling problem as a Mixed-Integer Linear Problem (MILP) and compute throughput-maximizing schedules. We demonstrate the throughput gain compared to the existing state of the art techniques and verify the solution in the simulator for various random instances of the grid network deployment.**

I. Introduction

Recent advances in underwater acoustic (UWA) networks have enabled applications such as environmental monitoring, scientific exploration, military surveillance and many commercial marine applications [1]. When operating over a large area, the multiline grid topology with multihop relaying can be considered for providing high-rate services [2]. Multiline grid topology considered in [2] consists of parallel lines with regularly placed nodes. Messages originate from the first node on each line and are destined to the final node on the same line. Intermediate nodes act as relay nodes which receive the incoming packets, decode them and retransmit them to the next hop, until they reach the final destination node. The throughput upper bound was established in [2] for a particular network setting with a regular N-node multiline grid network. The spacing between neighboring nodes on the same line was considered to be one unit and the distance separating every two adjacent lines to be two units.

Although, the study in [2] provides significant insights into exploiting the large propagation delay in regular multiline grid network, there are a few assumptions which when generalized, lead to more complex and interesting problems, the solution of which cannot be directly obtained from the techniques presented in [2]. For example, the deployment of the nodes while maintaining the regularity of the grid networks is extremely difficult. After deployment, the distance matrix is usually close to the desired matrix but not exactly the same. The assumptions on the regular spacing between the nodes will no longer be valid. This results in non-integer propagation delays among the nodes in contrast to the integer propagation delays considered in [2]. The schedules can be computed by approximating the non-integer propagation delays to the nearest integers as proposed in [3]. However, due to these approximations, we have to allocate sufficient guard times at the start and end of the time slots to prevent early and delayed receptions within the time slots [3], [4]. The approximations are the cause for inefficient utilization of slots thereby reducing the throughput. The time-slotted nature of the solution constrains the transmissions to be strictly within the time slots. We propose unslotted transmission schedules for the practical grid networks considered by formulating the scheduling problem as a Mixed-Integer Linear Problem (MILP) and compute schedules which do not require explicit transmission slots to transmit. Different from the objectives considered in [5], [6] to minimize the energy consumption by avoiding collisions and minimizing frame length in [7], we consider minimization of idle time in frame as the objective in the MILP formulated. Section II introduces the system model and assumptions followed by an algorithm to compute the time-slotted schedules in Section III. Section IV presents the MILP formulation. In Section V, we show the simulation results and demonstrate the capabilities of the algorithm to perform better than existing algorithms and in Section VI we present the conclusion.

II. System Model & Assumptions

We consider N-node multiline grid network with multihop relaying and non-negligible propagation delay among the nodes (see Fig. 1). Each node $i \in \{1, \cdots, N\}$ is a half-duplex underwater acoustic modem, i.e., the node cannot receive and transmit simultaneously. All the transmissions are assumed to be unicast and intended to their corresponding destinations. The number of independent node lines in the network is $\eta \geq 1$. Information-bearing data packets originate from the source nodes $1, 2, \cdots, \eta$ at one end of the line and are relayed hop by hop to the destination nodes $N-\eta+1, N-\eta+2, \cdots, N$ at the other end of the line as illustrated in Fig. 1. Due to the architecture of the network and the information flow, the links considered for the transmission of the data packets are fixed, given N and η. We denote the set of these directed links by $\mathcal{L}_s$ and the set of all possible links in the network by $\mathcal{L}$. Each link in the set $\mathcal{L}$ or $\mathcal{L}_s$ can be explicitly written as a 2-tuple (j, k) which represents a link where node j is the transmitter and node k is the receiver. The propagation delay corresponding to link (j, k) is D_{jk} units. The nodes in the

