Monostatic Sonar Performance Using Pulse Compression Waveforms

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***Abstract*—We compare pulse compression waveforms of different categories in terms of their ambiguity function characteristics and performance in detection and range estimation. How well these characteristics translate into sonar performance in shallow waters is studied via simulations. We select three operating scenarios to compare the performance of these waveforms, namely, short range, long range and detection of multiple targets that are close to each other. Our comparison indicates the relative strengths and weaknesses of these waveforms. The results indicate that some aspects of the relative sonar performance of these waveforms can be gauged from their ambiguity function characteristics, though not always in a straightforward manner. Thus, this gives us a methodology to select suitable waveforms for sonar operation without the need to perform computationally intensive Monte Carlo simulations.**

I. Introduction

Underwater active sonar involves transmitting sound waves and observing received echoes for target detection or range/speed estimation. The signals used for transmission can usually be categorized into continuous wave (CW), frequency hopped, frequency modulated (FM) or pseudo random noise (PRN) types. Challenges associated with shallow water sonar processing arise from factors such as multipath caused by the ocean channel, ambient noise and reverberation. The type of signal used for transmission and signal processing at the receiver plays a crucial role in system performance.

Various works have focused on signal design in order to improve sonar system performance. In [1], the authors compare the CW, FM and PRN categories of signals and some of their relative advantages and disadvantages in simulation using wideband ambiguity function. The authors also present a new type of Doppler-selective signal called cutFM for the underwater scenario, and test its performance to detect moving objects in air. FM signals such as the linear FM (LFM) and hyperbolic FM [1] have been studied for quite some time due to their high time-bandwidth product (TBP) and good range resolution. PRN such as M-sequences, Gold codes and Kasami codes [1], [2] have also received some attention, and have been deployed in real scenarios as well [3]. PRN design techniques for customizing the ambiguity function (AF) have also been developed. The authors in [4] designed a cyclic algorithm-new sequence which has good autocorrelation properties with lower sidelobes. They show that when nearby targets are in the same Doppler bin and have negligible Dopplers, it is better than PRN and random phase sequences for wideband scenario. Authors in [5] discussed frequency hopped waveforms based on Costas codes. They claim that a waveform consisting of frequency-hopped CW sub-pulses based on Costas code (CCW) has range and Doppler resolutions comparable to FM pulse of the same bandwidth and CW waveform of the same duration respectively. They tested the CCW using a broadband propagation model in the presence of white Gaussian noise (WGN) for shallow and deep water environments and also using data from sea trials. They concluded that the CCW can provide good range and Doppler resolution in real-world scenarios. In [6] the CCW was modified to obtain a signal consisting of frequency hopped LFM sub pulses based on Costas code (CLFM). The authors claim it has better range and Doppler resolutions and a high TBP with a smaller code length. Some works have attempted to design composite waveforms, such as [7], which combined a binary phase shift keying (BPSK) and LFM signal sequentially. However, the performance of this signal was not tested for realistic underwater channel settings.

Though a lot of work in the sonar literature focuses on signal design, there is scant work on comparing their performance [1], [8], [9]. The report in [8] discusses some categories of sonar pulses and their relative advantages and applications, without delving into a numerical evaluation. In [9], CW, LFM and Barker-coded BPSK pulse are evaluated in terms of their detection performance in high frequency shallow water channel with WGN. The paper shows that MF gain plays an important factor in determining waveform performance.

Though signal design approaches often focus on enhancing certain properties and metrics based on the signal AF, this does not necessarily translate into the expected performance. One main reason for this is that practical scenarios may not adhere to the idealistic assumptions used in predicting performance from signal properties. The increased challenges that occur in a realistic setting need to be considered in performance comparison. Moreover, the effect of signal properties varies in different application scenarios, and these factors must be taken into account. We undertake a comparison of different categories of signals in terms of their AF properties, and investigate how this translates into performance in a warm shallow water operating scenario, based on a simulation study. In particular, we focus on the signals' performance in terms of detecting submerged targets and estimating their range.

