Near-field/far-field source localization in ocean with an acoustic vector sensor array using polynomial rooting

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Abstract— In this paper, we consider the problem of three-dimensional (3D - azimuth, elevation/depth and range) localization of a single acoustic source using an array of acoustic vector sensors (AVS). Localization algorithms such as multiple signal classification (MUSIC) require a 3D search over the location parameter space which is computationally expensive. Several methods have been proposed based on arrays of acoustic pressure sensors (APS) [1], [2] which simplified this search to polynomial rooting (PR) combined with a 2D search in the range-elevation/depth space. However, the computational complexity remains high for such methods since 2D search is involved. We present a method to localize a source with an array of AVS via a decoupled estimation of azimuth, and a 1D range search combined with PR to estimate the elevation/depth. This is computationally simpler than, and performs as well as PR with 2D searching and 3D MUSIC.

***Index Terms*— acoustic vector sensor, MUSIC, localization, polynomial rooting.**

1. Introduction

Localization of acoustic sources is a crucial research problem that finds extensive applications in fields such as sonar, seismic exploration and room acoustics. It implies estimation of the 3D location coordinates (azimuth, elevation/depth and range) of a source. Localization using efficient methods such as maximum likelihood estimation and MUSIC involves a computationally expensive search for the coordinates in a 3D space. In order to mitigate this complexity, several methods were suggested to convert these searches into polynomial rooting (PR) procedures [1–4]. These algorithms are applicable for measurements obtained from an array of acoustic pressure sensors (APS).

Recently, a type of sensor known as an acoustic vector sensor (AVS) has been shown to perform superior to APS in localization [5]. AVS can measure the particle velocities as well as pressure at a point in space and estimate the direction of arrival (DOA) of a source unambiguously. The directionality of the AVS gives it an edge over APS in DOA estimation [5-7], detection [8],[9], tracking [10],[11] and communication [12] and its effectiveness has been demonstrated experimentally [13].

Some AVS-based algorithms are able to perform source localization without using a complex search. The methods in [6], [14], [15] are developed to use a single AVS. An iterative SAGE algorithm was presented for source localization in non-Gaussian noise [16]. An L-shaped AVS array was used for DOA estimation in [17] wherein the authors introduce an ingenious technique to reduce the DOA search into rooting procedures. The above mentioned algorithms suffer from one or more limitations: (i) not applicable to more than one AVS, (ii) computationally expensive or (iii) necessity to use an L-shaped array that may not always be feasible in practical implementations.

In this paper, we present a computationally simple MUSIC-based approach for AVS array-based localization in an ocean, that is inspired from and a simplified form of algorithms in [1],

[2]. Localization methods are presented in [1] to estimate spherical coordinates of near-field sources in homogeneous space, and in [2] for cylindrical coordinates of far-field sources in shallow ocean. These methods are faster than 3D MUSIC, but still considerably computationally complex due to the PR combined with search in a two-dimensional (2D) domain.

We present a novel technique by which a vertical linear array (VLA) of AVS can be used for acoustic source localization in a computationally simple way. We demonstrate this technique by deriving algorithms for three cases of localization in the ocean, encompassing near-field/far-field scenarios. A VLA is practical to construct and deploy, and is often more effective than a horizontal array for localization [16]. The method presented in this paper consists of a simple decoupled estimation of the source azimuth using a closed form expression. This is followed by estimation of source depth/elevation and range using PR combined with a 1D search over the source range space. In this paper, the unique directional manifold of an AVS is utilized to reduce the search complexity by one order of magnitude as compared to [1], [2]. Thereby, we tap into the directionality of the AVS to reduce the complexity, which cannot be done using the omni-directional APS. The presented method is also much simpler than a 3D search for the location estimates. The paper is organized as follows. Section 2 presents the data models for the problem of near-field/far-field localization of an underwater acoustic source. Section 3 presents the conventional 3D MUSIC method of localization. In section 4, we present our method for localization using an array of AVS, and in section 5 we present simulation results and conclusions regarding our method.

2. Data Model

We consider the problem of localization of a narrowband acoustic source using a VLA of N vector sensors. The topmost sensor (designated

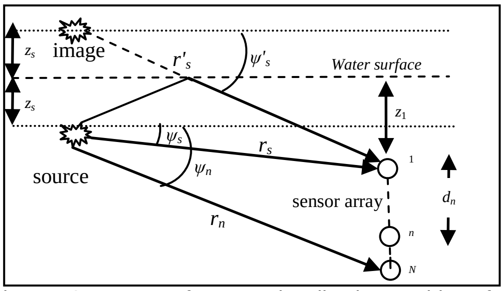

Fig. 1: Geometry of source localization problem for case (ii) when source/sensors are close to sea-surface. Reduces to case (i) when z_s, $z_1 \rightarrow \infty$