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network are intended to be deployed such that all the links in a line have a unit propagation delay and the adjacent node lines are separated by two units of propagation delay (similar to [2]) as shown in Fig. 1. However, in actual deployment, the nodes are slightly displaced from the regular grid alignment (see Fig. 1). The propagation delay D_{jk} corresponds to the measured delay after the deployment of nodes, i.e., D_{jk} is close to 1 unit but not exactly 1:

$$\lfloor D_{jk} \rceil = 1 \ \forall \ (j,k) \in \mathcal{L}_s \tag{1}$$

where $\lfloor \cdot \rceil$ denotes the operation to round off the value to the closest integer. We partition time into frames, but do not further partition into slots as is commonly done [2], [3], [5]–[8]. The frame length of the schedule (also termed as period of the schedule in [2], [3]) is T. The time, relative to the start of the frame, at which node j starts transmitting a packet to node k, is t_{jk}. The packet/transmission duration corresponding to the packet transmission on link (j,k) is τ_{jk}. As in [2], we adopt a protocol channel model [9] and denote by α, the ratio of interference range to the communication range. The protocol channel model assumes that, if two packets partially overlap in time at the receiver node, then the receiver will be unable to receive either of the packet successfully. In wireless radio networks, the interference range is often considered to be approximately twice the communication range [10], [11]. Hence, we set α to 2 similar to the setting considered in [2]. Moreover, we assume that nodes transmit with variable power which is just enough to include the desired destination node in the communication range of the transmitter.

III. Time-Slotted ρ-Schedules

The schedules computed utilizing the long propagation delay in UWA networks using algorithms presented in [2], [3] are time-slotted. Moreover, these algorithms only accept integer propagation delays. The ρ-Schedule was first defined in [3, Section III] for a network with non-integer propagation delays. The network geometry is represented in the form of a delay matrix $\mathbf{D}$ in [3, Section III], where each element of the delay matrix contains the propagation delay between the corresponding transmitter-receiver pair. For a non-integer delay matrix $\mathbf{D}$, the elements can be rounded off to yield an integer delay matrix $\mathbf{D}'$. The approximated integer delay matrix $\mathbf{D}'$ is then used to compute the time-slotted ρ-Schedules. However, the ρ-Schedule presented in [3] assumes a single collision domain network whereas here we consider a multihop network with multiple partially overlapping collision domains.

Each transmitter node j on link (j,k) is associated with its collision domain. There are $N-\eta$ partially overlapping collision domains (also illustrated in [2, Section II]) due to $N-\eta$ transmitters. For each link $(j,k) \in \mathcal{L}_s$, we enumerate all the nodes in its corresponding collision domain and denote the set of these nodes by $\mathcal{I}_{jk}$. Hence, $\mathcal{I}_{jk}$ contains a list of nodes which lie in the interference range corresponding to node j's transmission to node k. Given the set, $\mathcal{I}_{jk} \ \forall (j,k) \in \mathcal{L}_s$, the values of ρ^+ and ρ^- are computed as follows:

$$\rho^+ = \max_{\forall (j,i)} (D_{ji} - D'_{ji}) \ \forall i \in \mathcal{I}_{jk} \text{ and } \forall (j,k) \in \mathcal{L}_s \tag{2}$$

$$\rho^- = -\min_{\forall (j,i)} (D_{ji} - D'_{ji}) \ \forall i \in \mathcal{I}_{jk} \text{ and } \forall (j,k) \in \mathcal{L}_s \tag{3}$$

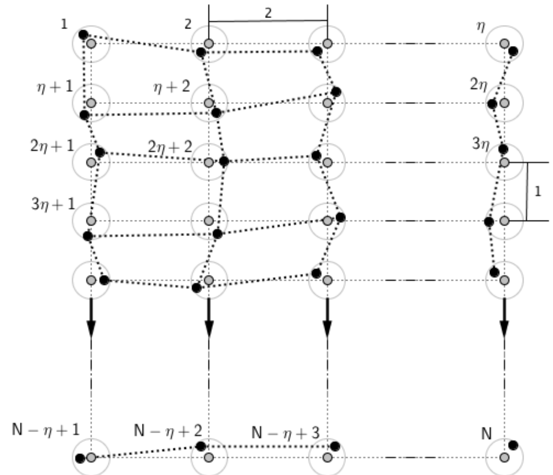

Fig. 1. The deployment of regular multiline grid networks at sea may result in slight irregularities in the grid network. The nodes are assumed to lie in a circle of radius r centered at the expected deployment locations.