We compare waveforms with high TBP, also called pulse compression waveforms, in terms of their AF properties and performance in detection and range estimation. These waveforms are often used in active sonar scenarios as they can achieve high SNR at the output of the MF by frequency or phase modulation that gives them better MF gain [10]. They are generally able to achieve better range resolution due to larger bandwidths. The four waveforms we compare are: M-sequence (representative of the PRN category of signals), LFM (representative of FM signal category) [1], CCW and CLFM (from the category of frequency hopped signals) [11].

The layout of the paper is as follows. In section II, we define and discuss some AF characteristics that help in gauging sonar performance of these waveforms. In section III we show results from simulations undertaken to evaluate some sonar performance metrics of these signals. Finally we conclude the paper in section IV.

II. Ambiguity function characteristics

An AF is a signal's autocorrelation output as a function of delay and Doppler. In many active sonar systems, the bandwidth of the transmitted signal is a considerable fraction of the carrier frequency. In these scenarios the effect of target's Doppler cannot be approximated by simple frequency shift therefore the wideband AF is defined in terms of delay, τ and Doppler scaling factor, η [12]. The scaling factor η represents compression (or stretching) of a transmitted signal, after reflection from a moving target [1]. For wideband signal $s(t)$, assuming receiver is static, AF is defined as follows.

$$\chi_s(\tau,\eta) = \int_{-\infty}^{\infty} s(t)s^*(\eta(t-\tau))\mathrm{d}t \tag{1}$$

$$\eta = \frac{1+v/c}{1-v/c} \approx 1 + \frac{2v}{c} \tag{2}$$

where v is the radial velocity of the target towards the receiver and c is the speed of sound in water. The magnitude of the AF i.e., $|\chi_s(\tau,\eta)|$ is often of more relevance to us. The AF magnitude plot versus delay and Doppler is shown in Figs. 1–4 for the CCW, CLFM, LFM and M-sequence signals of equal bandwidths and equal pulse-lengths. The parameters used to generate these signals, are discussed in detail in the following subsection II-A.

Some AF characteristics of the above-selected waveforms are as follows. The LFM pulse yields good range resolution but suffers from a range-Doppler ambiguity. It has a thin ridge over the velocity-range space and hence it is Doppler invariant. M-sequence offer nearly a thumbtack-like response in range-velocity plane but suffer from sidelobes in their MF response. Frequency hopped signals don't have a true thumbtack AF, but rather have a narrow peak at the origin and low sidelobes elsewhere. The AF of CCW and CLFM have small ridges along the Doppler axis which fall off fast. Frequency hopped signals like CCW and CLFM provide a good trade-off between LFMs and M-sequences in terms of the AF properties.

A. System parameters

The simulation parameters are as follows. All the signals have the same length i.e., $l = 0.5$ s and bandwidth, $B = 512$ Hz. The carrier frequency is $f_c = 2.56$ kHz and the sampling frequency is $f_s = 25.6$ kHz. For CCW, for each of the sub-pulse duration t_p, a single frequency tonal f_m is

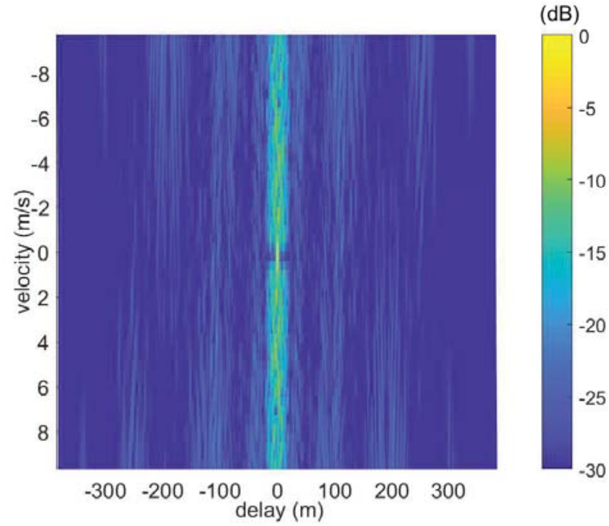

Fig. 1. Ambiguity diagram of CCW.

