A novel formulation of the Bayes recursion for single-cluster filtering

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Abstract— In this paper we address the problem of tracking several moving targets with a sensor whose location and orientation are uncertain. This is a generalization of the wellknown problem of feature-based simultaneous localization and mapping (SLAM). It is also a generalization of multitarget tracking (MTT) in general, and related to sensor bias estimation. We address such problems from the perspective of finite set statistics (FISST) and point process theory, and develop general expressions for the posterior multiobject density, as represented by probability-generating functionals (p.g.fl.'s). We discuss how this general solution relates to approximative solutions previously suggested in the literature, and we also discuss how the p.g.fl. should be defined for such problems. To the best of our knowledge, this is the first paper to outline a FISST-based treatment of explicit data association for SLAM and related problems.

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1. INTRODUCTION

The problem of tracking multiple targets using a sensor with unknown position and orientation (together known as pose) arises in a number of contexts. Our research on this problem is in particular motivated by simultaneous localization and mapping (SLAM), which is a special case of this problem. Similar problems also arise in sensor bias estimation, and in various pursuit-evasion scenarios. Finally, multi-target tracking (MTT) itself is a special case of this problem.

The MTT problem with sensor uncertainty is more complex than standard MTT problems because one must jointly estimate state vectors of both targets and sensor pose. This entails accounting for correlations between the sensor pose and the target state vectors. Furthermore, data association becomes problematic in the presence of sensor uncertainty. In conventional MTT, standard independence assumptions make it possible to decompose the probabilities of an association hypothesis into a sum of contributions from all the targets. In contrast, when sensor uncertainty is an issue, posterior association probabilities can only be found through an integral over the (unknown) sensor pose [1].

Recent research has suggested that such problems may be formulated in terms of a single-cluster process [2, 3], which is a special case of more general cluster point processes [4, 5, 6, 7, 8], which again can be viewed as a generalization of Mahler's finite set statistics (FISST) [9]. In a single-cluster process we represent the sensor pose by a parent process, while the targets (landmarks in SLAM) are represented by a daughter process. This emphasizes that the daughter process is specified conditional on the parent process.

FISST is a reformulation of point process theory tailored to MTT [10]. In FISST, both targets and measurements are generally treated as random finite sets, i.e., as set-valued random variables. This allows one to express a well-defined posterior distribution for the full tracking problem, involving data association and existence uncertainty, using a single prediction equation and a single update equation. Mahler's research have focused on the development of approximative solutions which avoid *explicit* data association. Examples are the probability hypothesis density (PHD) filter, the cardinalized probability hypothesis density (CPHD) filter and the multitarget multi-Bernoulli (MeMBer) filter [9, 11].

Recently, there has been an increased interest for analyzing the full multi-object posterior of MTT without approximations. Different conjugate prior solutions² to MTT have been proposed in [12] and [13]. Roughly speaking, these solutions express the multi-object posterior as a linear mixture over association hypotheses. Recent research by the authors [14] has found that the multi-object posterior under appropriate assumptions indeed is a mixture over association hypotheses similar to those used in Reid's [15] and Mori's [16] multihypothesis trackers (MHTs). A similar investigation has been carried out for a simplified SLAM-problem in [1] and [17].

Investigation of the full multi-object posterior for recursive SLAM remains an open research question. Previous advances on this problem have been reported by Kalyan et al. in [18], by Mullane et al. in [19] and [20] and by Lee et al. in [2] and [3]. All of these solutions have utilized PHD-approximations, which essentially imply that the daughter process is approximated by a Poisson process. None of these papers have attempted to venture beyond the PHD-

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²In such a solution both the prediction and posterior are of the same form.

approximation. Many other papers such as [21], [22] and [23] have addressed SLAM, including data-association, in a Bayesian context. However, these references have not even discussed the concept of a multi-object posterior density.

Investigation of the full multi-object posterior for recursive SLAM, and the more general problem of MTT with unknown sensor pose, is important for several reasons. First, PHD-approximations are known to suffer from misdetection problems [24,25]. This may not a be serious issue if our primary purpose is to estimate the sensor pose, but it can be problematic if our aim is to reliably discover all targets/landmarks. Second, addressing the full multi-object posterior may be useful in order to extend other approximations such as MeM-Ber filters from conventional MTT to SLAM and to MTT with sensor uncertainty. Third, we would like to answer the following question: When we approximate the multi-object posterior in SLAM, what exactly is it that we approximate?

Probability-generating functionals (p.g.fl.'s) play a central role in FISST and point process theory. These provide a transform-domain representation of multi-object densities: For any multi-object density, the corresponding p.g.fl. is found as a particular set-integral of the multi-object density (cf. Section 4). Conversely, for any p.g.fl., the corresponding multi-object density is found as a functional derivative of the p.g.fl. The usage of p.g.fl.'s can often simplify expressions and calculations in multi-object filtering. This is exploited in for example [13], where the developments are almost exclusively done in the p.g.fl. domain. It is, however, not immediately clear how the concept of a p.g.fl. should be generalized from ordinary point process to cluster processes. At least four different definitions have been employed in various references, such as [4], [8], [26] and [27].

In this paper we investigate a definition of the p.g.fl. for a cluster process which recently was suggested in [27], and we argue that it leads to a tractable representation of singlecluster processes. Then we establish a formal solution to single cluster filtering. This solution is a generalization of the MTT solution proposed in [13].

This paper is organized as follows. First of all, Section 2 gives an overview of notation used. Particular attention is given to the concept of association hypotheses in Section 3, before a brief introduction to FISST is given in Section 4. In Section 5 we discuss various definitions of the p.g.fl. of a cluster process. Assumptions underlying our treatment of single-cluster processes are given in Section 6. The main results of this paper, heavily influenced by [3], [13] and [28], are given in Section 7. We establish the relationship between this solution and previous PHD-based solutions in Section 8. In Section 9 we discuss possible multi-hypothesis implementations, and present some preliminary results on sonar data. Finally, we summarize our findings in Section 7, the reader is referred to the Appendix and to [13].

2. NOTATION

In this section we provide an overview of notation used, before we properly introduce FISST and assumptions employed in the subsequent sections.

Random finite sets

We use the terms "(point) process" and "random finite set" (RFS) interchangeably. A cluster process X can be decom-

posed into a parent process Υ and a daughter process Ξ , with respective realizations E and X. In the single-cluster context we have $E = \{\eta\}$ where η is a random vector. For reasons that will be explained, we distinguish sensor pose p from the parent η . Measurement sets are denoted Σ with realizations Z. Elements of X are denoted x, while elements of Z are denoted z. Other sets that we will encounter include the RFS of newborn targets Γ , the RFS of surviving targets Ψ , the RFS of false alarms K and the RFS Θ of measurements originating from a particular target.

Pdf's and multi-object densities

The stochastic properties of random vectors or RFS's are given by probability density functions (pdf's) or multi-object densities, respectively. The predicted and posterior pdf's of the parent process are written $f_{\Upsilon,k|k-1}(\eta_k)$ and $f_{\Upsilon,k}(\eta_k)$, respectively. Generic, predicted and posterior multiobject densities of, say, the RFS Ξ are denoted $f_{\Xi}(X)$, $f_{\Xi,k|k-1}(X)$ and $f_{\Xi,k}(X)$, respectively. Various pdf's that we encounter include the parent Markov pdf $f_{\eta}(\eta_k|\eta_{k-1})$, the pose Markov pdf $f_p(p_k|p_{k-1})$, the daughter Markov pdf $f_x(x_k|x_{k-1})$, the single-target likelihood $f_z(z_k | x_k, p_k)$, the single-track predicted pdf $f_{k|k-1}^{i,a_k^i}(x_k | \eta_k)$ and the single-track posterior pdf $f_k^{i,a_k^i}(x_k | \eta_k)$. Poisson-processes are given in terms of their intensities, such as $\lambda(z)$, v(x) or $\mu(x)$.

Probability-generating functionals

Alternatively, RFS's are also represented by p.g.fl.'s. The general notation for p.g.fl.'s are using capital G and square brackets, e.g., $G_{k-1}[g,h]$, $G_{\Xi,k}[h \mid \eta_k]$, etc. We reserve the notation F[l,g,h] for a particular joint p.g.fl. that encompasses both multi-object likelihood, daughter process and parent process. The notation $H[h|\eta_k]$ denotes an unnormalized functional whose normalized version is a p.g.fl.. Furthermore, we make extensive use of *linear* functionals which correspond to pdf's or Poisson intensities. These are always written using lowercase Latin or Greek letters:

$$\begin{split} f_k^{i,a^i}[h] &= \int h(x) f_k^{i,a^i}(x) \mathrm{d}x \\ v[h] &= \int h(x) v(x) \mathrm{d}x, \ \text{etc.} \end{split}$$

Test functions of functionals are always denoted g (for the parent process), h (for the daughter process) or l (for the measurement process). Variables bound within the scope of a functional are marked by a tilde. Such variables, e.g., $\tilde{\eta}_k$ and \tilde{x}_k , are always integrated out as part of the functional.

Time-, track- and measurement-indices

Time is accounted for by the time index k. The list of time steps $(l, l + 1, \ldots, k - 1, k)$ is in shorthand notation written as l:k. The notation p_k signifies sensor pose at time step k. The notation $\eta_k^{1:k-1}$ signifies the k-1 first pose vectors in the parent vector at time step k (See Section 6 for further discussion regarding the relationship between pose and parent vectors). The association hypotheses at time step k constitute a set \mathcal{A}_k , while the corresponding n_k track indices constitute a set \mathcal{T}_k (See Section 3). Measurement indices are generally denoted j, while track indices in most cases are denoted i. A single association hypothesis is denoted a. The notation a_k^i indicates the measurement claimed by track number i at time step k according to a. A notation such as $a_{1:k-1}^{1:n_{k-1}}$ indicates

the matrix of track-to-measurement associations for tracks 1 to n_{k-1} for time steps 1 to k-1 according to a.