where ρ^+ and ρ^- are the largest approximations made in the non-integer propagation delays while rounding-off to nearest integers. Since the schedules are time-slotted, let us denote the time slot length by τ. The duration for which the transmission slot is utilized for transmission in a ρ-Schedule was derived in [3], and is given by $t_{\text{pd}} = \tau(1-\rho^+-\rho^-)$. The guard times allocated at the start and end of the time slots to prevent collisions due to the approximations made in the propagation delays are $t_{\text{s}} = \tau\rho^-$ and $t_{\text{e}} = \tau\rho^+$ respectively. The throughput of a regular multiline grid N-node network with multihop relaying is upper bounded by $(N-\eta)/2$ [2, Th. 2], and there exists schedules which when adopted achieve this throughput. We can compute the throughput S_ρ of a ρ-Schedule as:

$$S_\rho = \left(\frac{N-\eta}{2}\right)\left(\frac{t_{\text{pd}}}{\tau}\right) = \frac{N-\eta}{2}(1-\rho^+-\rho^-). \tag{4}$$

IV. Problem Formulation

Consider a pair of links $(j,k),(l,i) \in \mathcal{L}_s$ such that node j starts transmitting a packet to node k at time t_{jk} with packet duration τ_{jk}, and node l transmits a packet to node i at time t_{li} with packet duration τ_{li}. Collision at receiver node i occurs if its desired message from transmitter node l overlaps with undesired message from node j's transmission to node k. Note that for node j's transmission not to interfere with node i's reception, either of the following conditions need to be satisfied [12]:

$$t_{jk} + \beta T + \tau_{jk} + D_{ji} \leq t_{li} + D_{li} \tag{5}$$

$$t_{jk} + \beta T + D_{ji} \geq t_{li} + \tau_{li} + D_{li} \tag{6}$$

$\forall \{(j,k),(l,i) \in \mathcal{L}_s | D_{ji} \leq \alpha D_{jk}\}$, where the condition $D_{ji} \leq \alpha D_{jk}$ is satisfied when node i lies in the interference range of node j. $\beta \in \mathbb{Z}$ is the integer constant which determines the number of adjacent frames in the past and future, that are taken into account. For example, setting $\beta = 0$ in (5) and (6) results in propagation delay constraints which avoid collisions within the current frame. In order to prevent the collisions from transmissions in previous frames, the constraints resulting from $\beta = -1$ can be considered. In [3], the optimal schedules for the network geometries result in the frame lengths which are at least greater than the girth G of the network: $G = \max_{(i,j)\in\mathcal{L}_s} \alpha D_{ij}$.With this reasonable assumption of limiting the value of frame length to be greater than the maximum girth among all collision domains, $T > G$, we limit our constraints to $\beta = 0$, $\beta = 1$ and $\beta = -1$, since in the worst case scenario, the interference is only limited within the next frame [12]. Note that, a collision domain can be identified with each transmitting node in the network. Since there are $N - \eta$ links to be scheduled in the set $\mathcal{L}_s$, there are $N - \eta$ partially overlapping collision domains associated with each transmitter. The message is considered as an interference at all other nodes (in the collision domain) except for the destination node. The transmission start times t_{jk}, t_{li} and corresponding packet duration τ_{jk}, τ_{li} must be chosen such that the desired message at node i is interference-free.