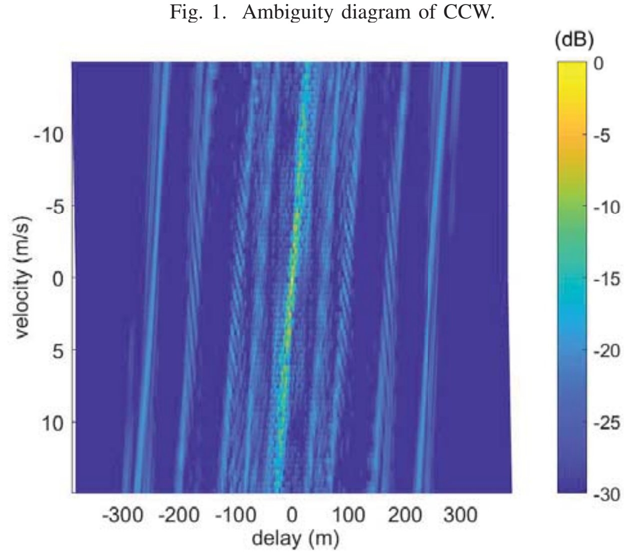

Fig. 2. Ambiguity diagram of CLFM.

emitted and used only once. The frequencies in CW sub-pulses are hopped based on the Costas code. We use a code of length $n = 16$. To design CLFM, the constraints on sub-pulse bandwidth and frequency spacing are determined to sure that the first grating lobe occurring due to overlap between non-orthogonal LFM sub-pulses, is eliminated by the m^{th} null of the the LFM sub-pulse's ACF [11]. The signal parameters such as bandwidth B_p, frequency spacing $\triangle f$ and m are designed based on the methodology outlined in [11]. These design procedure makes sure that grating lobes in the signal AF are minimized. We use a code length of $n = 10$ for CLFM waveform.

B. Signals' AF properties

Characteristics of the transmit signal such as its AF shape, range and Doppler resolutions can influence its performance. A signal's ability to resolve target is generally thought to be dependent on the widths of the mainlobe of the AF along the delay or Doppler axes. We study this further later in the

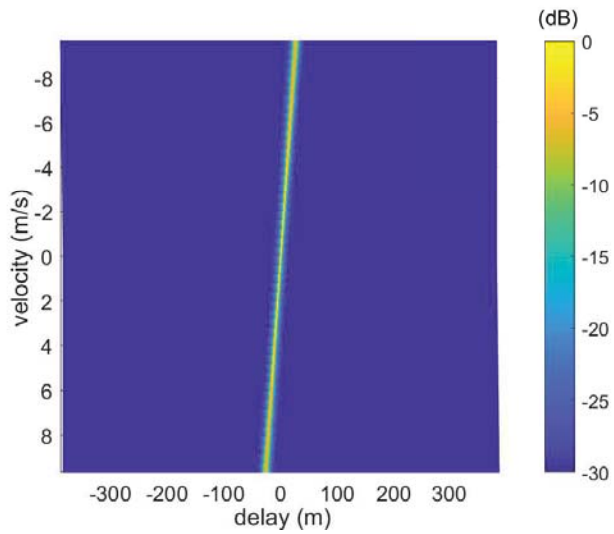

Fig. 3. Ambiguity diagram of LFM.

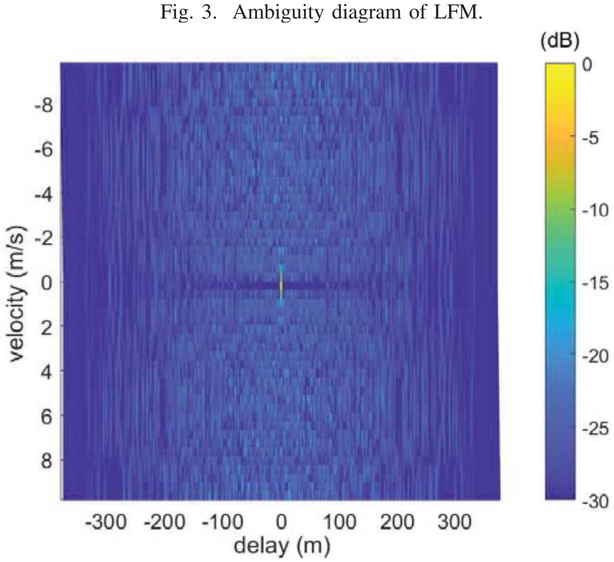

Fig. 4. Ambiguity diagram of M-sequence.

paper. The widths along these axes define resolution of a given waveform, and the height of the sidelobes affects its performance in clutter, or how well it can resolve multiple targets.