Miscellaneous notations

The dimensions of pose, targets (also known as daughters or landmarks) and measurements are τ , d and s, respectively. Thus, these vectors are elements of \mathbb{R}^{τ} , \mathbb{R}^{d} or \mathbb{R}^{s} , respectively. Let $\mathcal{F}(\mathbb{R}^{d})$ denote the set of all finite subsets of \mathbb{R}^{d} . Vertical concatenation of vectors is expressed using semicolon: $[\eta; x] = [\eta^{\mathsf{T}}, x^{\mathsf{T}}]^{\mathsf{T}}$. By $\mathbf{0}_{k \times n}$ we mean a $k \times n$ matrix consisting of zeros.

3. Association hypotheses

Any attempt at expressing the full multi-object density for conventional MTT problems must necessarily invoke association hypotheses in one way or another [1, 12, 13]. However, there is limited agreement as to how the fundamental concepts of *association hypotheses* and *tracks* should be understood and defined.

In [14] we suggest that a track can be understood as a sequence of measurements, while an association hypothesis is a multiset of tracks. This is in agreement with the classical papers [15] and [16]. In [12] Vo & Vo formulated a conjugate prior solution to standard MTT in terms of association hypotheses which accounted for track labels. In [13] Williams proposed a conjugate prior solution which does not need explicit track labeling. We will in the sequel refer to the hypotheses used in [13] as *W*-hypotheses. These hypotheses differ in some crucial aspects from the hypotheses used in the former references. In all the former hypothesis formulations, target existence uncertainty is fully subsumed by the hypothesis a has been specified, then we know how many targets there are conditional on a being true.

Williams' formulation, on the other hand, leaves existence probability to be specified conditional on any W-hypothesis. In other words, a W-hypothesis does not necessarily specify the number of targets present. The hypothesis representation in [13] is related to Kurien's track-oriented MHT [29]. In the formulation of [13], a track with origin in the measurement z_l^j is defined as the hypothesis tree of possible measurements that at later times $k \ge k$ may originate from the same target that produced z_l^j . Any sequence of "measurements" along the branches of this tree is then referred to as a single-target hypothesis³. We write "measurements" in quotation marks because both misdetection *and* the possibility that the target does not exist are included as possible measurements in this formalism. A W-hypothesis is then an entity which contains exactly one leaf node from each track.

As an alternative to this tree-based formalism, we will here suggest a matrix-based definition of W-hypotheses. Let us first define the linear index function $L(j;k) = j + \sum_{l=1}^{k-1} m_l$ with inverse $L^{-1}(i;k) = i - \sum_{l=1}^{k-1} m_l$. What L does is to give us the index of the track with origin in the measurement z_k^j , while its inverse gives the identity of the original measurement in track number i. The number of tracks at time k is $n_k = L(m_k;k)$. We can then define the set \mathcal{A}_k of W-hypotheses at time k as the set of mappings⁴ a :

$$\{1,\ldots,n_k\} \rightarrow \{0,\ldots,m_1\} \times \ldots \times \{0,\ldots,m_k\}$$
 such that

- R1: For any measurement index $j \in \{1, ..., m_k\}$ there exists one and only one track index $i \in \{1, ..., n_k\}$ such that $a_k^i = j$.
- that $a_k^i = j$. R2: There exist a hypothesis b in the previous hypothesis set \mathcal{A}_{k-1} such that $a_{1:k-1}^{1:n_{k-1}} = b$.
- R3: For any track index $i \in \{1, ..., n_k\}$, if there exists no l < k such that $a_l^i > 0$, and $i \le n_{k-1}$, then $a_k^i = 0$.
- R4: For any track index $i \in \{1, ..., n_k\}$, if there exists no l < k such that $a_l^i > 0$, and $i > n_{k-1}$, then $a_k^i = L^{-1}(i; k)$.

Requirement R1 means that all measurements must be accounted for. Requirement R2 means that any current hypothesis must have a parent hypothesis, i.e., that the collection of hypotheses is expanded in a recursive manner. Requirement R3 means that if the track with origin in z_k^i fails to claim z_k^i , then this track represents a non-existing target for all future. Requirement R4 means that any new track with index *i* at time step *k* must originate with the corresponding measurement at time step *k*.

It is of course possible to prune away low-probability tracks and to merge together different elements of A_k in order to reduce the complexity. Several strategies for how this can be done are proposed in [13]. We discuss possible extensions of such strategies to single-cluster filtering in Section 9. For future reference we denote the list of available tracks by T_k , and n_k is the cardinality of T_k .

4. KEY CONCEPTS IN FISST

FISST is a Bayesian formulation of point process theory. In FISST, the properties of RFS's are studied. An RFS is essentially the same as a point process [9 p. 708], and these terms are used interchangeably in this paper. FISST provides rules for how the random properties of RFS's can be represented in terms of so-called multiobject densities. This allows us to extend Bayesian probability calculus from the conventional domain of random vectors to the more complex domain of RFS's.

Multiobject densities are functional derivatives (see the next paragraph) of belief-mass functions, or more generally of p.g.fl.'s. The belief-mass function of a random finite set Ξ is the probability $\beta_{\Xi}(S) = \Pr(\Xi \subseteq S)$. Here S is a subset of the base space, i.e., if $x \in \Xi$ and $x \in \mathbb{R}^d$, then $S \subseteq \mathbb{R}^d$. FISST utilizes the so-called set integral [9 p. 361]. Let f(X) be a set function, i.e., a function of the finite set X. Then the set integral of f(X) over S is defined as

$$\int_{S} f(X)\delta X = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{S \times \dots \times S} f(\{x^{1}, \dots, x^{n}\}) \mathrm{d}x^{1} \dots \mathrm{d}x^{n}.$$

Here *n* is the cardinality of *X*, and $f(\{x^1, \ldots, x^n\}) = f(X)$ under the constraint that |X| = n. A multiobject density $f_{\Xi}(X)$ is a function which produces a non-negative real number from any realization *X* of Ξ , and which normalizes to one under the set integral: $\int_{\mathbb{R}} f_{\Xi}(X)\delta X = 1$. The multiobject density $f_{\Xi}(X)$ is related to the belief-mass function according to the requirement that $\beta_{\Xi}(S) = \int_S f_{\Xi}(X)\delta X$ for all $S \subseteq \mathbb{R}^d$.

³In [29 p. 53] these concepts are essentially defined reversely.

⁴Such a set of mappings between integers is obviously isomorphic to a

corresponding set of $k \times n_k$ -matrices.

Belief-mass functions are special cases of p.g.fl.'s. The p.g.fl. of a random set Ξ is defined as

$$G_{\Xi}[h] = \int h^X f_{\Xi}(X) \delta X$$

where $h : \mathbb{R}^d \to [0, \infty)$ is a test-function, and the notation h^X signifies the product $\prod_{x \in X} h(x)$. According to the fundamental theorem of multiobject calculus [9 p. 384] we can recover the multi-object density from the p.g.fl. by

$$f_{\Xi}(X) = \frac{\delta G_{\Xi}}{\delta X}[0].$$

Here $\frac{\delta G_{\Xi}}{\delta X} = \frac{\delta^n G_{\Xi}}{\delta x^1 \dots \delta x^n}$ is the iterated functional derivative of G_{Ξ} in the directions of the delta-functions $\delta_{x^1}(\cdot), \dots, \delta_{x^n}(\cdot)$ under the condition that $X = \{x^1, \dots, x^n\}$. These derivatives are found as the limit values

$$\frac{\delta G}{\delta x}[h] = \lim_{\epsilon \searrow 0} \frac{G[h + \epsilon \delta_x] - G[h]}{\epsilon}$$

Thus, p.g.fl.'s provide a transform-domain for the manipulation of multiobject densities. Many properties of p.g.fl.'s are listed in [9], and these properties often make manipulation of p.g.fl.'s easier than manipulation of the corresponding multiobject densities. An important property of p.g.fl.'s is that they normalize: For a p.g.fl. G_{Ξ} we have that $G_{\Xi}[1] = 1$.

5. REPRESENTATIONS OF CLUSTER PROCESSES

A cluster process is a point process where the points are grouped into clusters [4]. Any realization of a cluster process \mathbb{X} consists of a parent set $E = \{\eta^1, \ldots, \eta^n\}$ and a daughter set $X_{\eta^i} = \{x^1, \ldots, x^m\}$ for each parent η^i in the parent set. In this paper we are primarily concerned with singlecluster processes, where the parent set Υ have one and only one element with probability one, so that any realization of \mathbb{X} is a pair (η, X) . Let us assume that the base spaces are \mathbb{R}^{τ} and \mathbb{R}^d . That is, $\eta \in \mathbb{R}^{\tau}$ and $x \in \mathbb{R}^d$.

The representation of point processes in terms of p.g.fl.'s was introduced in [4], and has played a fundamental role in the development of FISST. It is, however, not immediately clear how this representation should be generalized to cluster processes. In order to represent a point process by means of a p.g.fl. we must define the domain of its test functions. In the literature, this has been done in at least four different ways. We will point out shortcomings with three these approaches, before we argue in favor of the fourth approach.