The average throughput S of a schedule with frame length T can be computed by summing the total reception (or equivalently transmission) time on all the nodes in the network in one frame duration T:

$$S = \frac{1}{T}\sum_{j=1}^{N}\Big[\sum_{(j,k)\in\mathcal{L}_s} \tau_{jk}\Big]. \tag{7}$$

A. Constraints

Each link $(j,k) \in \mathcal{L}_s$ must be scheduled atleast once in the frame. Note that the conditions listed in (5) and (6) form a set of disjunctive constraints which results in the feasible set forming a non-convex region over which the search for the solution is required. We use the Big-M transformation [13]–[15] to convert the disjunctive constraints (5) & (6) into conjunction and rearrange them as following for the values of $\beta = 0, 1$ and -1:

$$\begin{aligned}
t_{jk} - t_{li} + \tau_{jk} &\le -(D_{ji} - D_{li}) + Mp_{jk,li} && (8)\\
-t_{jk} + t_{li} + \tau_{li} &\le (D_{ji} - D_{li}) + M(1 - p_{jk,li}) && (9)\\
t_{jk} - t_{li} + \tau_{jk} + T &\le -(D_{ji} - D_{li}) + Mq_{jk,li} && (10)\\
-t_{jk} + t_{li} + \tau_{li} - T &\le (D_{ji} - D_{li}) + M(1 - q_{jk,li}) && (11)\\
t_{jk} - t_{li} + \tau_{jk} - T &\le -(D_{ji} - D_{li}) + Mr_{jk,li} && (12)\\
-t_{jk} + t_{li} + \tau_{li} + T &\le (D_{ji} - D_{li}) + M(1 - r_{jk,li}) && (13)
\end{aligned}$$

where $p_{jk,li}, q_{jk,li}$ and $r_{jk,li}$ are the binary variables[1] associated with each pair of disjunctive constraints considered $\forall (j,k),(l,i) \in \mathcal{L}$. A packet transmission in the current frame at time t_{jk}, in the worst case can cause interference till time $t_{jk} + \alpha D_{jk} + \tau_{jk}$. To prevent the interference caused by the packet transmission on the link (j,k) to cross the end of subsequent frame in future, we impose the following constraint:

$$t_{jk} + \alpha D_{jk} + \tau_{jk} < 2T. \tag{14}$$

[1]With the binary variable taking value 0 or 1 (e.g. $p_{jk,li} = 0$ or 1) along with a large enough value of parameter M, one of the constraints in the disjunctive pair becomes redundant. Note that smaller the value of M is, the tighter the Big-M reformulation can be. We select an arbitrarily large value of M for the transformation.

B. Objective function

In order to maximize the throughput and at the same time maintain the fairness among the nodes considered for scheduling, we can consider the objective of maximizing the minimum packet duration. Let the minimum packet duration be denoted by z, i.e., $z = \min\{\tau_{jk} | (j,k) \in \mathcal{L}_s\}$. Note that maximizing z, subject to condition $\tau_{jk} \ge z\ \forall (j,k) \in \mathcal{L}_s$, along with the propagation delay constraints will provide solutions in which the packet duration in the solution is unbounded above since $z \in \mathrm{R}^+$. To prevent this, we consider minimizing an objective similar to idle time [12] denoted by f as follows:

$$f = (N - \eta)T - 2(N - \eta)z. \tag{15}$$

The first term in (15) constitutes the total sum time available considering all $N-\eta$ transmitters in one frame and the second term constitutes the total sum time spent in transmissions and receptions on all the nodes in one frame for the worst case scenario. Based on the formalized objective and the constraints listed, the Mixed-Integer Linear Problem (MILP) is setup:

$$\begin{array}{llcl}
\min & & f & \\
\text{s.t.} & t_{jk} - t_{li} + \tau_{jk} - Mp_{jk,li} & \le & -(D_{ji} - D_{li})\\
& -t_{jk} + t_{li} + \tau_{li} + Mp_{jk,li} & \le & (D_{ji} - D_{li}) + M\\
& t_{jk} - t_{li} + \tau_{jk} + T - Mq_{jk,li} & \le & -(D_{ji} - D_{li})\\
& -t_{jk} + t_{li} + \tau_{li} - T + Mq_{jk,li} & \le & (D_{ji} - D_{li}) + M\\
& t_{jk} - t_{li} + \tau_{jk} - T - Mr_{jk,li} & \le & -(D_{ji} - D_{li})\\
& -t_{jk} + t_{li} + \tau_{li} + T + Mr_{jk,li} & \le & (D_{ji} - D_{li}) + M\\
& t_{jk} + \tau_{jk} - 2T & < & -\alpha D_{jk}\\
& -\tau_{jk} + z & \le & 0.
\end{array} \tag{16}$$

The optimization variables in (16) are T,z,t_{jk} and $\tau_{jk} \forall (j,k) \in \mathcal{L}_s$. The schedules computed by solving MILP result in the throughput S greater than the throughput S_ρ computed using ρ-Schedule, if and only if:

$$(1 - \rho^+ - \rho^-)T \le 2z. \tag{17}$$

To ensure $S \ge S_\rho$, the necessary condition (17) is derived using (4) and (7).

V. Results

We demonstrate the merits of the proposed unslotted schedules compared to the state of the art time-slotted ρ-Schedules, for practical implementations of UWA multiline multihop grid networks.

A. Demonstrating throughput gain

We generate 100 random instances of the grid networks with different number of nodes N with fixed $\eta = 3$. We vary the number of nodes N from 9 to 21. For each random instance of the grid network, the ρ-Schedule and its corresponding throughput is computed using (4). The MILP schedule is computed and the corresponding throughput is computed using (7). We use `MOSEK` optimizer with `MATLAB` on an iMac with 2.5 GHz Intel Core i5 quad-core processor to solve the MILP. The computation time was approximately 10 s for a grid network with 21 nodes and 18 links. The average throughput is calculated over 100 random instances and plotted in Fig. 2 as a function of N. The MILP schedules outperform the ρ-Schedules for all the network instances generated for different

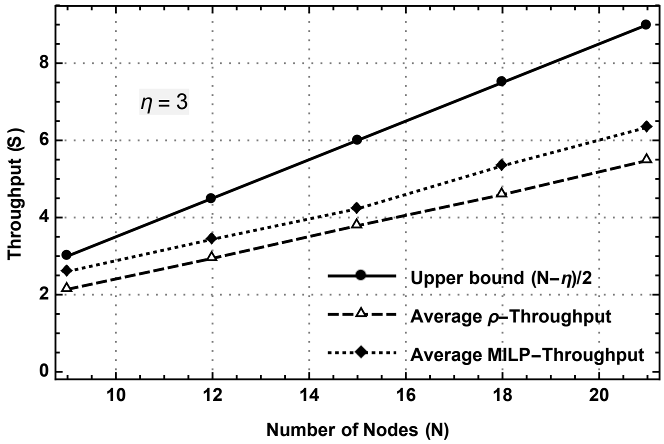

Fig. 2. Average throughput computed over 100 random instances of grid network for different number of nodes showing that the throughput achieved using MILP based unslotted schedules always outperform the time-slotted ρ-Schedules.

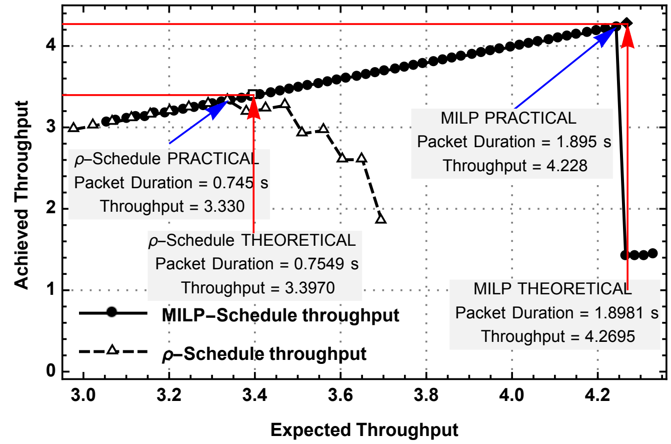

Fig. 3. Verification on UNET simulator for 12-node 3-line Multihop grid network.