We compare the AF properties of the four waveforms and compute their AF characteristics namely, range resolutions and sidelobe floor within an operating region of interest. These properties are further defined as follows:

- Range resolution: It is defined as the width of the main lobe along the delay axis or zero-Doppler cut of the AF. It is also a gauge of the waveform's ability to estimate the target range. We consider range resolution defined in terms of the 3 dB and 10 dB widths of the main lobe along the zero-Doppler cut of the AF.
- The sidelobe floor: It may influence multiple target detection as weak targets can be masked by the sidelobes of stronger targets and hence remain undetected in their vicinity. We compute three aggregate characteristics describing the sidelobe floor namely mean, median, and maximum sidelobe floor, within the delay span of 20 to 385 meters, where 385 metres corresponds to the maximum range of the pulse.

The theoretical expression for the range resolution of M-sequence and LFM is given as $0.88c/2B$ in [13]. The theoretical range resolution of CCW and CLFM is obtained from the expression for the composite ACF of these waveforms [11], described as

$$|\chi(\tau)| \simeq \left|\chi_s(\tau)\frac{\sin(n\pi\tau\triangle f)}{\sin(\pi\tau\triangle f)}\right|, |\tau| \leq t_p \tag{3}$$

where τ represents the time delay, t_p is the sub-pulse duration, $\chi_s(\tau)$ is the ACF of the CW or LFM sub-pulses, corresponding to the CCW or CLFM respectively. The signals parameters used by us to evaluate these expressions are described in subsection II-A.

The range resolutions computed from cuts on the the broadband ACF and the theoretical expressions, as well as the sidelobe floor of the four waveforms, are tabulated in Table I. We see that the true range resolution of these waveforms measured as 3 dB cuts, differs from the theoretically predicted values. This is because the theoretical predictions are based on the assumption of narrowband signals. However, this assumption does not work satisfactorily in the scenario currently considered by us. We note that though all the signals operate in the same bandwidth, they vary in their range resolution. The M-sequence has the best range resolution among all the four waveforms. CLFM and CCW have range resolutions that are close to the M-sequence and LFM has the poorest among all.

Doppler resolution is another signal property that is often desired when Doppler estimation is required. However, the performance metrics considered by us in this paper are not affected by this property, so we do not consider it in our analysis. We see that the order of performance of these waveforms is different if we consider median, mean or maximum aggregates to describe the sidelobe floor. In subsection III-D, we see that the median sidelobe floor is a good indicator of detection of multiple targets located nearby. Table I shows that the LFM has the lowest sidelobe floor in terms of all aggregates.

III. Sonar performance in different operational scenarios

We focus on warm shallow water operation of active sonar. This acoustic channel has two features, extensive time-varying multipath delays and high levels of non-Gaussian ambient noise due to snapping shrimp [14]. Three types of target scenarios are considered:

- Single target, at short range of 1 km.
- Single target, at long range of 6 km.
- Multiple targets located close to each other, at medium range of 4 km.

We further characterize the sonar performance degradation that can occur in a more realistic experimental scenario, as compared to an ideal favorable one, in the following two ways:

TABLE I
SIGNAL AF PROPERTIES. SIGNAL PARAMETERS: l = 0.5 S, f_c = 2.56 KHZ, B = 512 HZ, f_s = 25.6 KHZ

Signals	Range resolution (m)			Sidelobe level (dB)		
	Simulated		Theoretical			
	3 dB	10 dB	3 dB	median	mean	max
M-sequence	1.46	2.64	1.32	-31.6	-30.8	-18.4
LFM	1.81	2.77	1.32	-56.3	-51.7	-22.4
Costas CW	1.74	2.65	1.28	-33.8	-32.7	-16.9
Costas LFM	2.23	3.43	1.31	-36.3	-32.1	-16.1

- We undertake a performance analysis using not only with simulated noise, but also with recorded ambient noise from Singapore waters. The statistical characteristics of the noise in these waters deviate from conventional noise models as they are highly non-Gaussian [14].
- We incorporate channel variability due to multipath encountered in shallow water channels. We do this using a channel simulator that incorporates fluctuations caused by time-varying multipath and fading [15].