Marked process representation

Moyal's seminal paper [4] suggested that the test functions for a cluster process should be on the form $h_*(\eta, X)$. In other words, the test function h_* is a mapping from $\mathbb{R}^{\tau} \times \mathcal{F}(\mathbb{R}^d)$ to $[0, \infty)$. Such a formalism follows naturally from viewing the pair (η, X) as a point of \mathbb{X} , or from viewing the cluster process \mathbb{X} as a marked point process [30]: For a given "point" (η, X) , we can think of η as the *location* of the point, and Xas the *mark* of the point. Together they constitute the point, and the statistical properties of the underlying point process are naturally summarized by $G[h_*] = E_{\mathbb{X}}[h_*^{\mathbb{X}}]$ where $h_*^{\mathbb{X}}$ by definition is the product $h_*(\eta^1, X_{\eta^1}) \cdots h_*(\eta^n, X_{\eta^n})$. This representation is problematic for two reasons. First, it is quite intractable. The test-function h_* takes sets as arguments, and evaluating its expectation therefore becomes a complicated affair. Second, these test-functions appear to involve a large amount of redundancy. This can be seen by restricting attention to a particularly singular case of a singlecluster process. Assume that $\eta \in \mathbb{R}$ with pdf $\delta_0(\eta)$. That is, η is for all practical purposes fixed at the origin in \mathbb{R} , while the daughter process can be arbitrary. In this case, the cluster process is entirely given by its daughter process Ξ , which is a conventional point process. The p.g.fl. of this process is specified if we assign a real number to every test-function h: $\mathbb{R}^d \to [0,\infty)$. Moyal's cluster-p.g.fl., on the other hand, must be specified for any test-function $h_* : \mathbb{R}^{\tau} \times \mathcal{F}(\mathbb{R}^d) \to [0, \infty)$. Thus, at least for this example, it is possible to find a simpler p.g.fl. than a p.g.fl. working on test functions of the form h_* .

Representation in terms of bivariate test-functions

In [26] it was suggested that the p.g.fl. of a cluster process could be defined as a machine working on test-functions of the form $h(\eta, x)$ where $\eta \in \mathbb{R}^{\tau}$ and $x \in \mathbb{R}^{d}$. That is, in this definition h is a mapping from $\mathbb{R}^{\tau} \times \mathbb{R}^{d}$ to $[0, \infty)$. The p.g.fl. is then found as $G[h] = G_{\Upsilon}[G_{\Xi}[h(\tilde{\eta}, \cdot)]]$, where $G_{\Xi}[h(\tilde{\eta}, \cdot)]$, which we treat as an argument of G_{Υ} , is the p.g.fl. of the daughter process for any particular parent point $\tilde{\eta} \in \mathbb{R}^{\tau}$.

This representation, which uses a single bivariate testfunction, is more parsimonious than the marked process representation. However, redundancy is still present. This is especially evident in the case of single-cluster processes. Any single-cluster process with realizations (η, X) can be converted to a conventional point process by concatenating η to each daughter vector x. The converted process will then have realizations of the form $\{[\eta^1; x^1], \ldots, [\eta^n; x^n]\}$. However, since there is only a single parent, we must have $\eta^1 = \ldots = \eta^n$ with probability one. The Janossy densities [8 p. 125] of this modified process must therefore contain n-1 delta functions. This presence of delta functions is a warning sign that also this representation may exhibit excessive redundancy.

Representation in terms of a single univariate test-function

In the books [5] and [8], the p.g.fl. of a cluster process is defined using test functions of the form $h : \mathbb{R}^d \to [0, \infty)$ which operate solely on the daughter space. The p.g.fl. is then defined as a nested expectation $G[h] = G_{\Upsilon}[G_{\Xi}[h | \tilde{\eta}]]$.

This definition of the p.g.fl. is problematic because it is too simple to fully represent the statistical properties of arbitrary cluster processes. To give an example of this, we again restrict attention to a single-cluster scenario. Let the parent process Υ and the daughter process Ξ be independent. The p.g.fl. of the cluster process is then, according to this representation, found as $G[h] = \int f_{\Upsilon}(\eta)G_{\Xi}[h | \eta] d\eta =$ $\int f_{\Upsilon}(\eta)G_{\Xi}[h] d\eta = G_{\Xi}[h] \int f_{\Upsilon}(\eta)d\eta$. So, for example, if the daughter set Ξ is Poisson, then the *full* cluster p.g.fl. is given by the expression $G[h] = \exp(v[h - 1])$ for some intensity function v. This expression contains no information about the parent Υ , and cannot represent the entire cluster process.

A fourth representation

Based on the previous considerations, it seems reasonable to look for a representation which is simpler than the bivariate representation, but more complex than the univariate representation. Such a representation is obtained if we define the p.g.fl. of a cluster process as the nested expectation of two *separate* test functions: one for the parent and one for the daughter. That is,

$$G[g,h] = G_{\Upsilon}[g(\tilde{\eta})G_{\Xi}[h \,|\, \tilde{\eta}]].$$

Spelling out the details in terms of set-integrals and multiobject densities, we have

$$G[g,h] = \int f_{\Upsilon}(E) \prod_{\eta \in E} \left(g(\eta) \right)$$
$$\times \int f_{\Xi}(X \mid \eta) \prod_{x \in X} h(x) \, \delta X \delta E$$

where $f_{\Upsilon}(E)$ is the multiobject density of Υ and $f_{\Xi}(X | \eta)$ is the multiobject density of Ξ conditional on the parent η . Such a representation was recently also suggested in [27]. Investigations of recursive estimation for general cluster processes along the lines of this formulation are currently being conducted [28].

For a *single-cluster process* thus represented we can express joint and marginal densities in terms of functional derivatives

$$f_{\Upsilon,\Xi}(\eta, X) = \frac{\delta^2 G}{\delta \eta \delta X}[g, 0] \tag{1}$$

$$f_{\Xi|\Upsilon}(X|,\eta) = \frac{\delta G_{\Xi}}{\delta X} [0|\eta]$$
(2)

$$f_{\Upsilon}(\eta) = \frac{\delta G_{\Upsilon}}{\delta \eta}[g] = \frac{\delta G}{\delta \eta}[g, 1].$$
(3)

Notice that the value of g is irrelevant in these derivatives for single-cluster processes. This is because the parent p.g.fl. in this case always is a linear functional, whose derivative does not depend on the test-function [9 p. 378]. Expression (1) can be viewed as a definition of the joint density while expression (2) simply is the conventional relationship between p.g.fl. and multiobject density. The validity of expression (3) can be shown using relatively straightforward limit arguments.

For single-cluster processes the multi-object density of the parent reduces to a conventional pdf, and a case could be made that representation in terms of this pdf is simpler than representation in terms of p.g.fl.'s. We have nevertheless chosen to express the statistical properties of both the parent and the daughter in terms of p.g.fl.'s. This is done in order to present a treatment which is as systematic as possible, and which with minimal efforts can be generalized to general cluster processes.

6. SINGLE-CLUSTER ASSUMPTIONS

Having outlined a general framework for single-cluster filtering in Sections 4 and 5, we now state more precise, but still reasonably general, assumptions that pertain to SLAM problems and multi-target tracking with sensor pose uncertainty. We employ assumptions similar to those used in [3], which again can be viewed as a generalization of the standard MTT assumptions as used in references such as [9] or [13]. The assumptions are more general solely because we include the pose of the sensor as a random state to be estimated.

Let us first state the assumptions in a "colloquial" manner, before we in subsequent paragraphs translate the assumptions

into the language of FISST. We assume that an unknown number of targets $x_k^1, \ldots, x_k^n \in \mathbb{R}^d$ are observed by a sensor with pose p_k at time k. In the case of SLAM, we refer to the targets as landmarks. We receive a set of m_k measurements $z_k^1, \ldots, z_k^{m_k} \in \mathbb{R}^s$ of which an unknown number comes from the targets, and an unknown number are false alarms, also known as clutter. Any target x_k^i generates a single measurement with probability $P_{\rm D}(x_k^i, p_k)$, and no measurement with probability $1 - P_D(x_k^i, p_k)$. If a measurement z_k^j is generated by target x_k^i , then this measurement has the likelihood $f_z(z_k^j | x_k^i, p_k)$. Otherwise, z_k^j is a false alarm. The number of false alarms or clutter measurements in any region S of the sensor space \mathbb{R}^s is assumed Poisson-distributed with rate $\int_{S} \lambda(z_k) dz_k$ where λ is referred to as the clutter intensity. A target x_k^i continues to exist at the next time step with survival probability $P_{\rm S}(x_k^i)$. If so, then its transition density is $f_x(x_{k+1} \mid x_k)$. In any region R of the target state space \mathbb{R}^d , the number of newborn targets at time k+1 is Poissondistributed with rate $\int_{R} \mu(x_{k+1}) dx_{k+1}$ where μ is referred to as the birth intensity. The sensor pose has transition density $f_p(p_{k+1} | p_k)$ between time steps k and k+1.

The prior

In [13] it has been shown that for conventional MTT with Poisson birth- and clutter-processes, the p.g.fl. of the posterior multiobject density can be factorized into a Poissoncomponent and a MeMBer-component. For the single-cluster recursion, we can obtain a similar result if we condition on particular *trajectories* of the parent process. We explain why trajectory-conditioning is required in Remark 4. For now, let us merely require the parent state to be of the form $\eta_k = [p_1; \ldots; p_k] \in \mathbb{R}^{k\tau}$. In other words, any realization of the parent process Υ at time step k is a concatenated vector, also known as a trajectory, of k pose vectors.