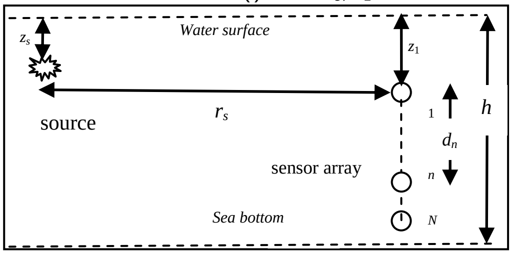

Fig. 2: Geometry of source localization problem in shallow ocean far-field scenario

as sensor 1) is the reference located at depth z_1. The n^{th} element of the VLA is located at a depth z_n and separated from the reference sensor by a vertical distance d_n. The source is located at a depth z_s, and an elevation ψ_n (measured with respect to the horizontal and positive when source is located higher than the sensor), range r_n and azimuth ϕ_s with respect to the n^{th} sensor. The range refers to absolute range in the case of spherical coordinates (for near-field), and horizontal range in cylindrical coordinates (far-field). The t^{th} snapshot of the $4N\times1$ measurement vector at the VLA is given as

$$\boldsymbol{y}(t) = [\boldsymbol{y}_1(t)^{\mathrm{T}}..\boldsymbol{y}_N(t)^{\mathrm{T}}]^{\mathrm{T}} = \boldsymbol{a}s(t) + \boldsymbol{e}(t),\ t=1,..,T, \quad (1)$$

where $\boldsymbol{y}_n(t)$ refers to the 4×1 measurement at the n^{th} AVS and $s(t)$ refers to amplitude of the source signal in the t^{th} snapshot. $\boldsymbol{e}(t)$ denotes a $4N\times1$ vector of additive environmental noise in the measurement of the t^{th} snapshot that is considered to be zero-mean white Gaussian noise. The steering vector $\boldsymbol{a}$ will be described now for three cases of the localization problem:

Case (i): Deep ocean near-field source

Assume the source is located in the near-field of the receiver array in an isotropic homogeneous medium away from the ocean boundaries (z_s, $z_1 \rightarrow \infty$). r_n and ψ_n are related to the location coordinates r_s and ψ_s of the source with respect to the reference sensor as

$$r_n(r_s,\psi_s) = \sqrt{r_s^2 + d_n^2 + 2r_s d_n \sin(\psi_s)} \ . \qquad (2)$$

In this case, the pressure measurement p_n at the n^{th} AVS in terms of a unit reference pressure can be found by assuming spherical spreading of the wave as [18]

$$p_n = e^{-jkr_n} / r_n \ , \qquad (3)$$

where k denotes the fundamental wavenumber of the acoustic wave. Let v_{xn}, v_{yn} and v_{zn} denote the measurements of the x, y and z particle velocities at the n^{th} AVS, scaled by the impedance ρc of the medium for homogeneity, where ρ is the density of water and c is the sound speed. There exists a relation between the scaled velocities and pressure measurement at the n^{th} sensor given by [18]

$$\begin{bmatrix} v_{xn} \\ v_{yn} \\ v_{zn} \end{bmatrix} = p_n \ . \ g(r_n) \begin{bmatrix} \cos(\phi_s)\cos(\psi_n) \\ \sin(\phi_s)\cos(\psi_n) \\ \sin(\psi_n) \end{bmatrix}, \qquad (4)$$

where

$$g(r_n) = \sqrt{1 + 1/(kr_n)^2}\, e^{-j\tan^{-1}(1/(kr_n))} . \qquad (5)$$

Thus, the 4×1 array manifold $\boldsymbol{a}_n$ of the field at the n^{th} sensor due to the source is given by

$$\boldsymbol{a}_n(r_n,\phi_s,\psi_n) = \begin{bmatrix} v_{xn} \\ v_{yn} \\ v_{zn} \\ p_n \end{bmatrix} = p_n \begin{bmatrix} g(r_n)\cos(\phi_s)\cos(\psi_n) \\ g(r_n)\sin(\phi_s)\cos(\psi_n) \\ g(r_n)\sin(\psi_n) \\ 1 \end{bmatrix} . \qquad (6)$$

From the geometry of the problem, it can be found that

$$\sin(\psi_n) = \frac{r_s \sin(\psi_s) + d_n}{r_n}, \cos(\psi_n) = \frac{r_s \cos(\psi_s)}{r_n} \ . \qquad (7)$$

Therefore from (2), (7), $\boldsymbol{a}_n(r_n, \phi_s, \psi_n)$ can be expressed as a function of spherical coordinates (r_s, ϕ_s, ψ_s) of the source. The overall $4N$×1 source steering vector of the VLA is

$$\boldsymbol{a}(r_s,\phi_s,\psi_s) = [\boldsymbol{a}_1^{\text{T}}(r_s,\phi_s,\psi_s),\ldots \boldsymbol{a}_N^{\text{T}}(r_N,\phi_s,\psi_N)]^{\text{T}} \quad (8)$$