number of nodes as shown in Fig. 2, i.e., $S_\rho \leq S$. To verify the schedules in the UNET simulator, we consider a particular random instance of 12-node 3-line multihop grid network and compute the ρ-Schedule and MILP schedule:

- *Computing ρ-Schedule*: The time-slotted schedule computed for a 12-node 3-line multihop grid network is shown in Table I. Note that the schedule has two transmissions per link in one frame in different time slots. The schedule can also be found in [2]. However, since the considered grid network for simulation does not have regular spacing as assumed in [2], the time slots cannot be fully utilized. ρ^+ and ρ^- are computed using (2) and (3). The values are computed as $\rho^+ = 0.0713$ and $\rho^- = 0.1738$ units of propagation delay. The corresponding packet duration is 0.7549 units. The frame length is $T = 4$ units and hence the throughput of the ρ-Schedule is $S_\rho = (\frac{12-3}{2})(1-0.0713-0.1738) = 3.3970$.
- *Computing MILP Schedule*: The solution to MILP results in unslotted transmission schedule presented in Table I. Note that in this case, there is only one transmission per link per frame with a packet duration greater than twice of each transmission duration when compared to the ρ-Schedule solution, also shown in Table I. The packet duration is computed and is equal to 1.8981 units for all scheduled links. Note that in an ideal scenario, the packet duration is 2 units. The frame length $T = 4.0011$ units is computed, the throughput S is computed to be 4.2695. There is a significant 25.7% increase in the throughput when compared to the throughput computed for ρ-Schedule. Fig. 2 shows this advantage as a function of the number of nodes.

B. Simulation setup

In the simulator, we consider the 12-node 3-line multi-hop grid network for which the solutions are presented in Table I. We set the sound speed $c = 1540$ m/s. For each transmission on a link (j, k) with distance d_{jk} between them, its interference extends up to a distance $2d_{jk}$. In simulation, the transmission schedule with packet duration 1.8981 units or less must not result in any packet collisions. To capture this, we increase the packet duration on all links expecting

TABLE I
ρ-SCHEDULE & MILP SCHEDULE

Link	ρ-Schedule		MILP Schedule
	First TX Start Time (s)	Second TX Start Time (s)	TX Start Time (s)
$(1,4)$	0	1	1.2008
$(2,5)$	2	3	3.0288
$(3,6)$	0	1	1.0058
$(4,7)$	0	3	3.9675
$(5,8)$	1	2	1.9670
$(6,9)$	0	3	0
$(7,10)$	2	3	3.0942
$(8,11)$	0	1	1.1116
$(9,12)$	2	3	2.8904

that the loss in throughput will occur due to packet collisions when the packet duration is increased beyond 1.8981 units for the MILP schedule. Note that increasing the packet duration increases the fraction of the total frame time that is utilized in transmissions. The packet duration is increased (see Fig. 3), the expected throughput is computed and the achieved throughput is observed. The throughput increases and reaches a different maximum value for the MILP schedule and the ρ-Schedule. The packet duration at which the maximum throughput is achieved matches the analytical throughput computed for both the cases as shown in Fig. 3. The sharp drop in throughput is due to the sudden increase in the number of packet collisions, attributed to the fact that even a partial overlap in the packets can cause a failure in the reception of both the packets. From Fig. 3, we can conclude that the unslotted schedule better utilizes long propagation delays in the grid network.

VI. CONCLUSION

We considered a practical multiline multihop underwater acoustic grid network with non-negligible propagation delay among the nodes and demonstrated that the MILP schedules resulting in unslotted transmission schedules are best suited to exploit large propagation delays and provide higher throughput when compared to the time-slotted ρ-Schedules.

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