Conditional on a trajectory η_{k-1} , the p.g.fl. of the daughter process Ξ at time k-1 consists of a Poisson part $G_{\Xi,k-1}^{\text{ppp}}[h \mid \eta_{k-1}]$ and a MeMBer part $G_{\Xi,k-1}^{\text{mbm}}[h \mid \eta_{k-1}]$. The former component is given by an intensity function $v_{k-1}(x_{k-1} \mid \eta_{k-1})$ which depends on the parent trajectory η_{k-1} . The latter component is a linear mixture over W-hypotheses a in the set \mathcal{A}_{k-1} , as defined above. Conditional on each W-hypothesis, the posterior density is a weighed MeMBer density, where each track i is characterized by

- A weight $w_{k-1}^{i,a^i}(\eta_{k-1})$
- An existence probability $r_{k-1}^{i,a^i}(\eta_{k-1})$
- A kinematic pdf $f_{k-1}^{i,a^i}(x_{k-1} | \eta_{k-1})$.

Based on this, we assume that the joint p.g.fl. at time step k-1 can be written as

$$G_{k-1}[g,h] = f_{\Upsilon,k-1}[g(\tilde{\eta}_{1:k-1})G_{k-1,\Xi}[h \,|\, \tilde{\eta}_{1:k-1}]]$$

where the parent and daughter p.g.fl.'s are given by

$$f_{\Upsilon,k-1}[g] = \int g(\eta_{k-1}) f_{\Upsilon,k-1}(\eta_{k-1}) d\eta_{k-1}$$
$$G_{\Xi,k-1}[h \mid \eta_{k-1}] = G_{\Xi,k-1}^{\text{ppp}}[h \mid \eta_{k-1}] G_{\Xi,k-1}^{\text{mbm}}[h \mid \eta_{k-1}]$$

and the Poisson and Multi-Bernoulli components of the

daughter p.g.fl. are given by

$$G_{\Xi,k-1}^{\text{ppp}}[h \mid \eta_{k-1}] = \exp(v_{k-1}[h-1 \mid \eta_{k-1}])$$

$$G_{\Xi,k-1}^{\text{mbm}}[h \mid \eta_{k-1}] = \sum_{a \in \mathcal{A}_{k-1}} \prod_{i \in \mathcal{T}_{k-1}} w_{k-1}^{i,a^{i}}(\eta_{k-1})$$

$$\left(1 - r_{k-1}^{i,a^{i}}(\eta_{k-1}) + r_{k-1}^{i,a^{i}}(\eta_{k-1})f_{k-1}^{i,a^{i}}[h \mid \eta_{k-1}]\right)$$

REMARK 1. The Poisson component $G_{\Xi,k-1}^{\text{ppp}}[h | \eta_{k-1}]$ accounts for targets which exist but have never been observed. However, as pointed out in [13], it is possible to "recycle" tracks on observed targets into this component. If we recycle *all* tracks into this component, then $G_{\Xi,k-1}[h | \eta_{k-1}]$ becomes identical to $G_{\Xi,k-1}^{\text{ppp}}[h | \eta_{k-1}]$. This is precisely what happens in a PHD-filtering approach such as the SLAMmethods of [3] and [19]. See Section 8 for further details.

The Markov model

The daughter RFS Ξ at time step k is a union of an RFS of newborn targets Γ and a collection of RFS's Ψ of surviving targets from time step k - 1. The former RFS has a Poisson p.g.fl. $G_{\Gamma,x}[h \mid x_{k-1}] = \exp(\mu[h - 1])$. For the latter RFS's, we assume a standard Bernoulli model. This is represented by the p.g.fl. $G_{\Psi}[h \mid x_{k-1}] = 1 - P_{\rm S}(x_{k-1}) + P_{\rm S}(x_{k-1})f_x[h \mid x_{k-1}]$ where the purely kinematic Markov target model is represented by the linear functional $f_x[h \mid x_{k-1}] = \int h(x_k)f_x(x_k \mid x_{k-1}) dx_k$.

For the parent process, the Markov model should encapsulate the concatenation of current sensor pose with previous sensor poses (see Remark 4). That is, the previous parent will generally have a realization $\{\eta_{k-1}\}$ of the form $\eta_{k-1} = [p_1; \ldots; p_{k-1}]$ and the new parent will generally have a realization $\{\eta_k\}$ of the form $\eta_k = [p_1; \ldots; p_{k-1}; p_k]$ where the first (k-1)d entries in η_k are identical to the corresponding entries in η_{k-1} . Thus, we express the Markov model of the parent by the linear functional

$$f_{\eta}[g \mid \eta_{k-1}] = \int g(\eta_k) f_{\eta}(\eta_k \mid \eta_{k-1}) \mathrm{d}\eta_k$$

which is given by the pdf

$$f_{\eta}(\eta_{k} | \eta_{k-1}) = f_{p}\left(\eta_{k}^{(k)} | \eta_{k-1}^{(k-1)}\right) \prod_{l=1}^{k-1} \delta_{\eta_{k-1}^{(l)}}\left(\eta_{k}^{(l)}\right)$$

The delta-functions ensure that $\eta_k^{1:k-1} = \eta_{k-1}^{1:k-1}$, i.e., that the old part of a trajectory remains fixed when the trajectory is extended.

REMARK 2. In typical SLAM problems the targets are stationary landmarks, implying that $f_x(x_k | x_{k-1}) = \delta_{x_{k-1}}(x_k)$. In tracking problems with sensor uncertainty this is no longer the case, and such problems are therefore more complex.

The likelihood

The measurement set Σ at time k is a union of a targetoriginating measurements and false alarms. The latter constitutes an RFS K with Poisson p.g.fl.

$$G_K[l] = \exp(\lambda[l-1])$$

where $\lambda[l]$ is the linear functional corresponding to the clutter intensity $\lambda(z)$. For target-originating measurements, let Θ represent the measurement set corresponding to a target with state x_k . It has the Bernoulli p.g.fl.

$$G_{\Theta}[l \mid x_k, \eta_k] = 1 - P_{\rm D}(x_k, \eta_k^k) + P_{\rm D}(x_k, \eta_k^k) f_z[l \mid x_k, \eta_k^k]$$

where $f_z[l | x_k, \eta_k^k] = \int l(z) f_z(z | x_k, \eta_k^k) dz$. Notice that the detection probability $P_D(x_k, \eta_k^k)$ in contrast to the survival probability $P_S(x_k)$ depends on both x_k and $\eta_k^k = p_k$. Assuming conditional independence between the measurements of two different targets, the multiobject likelihood is fully specified by these two p.g.fl.'s. For the full measurement set Σ this leads to a multiobject likelihood $f_{\Sigma}(Z_k | X_k, \eta_k^k)$ which is a straightforward generalization of the multiobject likelihood developed in [9 p. 421].

REMARK 3. For the case of sensor bias estimation, we must in general have more than one sensor to obtain observability. This leads to a more complicated likelihood than the one outlined above. However, from a conceptual perspective this can easily be dealt with by stacking the sensor poses together into a "super-sensor" pose vector p_k , and simply multiplying together the likelihoods for the different sensors [9 p. 445]. Stationarity of the sensors imply that $f_p(p_k | p_{k-1}) =$ $\delta_{p_{k-1}}(p_k)$. This simplifies the problem, since conditioning on the current super-sensor pose p_k then carries exactly the same information as conditioning on the full trajectory η_k .

7. THE SINGLE-CLUSTER RECURSION

Having specified multiobject prior, Markov model and likelihood in the previous section, we can express the predicted and posterior multiobject densities or p.g.fl.'s for all time steps.

The prediction

Based on the prior and Markov model outlined above, the predicted p.g.fl. can be written as

$$G_{k|k-1}[g,h] = f_{\Upsilon,k|k-1}[g(\tilde{\eta}_k)G_{\Xi,k|k-1}[h \,|\, \tilde{\eta}_k]].$$

Here the predicted parent pdf (and thus its corresponding p.g.fl. as well) is given by

$$f_{\Upsilon,k|k-1}(\eta_k) = \int f_{\eta}(\eta_k \,|\, \eta_{k-1}) f_{\Upsilon,k-1}(\eta_{k-1}) \mathrm{d}\eta_{k-1} \quad (4)$$

while the predicted daughter p.g.fl. factorizes according to

$$G_{\Xi,k|k-1}[h \mid \eta_k] = G_{\Xi,k|k-1}^{\text{ppp}}[h \mid \eta_k] G_{\Xi,k|k-1}^{\text{mbm}}[h \mid \eta_k]$$
(5)

where

$$G_{\Xi,k|k-1}^{\text{ppp}}[h \mid \eta_k] = \exp(v_{k|k-1}[h-1|\eta_k])$$
$$G_{\Xi,k|k-1}^{\text{mbm}}[h \mid \eta_k] \propto \sum_{a \in \mathcal{A}_{k-1}} \prod_{i \in \mathcal{T}_{k-1}} w_{k|k-1}^{i,a^i}(\eta_k)$$
$$\left(1 - r_{k|k-1}^{i,a^i}(\eta_k) + r_{k|k-1}^{i,a^i}(\eta_k) f_{k|k-1}^{i,a^i}[h \mid \eta_k]\right)$$

and

$$v_{k|k-1}[h \mid \eta_k] = \mu[h] + v_{k-1}[P_{\mathcal{S}}(\tilde{x}_{k-1})f_x[h \mid \tilde{x}_{k-1}] \mid \eta_k^{1:k-1}]$$

$$w_{k|k-1}^{i,a^i}(\eta_k) = w_{k-1}^{i,a^i}(\eta_k^{1:k-1})$$

$$r_{k|k-1}^{i,a^i}(\eta_k) = r_{k-1}^{i,a^i}(\eta_k^{1:k-1})f_{k-1}^{i,a^i}[P_{\mathcal{S}}(\tilde{x}_{k-1}) \mid \eta_k^{1:k-1}]$$

$$f_{k|k-1}^{i,a^i}(x_k|\eta_k) = \frac{f_{k-1}^{i,a^i}[f_x(x_k \mid \tilde{x}_{k-1})P_{\mathcal{S}}(\tilde{x}_{k-1}) \mid \eta_{k-1}^{1:k-1}]}{f_{k-1}^{i,a^i}[P_{\mathcal{S}}(\tilde{x}_{k-1}) \mid \eta_{k-1}^{1:k-1}]}.$$

For derivations of these results, the reader is referred to Appendix A.