Case (ii): Near-field source close to sea surface

When the source or sensors are located in a deep ocean but close to the sea-surface, the waves from the source reach the receiver through an additional path which involves reflection from the sea-surface. The contribution to pressure measurement at the sensors can be treated as being due to an image source placed at range r'_s, azimuth ϕ_s and elevation ψ'_s with respect to the reference sensor (note that the azimuth of the image remains the same as the true source). The geometry of the problem is illustrated in Fig. 2. Note that case (ii) reduces to case (i) when z_s, $z_1 \rightarrow \infty$. The pressure measurement p_n at the n^{th} AVS in terms of a unit reference pressure can be found as [19]

$$p_n = (e^{-jkr_n} / r_n) - (e^{-jkr_n'} / r_n') \ . \qquad (9)$$

When the effect of the image source is taken into account, the manifold $\boldsymbol{a}'$ of the field at AVS array for this case is given by

$$\boldsymbol{a}'(r_s, \phi_s, \psi_s) = \boldsymbol{a}(r_s, \phi_s, \psi_s) - \boldsymbol{a}(r'_s, \phi_s, \psi'_s), \quad (10)$$

where $\boldsymbol{a}(r_s, \phi_s, \psi_s)$ is given by (8). (r'_s, ψ'_s) can be easily found from the geometry of Fig. 2 as a function of (r_s, ψ_s) and known receiver depths as

$$\psi_s'(r_s,\psi_s) = \arg\left(r_s \cos(\psi_s) + j\left(2z_1 - r_s \sin(\psi_s)\right)\right), \quad (11)$$

$$r_s'(r_s,\psi_s) = r_s \cos(\psi_s) / \cos(\psi_s') \ . \qquad (12)$$

Case (iii): Shallow ocean far-field source

When the source is located in the far-field of the receiver array in a shallow ocean channel, the pressure field at the array can be expressed as a sum of M normal modes [20]. We consider the simple case of a homogeneous water layer of depth h over a fluid bottom half space. Here, it is no longer meaningful to talk in terms of the absolute range or elevation of the source, and the problem is described in terms of cylindrical coordinates (r_s, ϕ_s, z_s). The geometry of the problem is illustrated in Fig. 2. The measured field at the AVS array can be found as [8] [20]

$\boldsymbol{a}''(r_s, \phi_s, z_s) = [\boldsymbol{a}''_1{}^T(r_s, \phi_s, z_s), \ldots \boldsymbol{a}''_N{}^T(r_s, \phi_s, z_s)]^T$,(13)

where

$$\boldsymbol{a}''_n(r_s,\phi_s,z_s) = \frac{2\sqrt{2\pi}e^{\frac{j\pi}{4}}}{h}\sum_{m=1}^{M} p_{mn}\begin{bmatrix}\sin(\gamma_m z_s)\frac{k_m}{k}\cos(\phi_s)\\ \sin(\gamma_m z_s)\frac{k_m}{k}\sin(\phi_s)\\ \cos(\gamma_m z_s).\frac{-j\gamma_m}{k}\\ \sin(\gamma_m z_s)\end{bmatrix}. \quad (14)$$

In (14), k_m refers to the horizontal wavenumber of the m^{th} normal mode, $\gamma_m = m\pi/h$ is the vertical wavenumber, and

$$p_{mn} = \sin(\gamma_m z_n)\, e^{(-\delta_m + jk_m)r_s} \Big/ \sqrt{k_m r_s} \;. \quad (15)$$

3. Localization using 3D MUSIC

The 3D source localization problem in the first two cases consists in finding the estimates $(\hat{r},\hat{\phi},\hat{\psi})$ of the location parameters (r_s, ϕ_s, ψ_s). Using the asymptotically efficient MUSIC algorithm, the estimates can be found as the minimizers of the normalized null spectrum [1]

$$P(r,\phi,\psi) = \frac{\boldsymbol{a}^H(r,\phi,\psi)\mathbf{U}\mathbf{U}^H\boldsymbol{a}(r,\phi,\psi)}{\boldsymbol{a}^H(r,\phi,\psi)\boldsymbol{a}(r,\phi,\psi)} \quad (16)$$
$$= \bar{\boldsymbol{a}}^H(r,\phi,\psi)\mathbf{U}\mathbf{U}^H\bar{\boldsymbol{a}}(r,\phi,\psi),$$

where $-\pi<\psi<\pi$, $\psi \notin \left(\sin^{-1}(z_1/r_s), \pi - \sin^{-1}(z_1/r_s)\right)$, $0<\phi<\pi$, and