The received echoes at the sonar are modeled based on a two-way propagation through the channel from transmitter to the target and back. The channel model is based on ray tracing and includes time-varying statistical effects in an iso-velocity channel of constant depth. The time-varying amplitude variation of each multipath is modeled by a Rayleigh random process and variations in each multipath's delays are modeled by a Gaussian random process [15]. The parameters of the channel used in simulations are as follows. The speed of sound in water is set at c =1540 m/s, speed of sound in sediment is set at c_1 =1650 m/s, channel depth is set at 20 m and source, receiver and target depths are set at 10 m. Attenuation factor in sediment is set at α = 0.1, density of sound in sediment is set as ρ_1 =1200 kg/m^3 and absorption coefficient in water is calculated from [16].

A. Performance metrics

For all the four waveforms and the three operational scenarios, we study and evaluate two main performance metrics, namely, range accuracy and probability of detection P_{D} elaborated below:

- Range accuracy: We analyze the signals in terms of their ability to accurately estimate the range of the target from the simulated received data. Root mean square error (RMSE) of the range estimate (in meters) is computed by employing Monte Carlo simulations.
- Probability of detection: We evaluate the P_{D} of targets at their true ranges, based on a basic binary hypothesis formulation [10].

For a fixed SL, the SNR of the received signal is computed from the active sonar equation [13] as, SNR = SL-2TL+TS-NL, where SL is the transmitted source level, TS is target strength (set to 10 dB), TL is the transmission loss and NL is the in-band noise level set to 98 dB re 1μPa as per the Wenz curves [17]. We consider a single receiving hydrophone with no directionality gain. As the channel exhibits variability, the TL and hence the received SNR are also variable. Hence, rather than considering SNR, we compute the performance metrics at certain selected values of the *average* SNR of the simulation setup, computed for the SL we set. For a short range (1 km), the average low SNR considered by us is -3 dB and SL required to achieve this average SNR is 182 dB. For long range (6 km), SL required to achieve the average SNR of -3 dB is 206 dB. The SNRs are obtained by adding WGN or ambient noise recorded in Singaporean waters, to the simulated received signal data in an appropriate ratio.

B. Short range scenario

In the short range scenario, the performance degradation may arise due to both low SNR or channel effects. Thus, we aim to study the relative influence of low and high SNR and channel effects, on the sonar performance. We characterize the range accuracy with and without channel effects in recorded ambient noise for two SNR cases. The scenario with no channel effects, captures an idealized situation where the signal's AF properties may translate into performance in a more straightforward way, whereas the scenario with channel effects is more realistic. We also compute the SL required to attain a P_{D} of 0.9, when channel effects and ambient noise are considered. These results are tabulated in Table II for all the waveforms.

Comparing Table II to Table I, we observe that the range resolution property of a signal influences the range accuracy performance at short range for the high SNR case. M-sequence has the best range accuracy and its range resolution is also the best amongst the signals considered. However, the LFM exhibits poor range accuracy, despite its range resolution being comparable to the M-sequence and CCW. This is due to its ridge-shaped AF which shows a strong range-Doppler ambiguity. Hence, apart from the range resolution, the shape of the AF can also influence range accuracy up-to a degree. We also note that the range estimation performance with channel effects is worse by up-to two orders of magnitude than when the channel effects are not considered. This is because of the effect of multipath which degrades the performance. In the low SNR case, the range estimation is not dictated by range resolution alone, and the signal's robustness against noise comes into play.

TABLE II
SIMULATED SHORT RANGE (1 KM) TARGET SCENARIO WITH RECORDED NOISE ADDED.

Signals	RMSE (m) Average SNR = 10 dB (high)		RMSE (m) Average SNR = -3 dB (low)		SL (dB) at $P_{\mathrm{D}} = 0.9$ and $P_{\mathrm{FA}} = 10^{-3}$
	with channel	without channel	with channel	without channel	with channel
M-sequence	1.37	0.03	266.6	9.46	203.9
LFM	5.12	0.21	135.9	9.49	202.8
Costas CW	1.79	0.03	188.9	17.1	203.5
Costas LFM	2.07	0.03	173.3	19.3	204.3

In terms of P_{D} we see that for short range of 1 km, LFM performs the best, 1.1 dB better than M-sequence, 0.7 dB better than CCW and 1.5 dB better than CLFM, indicating that its AF shape gives it an edge in detection though not in terms of range accuracy.