REMARK 4. Trajectory-conditioning is necessary for the factorization in (5). Only by conditioning on trajectories can we ensure that the predicted p.g.fl. (or multiobject density) of Ξ does not involve integration over previous sensor poses. Quantities such as $v_{k-1}[h \mid \eta_k^{1:k-1}]$ and $f_{k-1}^{i,a^i}[h \mid \eta_k^{1:k-1}]$ depend on the previous sensor poses $p_1, p_2, \ldots, p_{k-1}$, and this dependency is transferred to the corresponding predicted entities (e.g., $v_{k|k-1}[h \mid \eta_k]$ and $f_{k|k-1}^{i,a^i}[h \mid \eta_k]$). Removing this dependence would require integration over previous sensor poses, which would destroy the factorizations in the above equations.

The posterior

We can also express the posterior p.g.fl. in the general form

$$G_k[g,h] = f_{\Upsilon,k}[g(\tilde{\eta}_k)G_{\Xi,k}[h \,|\, \tilde{\eta}_k]]. \tag{6}$$

Both the update of the parent process, and the parentconditional update of the daughter process can be expressed in terms of an *un-normalized* version of the daughter p.g.fl. $G_{\Xi,k}[h | \eta_k]$, which we denote $H[h | \eta_k]$. This functional arises as a functional derivative of a parent-conditional joint daughter- and measurement-p.g.fl. The technical details of this are left for Appendix B. In broad lines, the measurement update consists of the following two steps:

1. We find the posterior daughter p.g.fl. in (6) as

$$G_{\Xi,k}[h \mid \eta_k] = \frac{H[h \mid \eta_k]}{H[1 \mid \eta_k]}.$$
(7)

In other words, the posterior daughter p.g.fl. is a normalized version of $H[h | \eta_k]$.

2. We obtain the posterior parent pdf in (6) as proportional to the product of the predicted parent pdf $p_{\Upsilon,k|k-1}(\eta_k)$ and $H[1 | \eta_k]$.

Since the posterior daughter p.g.fl. $G_{\Xi,k}[h | \eta_k]$ is proportional to $H[h | \eta_k]$ (treating η_k as fixed), we only need to specify $H[h | \eta_k]$ in order to establish the structure of $G_{\Xi,k}[h | \eta_k]$. Again, the daughter p.g.fl. can be factorized into a Poisson-component and a MeMBer-component

$$H[h \mid \eta_k] = H^{\text{ppp}}[h \mid \eta_k] H^{\text{mbm}}[h \mid \eta_k].$$
(8)

The Poisson-component is given by

$$H^{\text{ppp}}[h \mid \eta_k] = \exp(v_k[h - 1|\eta_k])$$

where the linear functional $v_k[h - 1 | \eta_k]$ is given by the corresponding intensity function

$$v_k(x_k \,|\, \eta_k) = (1 - P_{\mathrm{D}}(x_k, \eta_k^k)) v_{k|k-1}(x_k \,|\, \eta_k).$$

The MeMBer-component is more complex than the Poissoncomponent since it involves data association. We find it as a linear mixture of the form

$$H^{\text{mbm}}[h \mid \eta_{k}] = \sum_{a \in \mathcal{A}_{k}} \prod_{i \in \mathcal{T}_{k}} w_{k}^{i,a^{i}}(\eta_{k}) \\ \times \left(1 - r_{k}^{i,a^{i}}(\eta_{k}) + r_{k}^{i,a^{i}}(\eta_{k}) f_{k}^{i,a^{i}}[h \mid \eta_{k}]\right).$$
(9)

The sum in (9) ranges over all feasible W-hypotheses as generated by requirements R1-R4 in Section 3. The exact forms of the constituents $w_k^{i,a^i}(\eta_k)$, $r_k^{i,a^i}(\eta_k)$ and $f_k^{i,a^i}[h | \eta_k]$ depend on the structure of the underlying single-target hypothesis a^i . We have three possible cases: First, a^i may hypothesize that the target in track number *i* is not detected, in which case the target may or may not exist. Second, a^i may hypothesize that the target in (the previously established) track number i indeed is detected, in which case the target Third, track i may be a new track. must exist. This happens if the root measurement of track number i is in Z_k , which is equivalent to $i \in \mathcal{T}_k \setminus \mathcal{T}_{k-1}$. In this case, the corresponding target may or may not exist. Using the matrixbased hypothesis representation suggested in Section 3, these possibilities can be accounted for by partitioning the track indices $i \in \{1, \ldots, n_k\}$ into 4 sets:

$$N(a) = \{i \text{ s.t. } a_{1:k}^{i} = \mathbf{0}_{k \times 1}\}$$

$$B(a) = \{i \text{ s.t. } a_{1:k-1}^{i} = \mathbf{0}_{k-1 \times 1} \text{ and } a_{k}^{i} > 0\}$$

$$M(a) = \{i \text{ s.t. } a_{k}^{i} = 0 \text{ and } a_{l}^{i} > 0 \text{ for some } l < k\}$$

$$D(a) = \{i \text{ s.t. } a_{k}^{i} > 0 \text{ and } a_{l}^{i} > 0 \text{ for some } l < k\}.$$
 (10)

Here N(a) contains tracks on non-existing targets, which do not contribute in any way in (9). The set B(a) contains newborn tracks, for which

$$\begin{split} w_{k}^{i,a^{i}}(\eta_{k}) = &\lambda(z_{k}^{a_{k}^{i}}) \\ &+ v_{k|k-1}[f_{z}(z_{k}^{a_{k}^{i}} \mid \tilde{x}_{k}, \eta_{k}^{k})P_{\mathrm{D}}(\tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}] \\ r_{k}^{i,a^{i}}(\eta_{k}) = &v_{k|k-1}[f_{z}(z_{k}^{a_{k}^{i}} \mid \tilde{x}_{k}, \eta_{k}^{k})P_{\mathrm{D}}(\tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}] / \\ & \left(\lambda(z_{k}^{a_{k}^{i}}) \right. \\ &+ v_{k|k-1}[f_{z}(z_{k}^{a_{k}^{i}} \mid \tilde{x}_{k}, \eta_{k}^{k})P_{\mathrm{D}}(\tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}] \right) \\ f_{k}^{i,a^{i}}(x_{k}|\eta_{k}) = & \frac{f_{z}(z_{k}^{a_{k}^{i}} \mid x_{k}, \eta_{k}^{k})P_{\mathrm{D}}(x_{k}, \eta_{k}^{k})v_{k|k-1}(x_{k}|\eta_{k})}{v_{k|k-1}[f_{z}(z_{k}^{a_{k}^{i}} \mid \tilde{x}_{k}, \eta_{k}^{k})P_{\mathrm{D}}(\tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}]}. \end{split}$$

The set M(a) contains previously established tracks which according to the hypothesis a did not yield any detections at the current time step. For these tracks we have

$$\begin{split} w_{k}^{i,a^{i}}(\eta_{k}) &= w_{k|k-1}^{i,a^{i}}(\eta_{k}) \left(1 - r_{k|k-1}^{i,a^{i}}(\eta_{k}) + r_{k|k-1}^{i,a^{i}}(\eta_{k}) + r_{k|k-1}^{i,a^{i}}(\eta_{k}) f_{k|k-1}^{i,a^{i}}[1 - P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k}) | \eta_{k}]\right) \\ r_{k}^{i,a^{i}}(\eta_{k}) &= r_{k|k-1}^{i,a^{i}}(\eta_{k}) f_{k|k-1}^{i,a^{i}}[1 - P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k}) | \eta_{k}] / \\ \left(1 - r_{k|k-1}^{i,a^{i}}(\eta_{k}) + r_{k|k-1}^{i,a^{i}}(\eta_{k}) f_{k|k-1}^{i,a^{i}}[1 - P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k}) | \eta_{k}]\right) \\ f_{k}^{i,a^{i}}(x_{k}|\eta_{k}) &= \frac{(1 - P_{\mathrm{D}}(x_{k},\eta_{k}^{k})) f_{k|k-1}^{i,a^{i}}(x_{k} | \eta_{k})}{f_{k|k-1}^{i,a^{i}}[1 - P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k}) | \eta_{k}]}. \end{split}$$

The set D(a) contains previously established tracks which according to the hypothesis a produced detections at the

current time step. For these tracks we have

$$w_{k}^{i,a^{i}}(\eta_{k}) = w_{k|k-1}^{i,a^{i}}(\eta_{k})r_{k|k-1}^{i,a^{i}}(\eta_{k})$$
$$f_{k|k-1}^{i,a^{i}}[f_{z}(z_{k}^{a^{i}_{k}} \mid \tilde{x}_{k}, \eta_{k}^{k})P_{D}(\tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}]$$
$$r_{k}^{i,a^{i}}(\eta_{k}) = 1$$
$$f_{k}^{i,a^{i}}(x_{k}|\eta_{k}) = \frac{f_{z}(z_{k}^{a^{i}_{k}} \mid x_{k}, \eta_{k}^{k})P_{D}(x_{k}, \eta_{k}^{k})f_{k|k-1}^{i,a^{i}}(x_{k}|\eta_{k-1})}{f_{k|k-1}^{i,a^{i}}[f_{z}(z_{k}^{a^{i}_{k}} \mid \tilde{x}_{k}, \eta_{k}^{k})P_{D}(\tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}]}$$

This completes the specification of the posterior parentconditional daughter p.g.fl. Notice that the weights $w_k^{i,a^i}(\eta_k)$ are not normalized. In practice, one may want to replace the MeMBer-component (9) by something simpler. We discuss possible approximation strategies in Sections 8 and 9.