$$\bar{\boldsymbol{a}}(r,\phi,\psi) = \boldsymbol{a}(r,\phi,\psi) \Big/ \sqrt{\boldsymbol{a}^H(r,\phi,\psi)\boldsymbol{a}(r,\phi,\psi)} \quad (17)$$

is the normalized steering vector. $\mathbf{U}$ is the $4N\times(4N\text{-}1)$ matrix of noise eigenvectors obtained from the eigen-value-decomposition (EVD) of the data covariance matrix $\hat{\mathbf{R}}$ and corresponds to the ($4N$-1) lowest eigenvalues. The reason for choosing to normalize the range search spectrum is because if the un-normalized spectrum is used to estimate the range of the source, the estimates will be biased towards peaks corresponding to possible nearer sources due to spherical spreading [1]. This bias is corrected by scaling the null spectrum in accordance with the variation caused by spherical spreading. In the case (iii), the search involves a search over the depth $0<z<h$ instead of the elevation ψ.

The 3D MUSIC location estimation method described above involves minimization of (r, ϕ, ψ/z) in a 3D space. The complexity of the estimation process (ignoring preprocessing steps such as EVD) can be quantified in terms of the big O notation as O(JKL), where J, K and L denote the number of search points in the range, azimuth and elevation/depth space respectively. Since many search points are needed for higher accuracy, the 3D MUSIC algorithm is complex and impractical to implement.

4. Localization using simplified polynomial rooting

In [1] and [2], the authors present a method in which PR is used to estimate the elevation and depth respectively, and the azimuth and range are estimated by a 2D search. These methods are still somewhat tedious because in order to estimate elevation/depth, one has to perform a large number of PRs for every value of range and azimuth on a search grid. The complexity can be quantified as O(JK.rooting(Q)), where rooting(Q) represents complexity of rooting a Q^{th} order polynomial to replace an elevation/depth search. In this section, we present a novel technique for localizing a source using a procedure referred to as simplified polynomial rooting (SPR). First, the azimuth estimation will be dealt with. We then describe the estimation of the coordinates (r_s, ψ_s) for cases (i) and (ii), and the estimation of coordinates (r_s, z_s) for case (iii).

4.1 Azimuth estimate (all cases)

We propose further simplification of the localization from that in [1]/[2] by decoupling the azimuth estimation from the elevation/depth and range estimates. This decoupling makes use of the unique manifold of the AVS to perform separate estimation of the azimuth. Once this decoupling is done, the estimation of other location parameters using PR becomes simpler.

The decoupled azimuth estimate $\hat{\phi}$ is found using the closed form expressions in [15], which are originally presented for measurements from a single AVS. This method estimates the 4×4 covariance matrix $\mathbf{R}_4$ at a single sensor and uses it for localization. In the present context, we are dealing with data from an array of AVS and not a single AVS. Hence the first part of the method in this paper consists of estimating the azimuth, by adapting the expressions in [15] to use array data. This is described below.

Note that in case of data from a VLA, information on the azimuth is only embedded in the directional terms in the horizontal velocity measurements v_{xn} and v_{yn} and there is no change in azimuth with sensor number. Hence the azimuth can be estimated by treating the 4×1 measurement $\boldsymbol{y}_n(t)$ from each AVS as a separate data vector. Each 4N×1 array snapshot $\boldsymbol{y}(t)$ is split into N separate 4×1 data vectors $\{\boldsymbol{y}_1(t)..\boldsymbol{y}_N(t)\}$. Since T snapshots of $\boldsymbol{y}(t)$ are collected, we obtain a total of TN data vectors $\{\boldsymbol{y}_n(t), n = 1\ldots N, t = 1\ldots T\}$ which are used to estimate the 4×4 measurement covariance matrix at an AVS as

$$\hat{\mathbf{R}}_4 = \frac{1}{TN}\sum_{t=1}^{T}\sum_{n=1}^{N}\boldsymbol{y}_n(t)\boldsymbol{y}_n^H(t). \quad (18)$$

In this way, the data from an N sensor array is treated as N times the data from one sensor. Azimuth estimate $\hat{\phi}$ is obtained by decoupling it from the overall 3D search. This is facilitated by restricting the measurements used to v_{xn} and v_{yn}. The closed form estimate of azimuth is [15]:

$$\hat{\phi} = \tan^{-1}(l + \sqrt{l^2+1}),$$

$$\text{where } l = 0.5\ (|u_2|^2 - |u_1|^2)\big/ Re(u_1^* u_2), \quad (19)$$

where u_1 and u_2 correspond to first two elements of the signal eigen-vector $\boldsymbol{u}_\varphi = [u_1\ \ldots\ u_4]^T$ that corresponds to the largest eigen-value of $\hat{\mathbf{R}}_4$. The accuracy of the estimates is retained in spite of the decoupling [15]. This decoupled estimation is possible only due to the directional manifold of a VLA of AVS in which ϕ_s can be estimated from the horizontal velocities alone.