When the SNR is low, ambient noise and the channel multipath, both affect the detection performance. It is hard to broadly conclude which of these is a more serious issue for the short range scenario. However, the SNR can be expected to be high when dealing with practical short range scenarios, and sonar systems are more affected by channel multipath.

C. Long range scenario

In the long range scenario, detection of echoes from the target is usually limited more seriously by high noise levels (low SNRs). Thus, in this scenario, we aim to characterize the effect of noise and robust receiver processing on the sonar performance. With this in mind, we consider two scenarios, namely the case when generated WGN is added, and the case when noise recorded in Singapore waters is added to the data. All the results are computed with the channel effects incorporated. In Table III, we tabulate simulation results for the long range scenario, in terms of the RMSE of the range estimate and the SL required to attain a P_{D} of 0.9.

We note that signals' range resolution property translates into the range accuracy in the WGN noise case, similar to that observed in the short range scenario. However, we note that the performance difference between M-sequence, CCW and CLFM is lesser in this case than it was for the short range scenario. This highlights that the effect of multipath in long range scenario is being a more detrimental factor for range accuracy, than the range resolution.

When recorded noise is considered, the range accuracy using the conventional MF is severely degraded. In this case, the presence of non-Gaussian noise further impedes the range accuracy for all the waveforms, even if the average SNR considered is the same as that for WGN case. In order to confirm whether the impact can be attributed to the non-Gaussian noise, we consider an additional set of results where we perform detection of signal in recorded noise, using a detector that is robust to the non-Gaussian noise, namely the sign detector (SD) [14]. It is interesting to note that when we

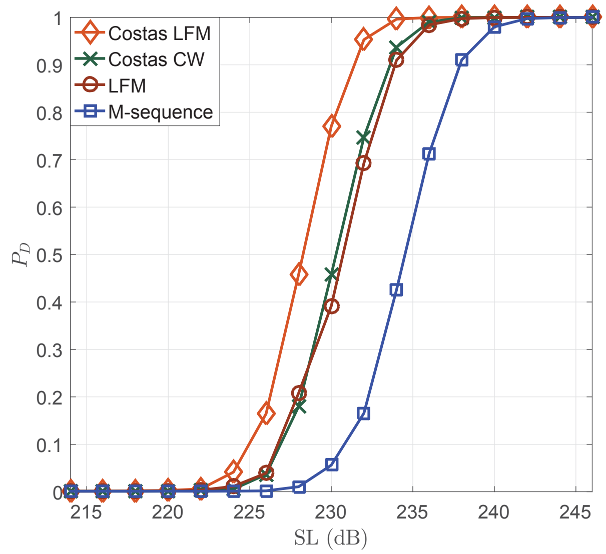

Fig. 5. Comparison of P_{D} vs. SL of signals for long range single target detection.

TABLE III
SIMULATED LONG RANGE (6 KM) TARGET SCENARIO WITH CHANNEL EFFECTS INCORPORATED.

Signals	RMSE (m) Average SNR= -3 dB			SL (dB) at $P_{\mathrm{D}} = 0.9$ and $P_{\mathrm{FA}} = 10^{-3}$
	WGN (MF)	recorded noise (MF)	recorded noise (SD)	recorded noise
M-sequence	0.74	809.5	0.75	237.8
LFM	2.13	618.6	2.09	233.8
Costas CW	0.76	507.8	0.77	233.4
Costas LFM	0.74	642.5	0.77	231.2

use the SD on the data with the recorded noise, the results are quite comparable to that observed using the MF for WGN. This shows that the non-Gaussian nature of the noise plays a significant part in the degradation of sonar performance. This is in agreement with work done on detection in impulsive noise described in [15], [18].

The detection performance with MF for the long range scenario is shown in Fig. 5, for the four waveforms. The differences in performances of the waveforms is more stark as compared to the short range scenario. From the Table III, the CLFM performs the best for long range detection, indicating its robustness to noise, whereas the performance of the M-sequence is lagging behind that of other waveforms by 4-6 dB.