Let us then look at the parent update. Based on the formal definition of $H[h | \eta_k]$, given in Appendix B, it can be shown that $H[h | \eta_k]$ satisfies the requirement

$$G_k[g,h] \propto f_{\Upsilon,k|k-1}[g(\tilde{\eta}_k)H[h\,|\,\tilde{\eta}_k]]. \tag{11}$$

Assuming that $H[h | \eta_k]$ is given, we can obtain the posterior parent pdf as follows: If we equate the expressions in (6) and (11), and calculate their functional derivatives with regard to η_k , we obtain

$$f_{\Upsilon,k}(\eta_k)G_{\Xi,k}[h \mid \eta_k] \propto f_{\Upsilon,k|k-1}(\eta_k)H[h \mid \eta_k].$$

It follows from (7) that $G_{\Xi,k}[1 | \eta_k] = 1$, as one would expect. Based on this we can update the parent pdf according to

$$f_{\Upsilon,k}(\eta_k) \propto H[1 \mid \eta_k] f_{\Upsilon,k|k-1}(\eta_k) \tag{12}$$

where

$$H[1 \mid \eta_k] = \sum_{a \in \mathcal{A}_k} \prod_{i \in \mathcal{T}_k} w_k^{i,a^i}(\eta_k).$$
(13)

In other words, the normalization constant in (7) is the likelihood for the parent. This completes the formal development of the single-cluster recursion in terms of p.g.fl.'s. It is readily apparent that the the posterior p.g.fl. is of the same form as the prior p.g.fl.

8. RELATIONSHIP TO PHD FILTERS

In the previous section we established an exact (although computationally intractable) solution to the single-cluster filtering problem. To the best of our knowledge, all previous attempts at addressing this problem in a similarly rigorous manner have utilized PHD approximations of the daughter process. In this section we will compare the general formulation with two such approximations, proposed in [3] and [19].

The PHD approximation of Lee et al.

The SC-PHD filter proposed in [2, 3] can be derived from our general formalism if two approximations are employed. First, the daughter process $G_{\Xi,k-1}[h | \eta_{k-1}]$ at time step k-1is assumed to be Poisson (i.e. no MeMBer-component). Second, the full MeMBer-Poisson measurement update is replaced by a standard PHD filter update for the daughter process. To make all this more precise, we assume that the prior p.g.fl. is on the form

$$G_{k-1}[g,h] = f_{\Upsilon,k-1}[g(\tilde{\eta}_{k-1})G_{\Xi,k-1}[h \mid \tilde{\eta}_{k-1}]]$$

where

$$G_{\Xi,k-1}[h \,|\, \eta_{k-1}] = \exp(v_{k-1}[h-1 \,|\, \eta_{k-1}])$$

is a Poisson p.g.fl. for some intensity function $v_{k-1}(x_{k-1} | \eta_{k-1})$ which depends on the previous parent η_{k-1} , and where $g : \mathbb{R}^{(k-1)\tau} \to [0,\infty)$. Propagating this to the next time step we obtain

$$G_{k|k-1}[g,h] = f_{\Upsilon,k|k-1}[g(\tilde{\eta}_k)G_{\Xi,k|k-1}[h \,|\, \tilde{\eta}_k^{k-1:k}]]$$

where we now have $g: \mathbb{R}^{k\tau} \to [0, \infty)$,

$$G_{\Xi,k|k-1}[h \mid \eta_k] = \exp(v_{k|k-1}[h-1 \mid \eta_k^{1:k-1}])$$
$$v_{k|k-1}(x_k \mid \eta_k^{1:k-1}) = \mu(x_k) + \int v_{k-1}(x_{k-1} \mid \eta_k^{1:k-1})$$
$$P_{\mathrm{S}}(x_{k-1})f_x(x_k \mid x_{k-1})\mathrm{d}x_{k-1} \quad (14)$$

and $f_{\Upsilon,k|k-1}(\eta_k)$ is as given in (4).

In the measurement-update, SC-PHD approximates the posterior daughter p.g.fl. by a Poisson-model

$$G_{\Xi,k}[h \mid \eta_k] = \exp(v_k[h(\tilde{x}_k) - 1 \mid \eta_k])$$

where the underlying intensity $v_k(x_k \mid \eta_k)$ is found through a standard PHD-filter update

$$v_{k}(x_{k} \mid \eta_{k}) = v_{k|k-1}(x_{k} \mid \eta_{k}) \left((1 - P_{\mathrm{D}}(x_{k}, \eta_{k}^{k})) + \sum_{z \in Z_{k}} \frac{P_{\mathrm{D}}(x_{k}, \eta_{k}^{k}) f_{z}(z \mid x_{k}, \eta_{k}^{k})}{\lambda(z) + v_{k|k-1} [P_{\mathrm{D}}(\tilde{x}_{k}, \eta_{k}^{k}) f_{z}(z \mid \tilde{x}_{k}, \eta_{k}^{k}) \mid \eta_{k}]} \right).$$
(15)

The parent update is similar to (12). Since there are no previous tracks, the functional $H[h | p_k]$ only contains components corresponding to the Poisson-component $H^{\text{ppp}}[h | \eta_k]$ and to the set B(a) of newborn tracks. Mathematically, we can express this as

$$f_{\Upsilon,k}(\eta_k) \propto f_{k|k-1}(\eta_k) H^*[1 | \eta_k].$$
 (16)

where

$$H^*[1 | \eta_k] = \exp(v_{k|k-1}[(1 - P_{\mathrm{D}}(\tilde{x}_k, \eta_k^k)) | \eta_k]) \\ \times \prod_{z \in Z_k} (\lambda(z) + v_{k|k-1}[P_{\mathrm{D}}(\tilde{x}_k, \eta_k^k) f_z(z | \tilde{x}_k, \eta_k^k) | \eta_k]).$$

The PHD approximation of Mullane et al.

Mullane et al. [19, 20] also proposed a solution to SLAM based on the PHD approximation. The prediction and update of the daughter distribution was done using standard PHD techniques similar to (14) and (15). The parent update did, however, differ from the framework suggested here and in [2]. As pointed out at the end of Section 7, the parent likelihood is proportional to the normalization constant in the Bayes-update of the daughter process. A similar observation

was made in [19]. Using only manipulations of multiobject densities, Mullane showed that the normalization constant

$$f_{\Sigma}(Z_k \mid Z_{1:k-1}, \eta_k) = \int f_{\Sigma}(Z_k \mid X_k, \eta_k^k) \\ \times f_{\Xi,k|k-1}(X_k \mid \eta_k) \, \delta X_k \quad (17)$$

plays the role of the likelihood during the vehicle pose update. In order to obtain an expression for this quantity, Mullane used the following relationship:

$$f_{\Sigma}(Z_k \mid Z_{1:k-1}, \eta_k) = \frac{f_{\Sigma}(Z_k \mid X_k, \eta_k^k) f_{\Xi,k|k-1}(X_k \mid \eta_k)}{f_{\Xi,k}(X_k \mid \eta_k)}$$

Here, the likelihood $f_{\Sigma}(Z_k | X_k, \eta_k^k)$ is a standard multiobject likelihood, similar to (12.139) in [9], while $f_{\Xi,k|k-1}(X_k | \eta_k)$ and $f_{\Xi,k}(X_k | \eta_k)$ are the predicted and posterior multiobject Poisson densities of the daughter process, which correspond to the p.g.fl.'s $G_{\Xi,k|k-1}[h | \eta_k]$ and $G_{\Xi,k}[h | \eta_k]$.

Thus, if expressions for $f_{\Xi,k|k-1}(\cdot | \eta_k)$ and $f_{\Xi,k}(\cdot | \eta_k)$ are available, then any dummy realization X_k of the daughter RFS can be inserted in these so that $f_{\Sigma}(Z_k | Z_{1:k-1}, \eta_k)$ is obtained. The updated trajectory pdf is then found as

$$f_{\Upsilon,k}(\eta_k) \propto f_{\Sigma}(Z_k | Z_{1:k-1}, \eta_k) f_{\Upsilon,k|k-1}(\eta_k).$$

Since any dummy set can be used, Mullane suggested very simple candidates for the dummy set such as the empty set. Furthermore, both $f_{\Xi,k|k-1}(\cdot | \eta_k)$ and $f_{\Xi,k}(\cdot | \eta_k)$ were approximated by multi-object Poisson densities as given by the PHD prediction and update, respectively.

However, from the results in Section 7 it follows that $f_{\Sigma}(Z_k | Z_{1:k-1}, \eta_k)$ is proportional to $H[1 | \eta_k]$. Since we have a closed-form expression for $H[1 | \eta_k]$ both in the general case and under the assumption that $f_{\Xi,k|k-1}(\cdot | \eta_k)$ is Poisson, the utility of dummy sets in the parent likelihood appears questionable.

9. PRELIMINARY IMPLEMENTATION

Practical implementation of the formalism developed in Section 7 is currently a topic of active research. In this section we make some general remarks about this, before we outline a preliminary implementation in the context of SLAM for data recorded by a blazed array sonar mounted on an autonomous underwater vehicle (AUV). This data set has previously been discussed in [31].

General considerations

Clearly, evaluation of the joint posterior density, as given by $G_k[g, h]$, is computationally infeasible, even more so than for conventional multi-target tracking. Several approximative strategies are conceivable:

- A1 One can sample random parent trajectories, and evaluate a multi-hypothesis solution for the daughter process conditional on such samples.
- A2 One can sample both association hypotheses and parent trajectories at random, and evaluate a kinematic pdf of the daughter process conditional on such samples.
- A3 One can evaluate a multi-hypothesis solution for joint parent-and-daughter vectors.