4.2 Elevation-range estimate (near-field case)

Once the azimuth estimate is obtained using (19), the problem reduces to estimation of ψ_S and r_S from the VLA data. This can be done by extending the approach presented in [3] which is described as follows. From the data models in sections 2, we observe that r_n, r'_n , ψ_n and ψ'_n are all periodic functions of ψ_s. Hence, the normalized steering vector $\bar{\boldsymbol{a}}(r,\hat{\phi},\psi)$ (for any r and ψ) is also a periodic function of ψ. Thus it can be expressed in a Fourier series of ψ as [3]

$$\bar{\boldsymbol{a}}(r,\hat{\phi},\psi) = \sum_{m=-\infty}^{\infty}\boldsymbol{c}_m(r,\hat{\phi})e^{jm\psi}, \quad (20)$$

where the 4N×1 Fourier coefficients $\boldsymbol{c}_m(r,\hat{\phi})$ are found as

$$\boldsymbol{c}_m\left(r,\hat{\phi}\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\bar{\boldsymbol{a}}\left(r,\hat{\phi},\psi\right)e^{-jm\psi}d\psi\ . \quad (21)$$

Since $\bar{\boldsymbol{a}}(r,\hat{\phi},\psi)$ is a smooth function of r, it is enough to use few Fourier coefficients to represent it. Let us assume 2M+1 coefficients are needed to satisfactorily represent $\bar{\boldsymbol{a}}$. Thus

$$\bar{\boldsymbol{a}}(r,\hat{\phi},\psi) \approx \sum_{m=-M}^{M}\boldsymbol{c}_m(r,\hat{\phi})e^{jm\psi}\ . \quad (22)$$

Construct the 4N×(2M+1) matrix

$$\mathbf{C}(r,\hat{\phi}) = [\boldsymbol{c}_{-M}(r,\hat{\phi}),\boldsymbol{c}_{-M+1}(r,\hat{\phi}),..\boldsymbol{c}_M(r,\hat{\phi})]. \quad (23)$$

From (22), $\bar{\boldsymbol{a}}(r,\hat{\phi},\psi)$ can be expressed as

$$\bar{\boldsymbol{a}}(r,\hat{\phi},\psi) = \mathbf{C}(r,\hat{\phi})\,\boldsymbol{b}(\psi), \quad \text{where} \quad (24)$$

$$\boldsymbol{b}(\psi) = [e^{-jM\psi},\ldots e^{jM\psi}]^T = [s^{-M},\ldots s^M]^T, \quad (25)$$

and $s = e^{j\psi}$. From (24) and (17), the normalized MUSIC spectrum in (16) can be expressed as

$$P(r,\psi) = \boldsymbol{b}^H(\psi)\mathbf{C}^H(r,\hat{\phi})\mathbf{U}\mathbf{U}^H\mathbf{C}(r,\hat{\phi})\boldsymbol{b}(\psi) \quad (26)$$

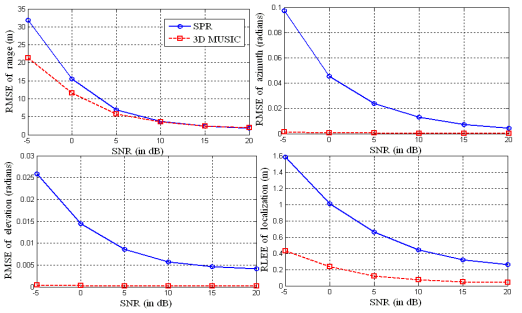

Fig. 3: RMSE of (a) range,(b) azimuth, (c) elevation estimates, (d) RLEE of overall localization vs. SNR, using 3D MUSIC and SPR in case (i):near-field.

or equivalently

$$P(r,s) = \boldsymbol{b}^T (1/s)\mathbf{C}^H(r,\hat{\phi})\mathbf{U}\mathbf{U}^H\mathbf{C}(r,\hat{\phi})\boldsymbol{b}(s). \quad (27)$$

Thus P can be expressed as a $2M^{\text{th}}$ order polynomial in s. In the case of infinite data snapshots (i.e., exact covariance matrix) and infinite number of terms of the Fourier series, the roots of $P(r, s)$ for the true value of range $r = r_s$ lie on the unit circle and correspond to $s = e^{j\psi_s}$. Hence the angles of the roots of (27) that are on the unit circle would correspond to the directions for which $P(r, s)$ is minimized. However, the roots may move off the unit circle due to the use of finite snapshots and the truncation of the Fourier series as done in (22). The elevation estimate $\hat{\psi}$ is thus obtained by rooting $P(r, s)$ and choosing the root closest to the unit circle which is the same as that obtained by minimizing $P(r, \psi)$ with respect to ψ.