D. Multiple target scenario

In order to test the effect of the sidelobe floor on detection, we consider two targets whose target strengths differ by 10 dB for a 4 km medium range scenario. The targets are placed within d meters of each other, where d is chosen randomly within the range from 20 to 385 meters, for each Monte

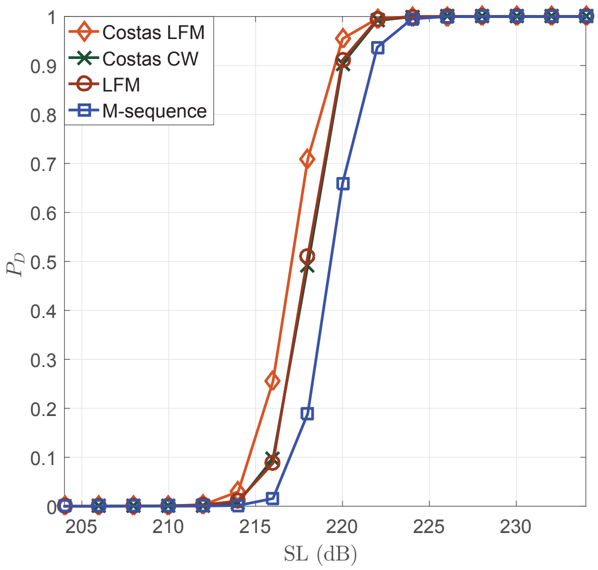

Fig. 6. Comparison of P_D vs. SL of signals for medium range (4 km) detection of multiple targets, with difference of 10 dB in target strengths.

Carlo simulation. This way, we capture the overall effect of sidelobes of a strong target on detecting a weaker target located anywhere within its clutter plateau, but beyond its main lobe. The detection performance is calculated in terms of the P_D for detecting two peaks simultaneously at the respective target locations. We tabulate the SL required to attain $P_\mathrm{D} = 0.9$ for the four signals in Table IV. In Fig. 6, the variation of P_D with SL is shown.

Firstly, we note that the CLFM performs the best in multiple target detection amongst all the waveforms considered, outperforming the M-sequence by 2.3 dB. Amongst the AF characteristics tabulated in Table I, the median sidelobe floor is the most effective aggregate indicator of multiple target detection when target two is located anywhere within the sidelobe floor of target 1.

Interestingly the conventional LFM signal, despite having minimal sidelobe levels in its AF, is marginally inferior to the CLFM in detecting multiple targets. This is because the performance in Table III depends not only on the sidelobe level, but also on the overall detection performance of the waveform for each target. The combined effect of sidelobe levels (Table I) and distant target detection with channel and noise effects (Table II) sees the CLFM performing better.

IV. Conclusions

We studied four waveforms in terms of their AF properties and their performance metrics for active sonar operation in a warm shallow water acoustic channel. The AF properties studied were range resolution and sidelobe floor, and the performance metrics considered were range accuracy and probability of detection.

The AF characteristics can be used as simple indicators of sonar performance without having to undertake computationally intensive simulations. For example, the median

TABLE IV
Simulated medium range (4 km) multiple target Scenario with channel effects incorporated and recorded noise added.

Signals	SL (dB) at $P_\mathrm{D} = 0.9$ and $P_\mathrm{FA} = 10^{-3}$
M-sequence	221.6
LFM	219.9
Costas CW	220.0
Costas LFM	219.3

sidelobe level computed from the AF strongly determines the detection of multiple targets located close to each other. In ideal scenarios, the range accuracy depends mostly on the waveform's range resolution. However, this is not true in practical scenarios. Channel effects can degrade range estimation performance by orders of magnitude. Moreover in low SNR situations, which are common in long range operational scenarios, the non-Gaussianity of noise is a bigger problem, and the use of robust estimators to tackle the noise encountered is of more practical importance.

Waveforms such as CCW and CLFM which are less studied or employed in trials, performed well in long range single target detection, apart from the conventionally known LFM signal. The CLFM also performed well in detecting multiple targets, thus highlighting its advantages as a good potential candidate for long range target detection applications. The M-sequence performed poorly in terms of detection, and its disadvantage was more apparent at longer target ranges. However, it yields good range estimation, especially at short ranges.

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