Strategy A1 is in some sense the most natural solution to the single-cluster filtering problem, and will be pursued below. In such a Rao-Blackwellized particle filter, following along the lines of [3], [18] and [19], each particle q contains a parent vector $\eta_k^{(q)}$ and a representation of the daughter process as given by some approximation of $G_{\Xi,k}[h \mid \eta_k^{(q)}]$. In a standard sequential importance-sampling resampling (SIR) implementation we assume that the particles after the update at time step k - 1 are distributed according to $f_{\Upsilon,k-1}(\eta_{k-1})$. We then propose new particles according to the Markov model, so that the particles at time step k, before the update, are distributed according to $f_{\Upsilon,k|k-1}(\eta_k)$. We then calculate the weight of particle q as proportional to $H[1 \mid \eta_k^{(q)}]$, resample, and proceed to the next cycle.

Strategy A2 may possibly lead to more economic implementations than A1. Indeed, the celebrated FastSLAM method [21] follows along this line of thought. A key difference between FastSLAM and the FISST-based machinery developed above is that FastSLAM does not utilize information about false alarm rate, detection probability etc. From an intuitive point of view, such information should affect our confidence in navigation and landmark estimates. The inclusion of such information in a FastSLAM-like method would be an interesting extension of the research reported in this paper.

Strategy A3 is a more conventional multi-hypothesis approach, which avoids trajectory sampling and particle filtering altogether. Such a solution may not be able to exploit the MeMBer-Poisson factorization. Instead, the joint posterior could be expressed in terms of conventional association hypotheses such as those used in [1, 15, 16]. Such an approach will arise if we combine our previous work on multi-hypothesis scan matching [1] with Mori's general MHT-formalism [16].

Assumptions and problem statement

We apply a Rao-Blackwellized particle filter based on strategy A1 to 8 scans of sonar data, recorded as part of the experiment reported in [31]. Mine-like objects were deployed in the area before experiments, and the goal is to discover these. For this problem the pose vector is $p_k \in \mathbb{R}^6$, containing northing, easting (together denoted ρ_k), vehicle orientation (denoted ψ_k), surge velocity, sway velocity and rotation rate. The landmarks are of the form $x_k^i \in \mathbb{R}^2$, containing north and east positions of the landmarks. Measurements are of the form $z_k^j \in \mathbb{R}^2$, containing range and bearing. We assume that such measurements have been extracted by means of a constant false alarm rate (CFAR) criterion followed by a clustering procedure [32].

If the measurement z_k^j originates from landmark x_k^i , then its likelihood is

$$f_z(z_k^j | x_k^i, p_k) = \mathcal{N}(z_k^j; \mathbf{h}(x_k^i, p_k), \mathbf{R})$$
(18)

where

$$\mathbf{h}(x_k^i, p_k) = \mathbf{f}(\mathbf{g}(x_k^i, p_k)) \tag{19}$$

$$\mathbf{f}\left(\left[\begin{array}{c}x\\y\end{array}\right]\right) = \left[\begin{array}{c}\sqrt{x^2 + y^2}\\\operatorname{atan2}(y,x)\end{array}\right]$$
(20)

$$\mathbf{g}(x_k^i, p_k) = \mathsf{R}(\psi_k)^{\mathsf{T}}(x_k^i - \rho_k), \qquad (21)$$

where $\mathbf{R} = \text{diag}([(0.1 \text{ m})^2, (0.29^\circ)^2])$ is the measurement

noise covariance and where R is a rotation matrix as given by

$$\mathsf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi\\ \sin\psi & \cos\psi \end{bmatrix}.$$
 (22)

The detection probability is given by

$$P_{\rm D}(x_k^i, p_k) = \begin{cases} 0.5 & \text{if } \mathbf{h}(x_k^i, p_k) \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases}$$

where $S = [1.24 \text{ m}, 40.0 \text{ m}] \times [-22.5^{\circ}, 22.5^{\circ}]$ is the field of view (FOV) in sensor coordinates, that is, polar coordinates. Both the birth density and the survival probability are fixed at $\mu(x) = 0$ and $P_{\rm S}(x) = 1$, respectively. In other words, we do not consider birth and deaths of landmarks. The false alarm intensity is

$$\lambda(z) = \begin{cases} P_{\rm FA}/(\Delta r \Delta \theta) & \text{if } z \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases}$$

where $\Delta r = 0.082 \,\mathrm{m}$ and $\Delta \theta = 0.088^{\circ}$ quantify the sensor resolution and $P_{\mathrm{FA}} = 10^{-5}$ is the false alarm rate. The initial PHD of unobserved targets is

$$v_0(x_0) = \begin{cases} 5.4 \cdot 10^{-4} \,\mathrm{m}^{-2} & \text{if } x \in [-100 \,\mathrm{m}, 10 \,\mathrm{m}] \\ \times [-10 \,\mathrm{m}, 320 \,\mathrm{m}] \\ 0 & \text{otherwise.} \end{cases}$$

This PHD, which corresponds to a belief that about 20 landmarks are present in the entire surveillance region, is represented by a discrete grid containing 100×100 cells. The initial MeMBer component $G_{\Xi,0}^{\text{mbm}}[h | \eta_0]$ contains only one hypothesis with zero tracks. The Markov model of the landmarks is a delta-function $f_x(x_k|x_{k-1}) = \delta_{x_{k-1}}(x_k)$, implying stationarity of the landmarks. Finally, the Markov model of the pose vector is given by $f_p(p_k|p_{k-1}) = \mathcal{N}(p_k; \bar{p}_{k|k-1}, Q)$ where

$$\bar{p}_{k|k-1} = \begin{bmatrix} \mathbf{I}_3 & \Delta t \begin{bmatrix} \mathsf{R}(\psi_{k-1}) & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & 1 \end{bmatrix} \\ \mathbf{0}_{3\times 3} & \mathbf{I}_3 \end{bmatrix} p_{k-1}$$

is the predicted pose vector conditional the previous pose vector, and Q is a process noise matrix, which is estimated from GPS-, DVL- and compass data.

Particle filter implementation and simplifications

In order to turn the general formalism of Section 7 into a tractable method, we invoke the following approximations and simplifications:

- S1 The vehicle path posterior $f_{\Upsilon,k}(\eta_k)$ is approximated by $N_p = 20000$ particles. We use a standard particle filter based on sequential importance-sampling resampling (SIR). Notice that this automatically provides sampling of the full vehicle trajectory η_k , and not just of the current pose vector p_k . For analysis we store the initial pose vector $p_0 = \eta_0$ of each trajectory.
- S2 The PHD of unobserved targets is evaluated over a discrete grid. Furthermore, we make the approximation $v_k(x_k | \eta_k) \approx v_k(x_k)$. That is, we treat the PHD of unobserved targets as independent of the vehicle pose, so that a single grid can be used instead of N_p grids.
- S3 The vehicle-conditional MeMBer-component of the landmark posterior is represented as a collection of tracks



Figure 1. Flowchart of the implemented algorithm.

with corresponding existence probabilities and Gaussian pdf's. Each particle contains the same number of tracks, in order to facilitate parallelization.

- S4 During each estimation cycle, associations between these tracks and the measurements give rise to several hypotheses. We enumerate all such hypotheses, subject to Simplification S5 below. This is in contrast to previous approaches [3, 18, 19, 21]. Nevertheless, for the sake of computational tractability, we merge this collection into a single hypothesis after each measurement update. The technique used for this is essentially the joint integrated probabilistic data association (JIPDA). The formulas used can be found in [33].
- S5 Validation gating with a threshold of 6 standard deviations [1] is employed to avoid the construction of hypotheses with unlikely measurement-to-track assignments.
- S6 Tracks with existence probabilities less than 10^{-4} are pruned.
- S7 Similar tracks are merged. This is done if the distance between the state estimates of two tracks is less than 1 m.

Of these simplifications, S4 and S7 are most drastic. Let us also mention that S4 does not necessarily have to be performed along the lines of the JIPDA. More refined graphtheoretical techniques have been proposed in [34], and future research may investigate application of such techniques in SLAM. S7 is, at least in the current implementation, largely heuristic. Future research may suggest more rigorous ways of performing S7.



Figure 2. Extracted measurements processed by the filter.

The workflow of the proposed method is illustrated in Figure 1. Previous measurements from GPS and DVL are used to establish a Gaussian prior $f_{\Upsilon,0}(p_0)$, from which the initial particles are drawn. During each iteration, new particles are proposed by means of the kinematic prior. For each particle, track and measurement, gating is carried out and likelihoods and extended Kalman filter (EKF) estimates are calculated. For brevity, the exact details are omitted. Based on the output from this procedure, all feasible W-hypotheses are established. The same hypotheses are utilized for all particles, although their kinematic details will differ. For each particle, hypothesis and track we then obtain trajectoryconditional kinematic pdf's as represented by expectations and covariances, as well as track weights and existence probabilities. Particle weights are then updated by means of the fundamental formula (13). Subsequently, each particle's hypothesis collection is merged into a single hypothesis, followed by track pruning and track merging.

Results

The 8 scans of extracted measurements are depicted in Figure 2. By visual inspection, it seems reasonably clear that there are 4 landmarks in this region.

Let us first take a birds-eye perspective. An overview of the general setup and estimation results after k = 8 time steps is displayed in Figure 3. The figure illustrates how the PHD of unobserved landmarks is strongly reduced inside the FOV. This is explained as follows: Since we already have searched for new targets in this region, we are reasonably confident that we are unlikely to find any more targets. On the other hand, we have no information about what lurks outside of this region, and the original PHD is therefore unaltered for grid cells that have not been touched by the FOV.