By replacing the elevation search with PR for every value of search range r, we can considerably reduce the amount of computation involved. Using PR instead of searching in the ψ domain also improves the ability of the estimation algorithm to resolve closely spaced sources. This is because in the case of estimation of elevation using search, the resolution is decided by the width of the search points into which the elevation space is divided. The matrices $\mathbf{C}(r,\hat{\phi})$ can be prepared in advance, and the necessary pre-computed matrix to be used can be selected when $\hat{\phi}$ has been determined by (19). Hence these matrices do not require hectic on-line computation. The complexity of this method can be quantified as O(J.rooting($2M$)), which is lower than that of 3D-MUSIC or the methods in [1] and [2].

4.3 Depth and range estimate (far-field case)

This section is applicable for case (iii). Once the azimuth estimate is obtained using (19), the estimation of z_S and r_S from the VLA data can be done by extending the approach in [2]. The steering vector in (14) can be rewritten as

$$\boldsymbol{a}''_n(r_s,\phi_s,z_s) = q\sum_{m=1}^{M} p_{mn} \begin{bmatrix} (e^{j\gamma_m z_s} - e^{-j\gamma_m z_s})\frac{k_m}{k}\cos(\phi_s) \\ (e^{j\gamma_m z_s} - e^{-j\gamma_m z_s})\frac{k_m}{k}\sin(\phi_s) \\ (e^{j\gamma_m z_s} + e^{-j\gamma_m z_s})\frac{\gamma_m}{k} \\ (e^{j\gamma_m z_s} - e^{-j\gamma_m z_s}) \end{bmatrix}, \quad (28)$$

where q is a complex constant. Setting $s = e^{j\pi z_s/h}$, we can represent the overall steering vector $\boldsymbol{a}''(r_s,\phi_s,z_s)$ in (13) as

$$\boldsymbol{a}''(r_s,\phi_s,z_s) = \mathbf{C}''(r_s,\phi_s)\,\boldsymbol{b}''(z_s). \quad (29)$$

In the above expression,

$$\boldsymbol{b}''(z_s) = [s^{-M},\ldots s^{-1}, s, \ldots s^{M}]^T, \quad (30)$$

and the $(n,m)^{\text{th}}$ block of the $4N\times 2M$ matrix $\mathbf{C}''(r_s,\phi_s)$ is

$$[\mathbf{C}''(r_s,\phi_s)]_{nm} = \boldsymbol{c}_{nm}, \quad (31)$$

where the 4×1 block $\boldsymbol{c}_{nm}$ is defined as

$$\boldsymbol{c}_{nm} = \begin{cases} \frac{-p_{mn}}{k}[k_m\cos(\phi_s), k_m\sin(\phi_s), -\gamma_m, k]^T, & m \le M \\ \frac{p_{mn}}{k}[k_m\cos(\phi_s), k_m\sin(\phi_s), \gamma_m, k]^T, & m > M \end{cases}. \quad (32)$$

Thus, from (29), the MUSIC spectrum can be expressed as

$$P''(r,s) = \boldsymbol{b}''^T(1/s)\mathbf{C}''^H(r,\hat{\phi})\mathbf{U}\mathbf{U}^H\mathbf{C}''(r,\hat{\phi})\boldsymbol{b}''(s) \quad (33)$$

which is a $2M^{\text{th}}$ order polynomial in s. This estimation through PR is possible because the steering vector is a periodic function of the depth z. The size of coefficient matrix $\mathbf{C}$ is $4N\times 2M$ where M is the known number of modes. The strategy to find the roots is similar to that described in

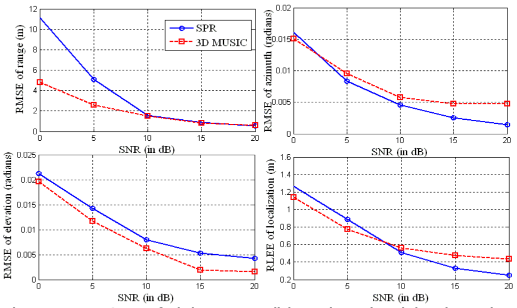

Fig. 4: RMSE of (a) range,(b) azimuth, (c) elevation estimates, (d) RLEE of overall localization vs. SNR, using 3D MUSIC and SPR in case (ii):near-field.

section 4.2. The estimate $\hat{z}$ of depth is found by finding the root *s* of the polynomial $P(r, s)$ lying closest to the unit circle but not exactly on it, and has a positive argument. From this, $\hat{z}$ is found from the relation $s = e^{j\pi z_s/h}$.

5. Results

We compare through simulations the performance of the SPR method presented in section 4 against that obtained from 3D MUSIC (section 3). We consider a VLA of N = 5 sensors located at azimuth 70.5^o with respect to a source of frequency 50 Hz. For cases (i) and (ii), we consider the source to be located at a range of 50 m and an elevation of -21^o. For case (iii), the source is at a range 4.5 km and depth of 5 m. The topmost sensor of the VLA is located at a depth of 5 m for cases (ii) and (iii). Estimation is done using 200 snapshots of data. In case (iii), a 3-channel AVS (consisting of pressure and horizontal velocity measurements) is used. The z-velocity measurement is not used as it does not contribute significantly to the performance and can be avoided [16].

In Fig. 3 we compare the performance of estimation of (a) range, (b) azimuth and (c) elevation in case (i), using 3D MUSIC and SPR. The comparison is done in terms of the variation of root mean square error (RMSE) of the estimates with variation in average signal-to-noise-ratio (SNR) at the array. The overall source localization performance gauged in terms of the relative location estimate error (RLEE) [15] is also plotted in Fig. 3 (d). It is observed that the SPR method presented in this paper yields good location estimates whose accuracy approaches that of 3D MUSIC as the SNR increases. It should be noted that the complexity of SPR is much lower than that of 3D MUSIC.

Figures 4 and 5 are similar to Fig. 3, but are plotted for cases (ii) and (iii) respectively. In Fig. 5(c), the RMSE of depth estimates is plotted instead of the elevation estimates. These figures show that in all the cases considered, SPR provides accurate location estimates which are comparable to 3D MUSIC and much simpler to execute. In Fig 4(b), RMSE of azimuth estimation of 3D MUSIC is worse than that of SPR at high SNR, because the accuracy of 3D MUSIC is limited by the resolution of its search bin (0.5^o in this case), whereas SPR has much better resolution. Thus, the RLEE of SPR is also better at high SNR in Fig. 4(d).

In general, when the steering vectors are periodic functions of location parameters, PR can be applied to simplify source localization. When an array of AVS are used, directionality of the sensors allows the azimuth to be estimated separately. This allows a faster localization algorithm with an array of AVS.

6. Conclusion

A computationally simple method to perform 3D localization of a source using an acoustic vector sensor array with simplified polynomial rooting is presented. This method is based on a decoupled estimation of the azimuth, and estimation of the elevation/depth and range using polynomial rooting combined with a 1D search. The computational complexity involved in this estimation is drastically reduced as compared to 3D MUSIC or polynomial rooting combined with 2D searching. The method is also efficient and accurate. The effectiveness of the algorithm for localization using a VLA is demonstrated for three cases of near-field/far-field sources. Note that the use of a VLA for localization is made possible due to the use of AVS, whereas in the case of a conventional

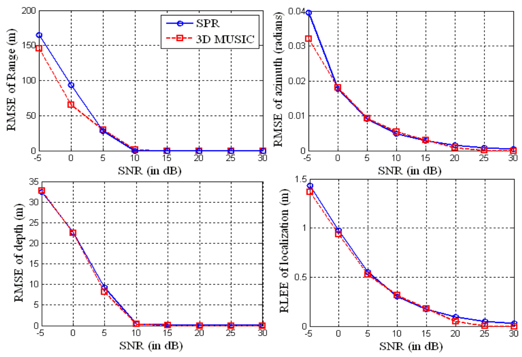

Fig. 5: RMSE of (a) range,(b) azimuth, (c) depth estimates, (d) RLEE of overall localization vs. SNR, using 3D MUSIC and SPR in case (iii): far-field.

APS array, it is not possible to employ a VLA as the azimuth cannot be estimated. This is an advantage of using AVS for localization, since a vertical array is convenient to deploy.

Our algorithm is presented for the example of Gaussian environmental noise. It can be extended to the case of impulsive noise that cannot be modeled as Gaussian, by using methods such as nonlinear preprocessors or fractional order statistics [10],[21] that are effective for environmental noise modeled as alpha-stable processes. It can also be extended to multiple sources, and be adapted for real-time implementation using fast recursive eigen-decomposition updating methods [22].

References

[1]H. S. Hung, S. H. Chang and C. H. Wu, “3-D MUSIC with polynomial rooting for near-field source localization,” in *Proc. ICASSP 1996*, vol. 6, pp. 3065–3068.

[2]J. Tabrakian and H. Messer, “Source localization in a waveguide using polynomial rooting,” *IEEE Trans. Sig. Proc.*, vol. 44, no. 8, pp. 1861–1871, Aug. 1996.

[3]A. J. Weiss and B. Friedlander, “Range and bearing estimation using polynomial rooting,” *IEEE J. Oceanic Engg.*, vol. 18, no. 2, pp. 130–137, Apr. 1993.

[4]M. Rubsamen and A. B. Gershman, “Root-MUSIC based direction-of-arrival estimation methods for arbitrary non-uniform arrays,” *Proc. ICASSP 2008*, no. 1, pp. 2317–2320.

[5]M. Hawkes and A. Nehorai, “Acoustic vector-sensor beamforming and Capon direction estimation,” *IEEE Trans. Sig. Proc.*, vol. 46, no. 9, pp. 2291–2304, Sep. 1998.

[6]Y. I. Wu and K. T. Wong, “Acoustic Near-Field Source-Localization by Two Passive Anchor-Nodes,” *IEEE Trans. Aero. & Elect. Sys.*, vol. 48, no. 1, pp. 159–169, Jan. 2012.

[7]M.Hawkes, A.Nehorai, “Wideband source localization using a distributed acoustic vector-sensor array,” *IEEE Trans. Sig. Proc*, vol. 51, no. 6, pp. 1479–1491, Jun. 2003.

[8]V. N. Hari, G. V. Anand and A. B. Premkumar, “Narrowband signal detection techniques in shallow ocean by acoustic vector sensor array”, *Dig. Sig. Proc.*, vol. 23 pp. 1645-1661, June 2013.

[9]V. N. Hari, G. V. Anand and A. B. Premkumar, “Narrowband detection in ocean with impulsive noise using an acoustic vector sensor array”, in *EUSIPCO*, 2012, pp. 1334–1339.

[10]X. Zhong, A. B. Premkumar and A. S. Madhukumar, “Particle Filtering and Posterior Cramér-Rao Bound for 2-D Direction of Arrival Tracking Using an Acoustic Vector Sensor,” *IEEE Sensors J.*, vol. 12, no 2, pp. 363–377, Feb. 2012.

[11]X. Zhong and A. B. Premkumar, “Particle filtering approaches for multiple acoustic source detection and 2-D direction of arrival estimation using a single acoustic vector sensor,” *IEEE Trans. Sig. Proc.*, vol. 60, pp 4719-4733, 2012.

[12]A. Song, A. Abdi, M. Badiey and P. Hursky, “Experimental Demonstration of Underwater Acoustic Communication by Vector Sensors,” *IEEE J. Oceanic Engineering*, vol. 36, no. 3, pp. 454–461, Jul. 2011.

[13]M. Porter, B. Abraham, M. Badiey, M. Buckingham, T. Folegot, P. Hursky, S. Jesus, K. Kim, B. Kraft, V. McDonald, C. DeMoustier, J. Presig, S. Roy, M. Siderius, H. Song, and W. Yang, “The Makai experiment: High frequency acoustics,” in *ECUA*, 2006, pp. 9–18.

[14]P. Tichavsky, K. T. Wong and M. D. Zoltowski, “Near-field/far-field azimuth and elevation angle estimation using a single vector hydrophone,” *IEEE Trans. Sig. Proc.*, vol. 49, no. 11, pp. 2498–2510, Nov. 2001.

[15]V. N. Hari, A. B. Premkumar and X. Zhong, “A Decoupled Approach for Near-Field Source Localization Using a Single Acoustic Vector Sensor,” *Circuts, Systems and Signal Proc.*, vol. 32, no. 2, pp. 843–859, Oct. 2012.

[16]Z. Madadi, G. V. Anand and A. B. Premkumar, “3-D source localization in shallow ocean with non-Gaussian noise using a linear array of acoustic vector sensors,” in *Proc. 11th ISSPA*, 2012, pp. 1353–1358.

[17]K. T. Wong and M. D. Zoltowski, “Root-MUSIC-based azimuth - elevation angle - of - arrival estimation with uniformly spaced but arbitrarily oriented velocity hydrophones,” *IEEE Trans. Sig.Proc*, vol. 47, no. 12, pp. 3250–3260, Dec. 1999.

[18]Y. I. Wu, K. T. Wong, and S. Lau, “The Acoustic Vector-Sensor’s Near-Field Array-Manifold,” *IEEE Trans. Sig. Proc.*, vol. 58, no. 7, pp. 3946–3951, Jul. 2010.

[19]F. B. Jensen, W. A. Kuperman, M. B. Porter, and H. Schmidt, *Computational Ocean Acoustics*. New York, NY: Springer New York, 2011.

[20]L. M. Brekhovskikh and Y. P. Lysanov, *Fundamentals of Ocean Acoustics*. Berlin, 1991.

[21]V. N. Hari, G. V. Anand and A. B. Premkumar, “Preprocessor based on suprathreshold stochastic resonance for improved bearing estimation in shallow ocean,” in *OCEANS 2009*, 2009, no. 65, pp. 1–8.

[22]M. Moonen, F. J. Vanpoucke, and E. F. Deprettere, “Parallel and adaptive high-resolution direction finding,” *IEEE Trans. Sig. Proc.*, vol. 42, no. 9, pp. 2439–2448, 1994