Since the particle filter naturally involves conditioning on vehicle trajectories, we can study the landmark multiobject density conditional on any vehicle pose $p_l = \eta_k^l$ for $l \in \{0, \ldots, k\}$ by averaging some kind of projection of the landmark statistics into the body frame given by $\eta_k^{l,(q)}$ over all the particles $q \in \{1, \ldots, N_p\}$. We can for example study the landmark PHD conditional on the initial vehicle pose p_0 , which formally is given by

$$\begin{split} &\int \frac{\delta}{\delta x} G_{\Xi,k}[h\,;\,\eta_k] \mathrm{d}\eta_k^{1:k} = \int v_k(x\,|\,\eta_k) \\ &+ \frac{1}{H[1|\eta_k]} \sum_{a \in \mathcal{A}_k} \sum_{i \in \mathcal{T}_k} w_k^{i,a^i}(\eta_k) r_k^{i,a^i}(\eta_k) f_k^{i,a^i}(x\,|\,\eta_k) \mathrm{d}\eta_k^{1:k} \end{split}$$

The function that results from this at k = 8 is displayed in Figure 4. Confident and very accurate estimates of all 4 landmarks are obtained. Also notice how the PHD of unobserved targets decreases as we enter the FOV. A similar, but slightly different, function is displayed in Figure 5, where the marginalization is done over the *entire* trajectory, including the initial pose $\eta_k^0 = p_0$. Thus, the landmark statistics are here visualized relative to the world frame. In this case, the uncertainty of the landmark estimates becomes considerably larger, reflecting the large prior uncertainty which simply cannot be eliminated by SLAM alone. Also notice that despite the large number of particles, a "shotgun effect" can be observed. This indicates that data-dependent sampling, along the lines of FastSLAM 2.0 [21], may be required for truly genuine evaluation of the joint posterior for a problem with model assumptions such as those stated above.

Let us then look at things from a more low-level perspective. At each time step, the posterior is a mixture over a multitude of W-hypotheses. Two such hypotheses, which both are reasonably plausible at k = 8, are (using the matrix representation proposed in Section 3) given by

$ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} $	$ \begin{array}{c} 2 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \\ 1 \end{array} $	$ \begin{array}{c} 3 \\ 2 \\ 3 \\ 3 \\ 4 \\ 3 \\ 4 \\ 3 \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	0 0 0 0 0 0 0 0 0	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	0 0 0 0 0 0 0 0 0	$egin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$	$egin{array}{c} 0 \\ 0 \\ 2 \\ 3 \\ 0 \\ 3 \\ 2 \end{array}$	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	····] ····] ····] ····]	
and by $\begin{bmatrix} 1\\1\\1\\1\\1\\1\\1\\1\\1\\0 \end{bmatrix}$	2 0 0 2 2 2	3 0 0 0 0 0 0 0		$ \begin{array}{c} 0 \\ 2 \\ 3 \\ 3 \\ 4 \\ 3 \\ 4 \\ 3 \\ 4 \\ 2 \\ 7 \end{array} $		$ \begin{array}{c} 0 \\ 0 \\ 2 \\ 0 \\ $	0 0 0 0 0 0 0 0		$ \begin{array}{c} 0 \\ 0 \\ 2 \\ 3 \\ 0 \\ 3 \\ 2 \end{array} $	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	 	

where both matrices contain a total of 23 columns. The first submatrix (in both matrices) corresponds to tracks established at k = 1, the second submatrix corresponds to tracks established at k = 2, and so on. According to both hypotheses there are four relatively stable tracks from k = 4 onwards. In the first hypothesis, the third track (which is on the target furthermost away from the AUV) is established with basis in z_1^3 . In the second hypothesis, the track originating with z_1^3 is not continued. However, a similar track is established with basis in z_2^2 instead. Thus the second hypothesis suggests that z_1^3 is a false alarm or a lone measurement from a target which thereafter is never observed again. Notice that the hypothesis does not specify whether z_1^3 is a false alarm or the lone measurement from a target that only is seen once.



Figure 3. Overview of estimation results after 8 time steps. Landmark estimates and FOV's of 10 randomly selected particles are shown. The PHD of unobserved landmarks almost vanishes inside the FOV.



Figure 4. Occupancy grid (PHD) for landmarks after 8 time steps in body frame conditional on p_0 .



Figure 5. Occupancy grid (PHD) for the landmarks after 8 time steps in the world frame.

$a_1 = [1]$	0	2	0	0]
$a_2 = [1]$	0	0	0	2]
$a_3 = [0]$	0	2	1	0]
$a_4 = [0$	0	0	1	2]

which constitute the hypothesis collection of the practical method at k = 2. In Figure 6 we take a closer look at the MeMBer weights for two particles at k = 2. Visual inspection reveals that particle number 1 (with parent vector $\eta_2^{(1)}$) has better match between its landmark estimates and the measurements than particle number 2 (with parent vector $\eta_2^{(2)}$). Consequently, the w_2^{i,a^i} for detected tracks $i \in D(a)$ are higher for particle number 1 than for particle number 2. Misdetected tracks $i \in M(a)$ do not affect the particle weights very much, and non-existing tracks $i \in N(a)$ do not contribute at all. Newborn tracks $i \in B(a)$ yield low values of w_2^{i,a^i} . This is partly so because most of the FOV at k = 2 has already been surveyed once at k = 1, and thus we may expect, with some probability, that any targets present already have been observed.

10. SUMMARY

In this paper we have suggested that the probability generating functional (p.g.fl.) for a cluster process should be defined as a functional working on two univariate test-functions: One for the parent process and one for the daughter process. We have used this, together with results from [13], to establish expressions for the full Bayes-recursion in single-cluster filtering. These expressions are, in particular, applicable to problems in simultaneous localization and mapping (SLAM).

Two important observations should be pointed out. First, the solution naturally involves conditioning on the entire parent trajectory, suggesting implementation in terms of Rao-Blackwellized particle filters. Second, the predicted density of the parent process is transformed into the posterior density of the parent process through multiplication by an unnormalized version of the daughter p.g.fl. We have discussed how the general formalism proposed here relates to the recent SLAM methods proposed in [3] and [19], and we have presented a preliminary and simplified implementation of the general formalism on sonar data.

In future work we intend to continue along this path by developing robust and practical approximations of the general formalism based on multi-hypothesis and multi-Bernoulli techniques. We also plan to investigate the extent to which significant performance loss is incurred by such approximations, or by the the approximations employed in [3] and [19]. Also, it is of great interest to extend the p.g.fl. formulation discussed in this paper to general cluster processes. Research in this direction is currently being conducted [28].



Figure 6. Interpretation of the track weights w_k^{i,a^i} at k = 2. Here we have displayed the landmark estimates (triangles) for two particles, together with the measurements (circles) projected into the world frame according to the two particles' pose vectors. The first particle (blue) gets a considerable score due to good match between its landmark estimates and its measurements under the first hypothesis a_1 . The second particle (red) provides an inferior match between landmark estimates and measurements, and its score therefore only comes from the pure misdetection hypothesis a_4 .

APPENDICES

A. DERIVATION OF THE PREDICTION STEP

The predicted joint p.g.fl. at time step k is given by

$$\begin{split} G_{k|k-1}[g,h] = & f_{\Upsilon,k-1}[f_{\eta}[g(\tilde{\eta}_{k}) \, G_{\Gamma}[h] \\ G_{\Xi,k-1}[G_{\Psi,k|k-1}[h \, | \, \tilde{x}_{k-1}] \, | \, \tilde{\eta}_{k-1}] \, | \, \tilde{\eta}_{k-1}]]. \end{split}$$

All the functionals involved in this expression have previously been defined as part of the prior or as part of the Markov model. Recall that Γ is the RFS of newborn daughters, while Ψ is the RFS of a daughter that with probability $P_{\rm S}(x_{k-1})$ survives from time step k – 1 to time step k. Let us define $G^*_{\Xi,k|k-1}[h|\eta_{k-1}] = G_{\Gamma}[h]G_{\Xi,k-1}[G_{\Psi,k|k-1}[h|\tilde{x}_{k-1}]|\tilde{\eta}_{k-1}]$. In other words, $G^*_{\Xi,k|k-1}[h|\eta_{k-1}]$ is the predicted daughter p.g.fl. conditional on the *previous* parent η_{k-1} . We then find the predicted joint p.g.fl. as

$$\begin{split} G_{k|k-1}[g,h] &= \int g(\eta_k) \int f_{\Upsilon,k-1}(\eta_{k-1}) f_{\eta}(\eta_k \mid \eta_{k-1}) \\ &\times G_{\Xi,k|k-1}^*[h \mid \eta_{k-1}] \mathrm{d}\eta_{k-1} \mathrm{d}\eta_k \\ &= \int g(\eta_k) \int f_{\Upsilon,k-1}(\eta_{k-1}) f_p\left(\eta_k^k \mid \eta_{k-1}^{k-1}\right) \\ &\times \prod_{l=1}^{k-1} \delta_{\eta_{k-1}^l}\left(\eta_k^l\right) G_{\Xi,k|k-1}^*[h \mid \eta_{k-1}] \mathrm{d}\eta_{k-1} \mathrm{d}\eta_k \\ &= \int g(\eta_k) f_{\Upsilon,k-1}(\eta_k^{1:k-1}) f_p\left(\eta_k^k \mid \eta_k^{k-1}\right) \\ &\times G_{\Xi,k|k-1}^*[h \mid \eta_k^{1:k-1}] \mathrm{d}\eta_k. \end{split}$$

If we now define

$$\begin{aligned} G_{\Xi,k|k-1}[h \mid \eta_k] = & G_{\Xi,k|k-1}^*[h \mid \eta_k^{1:k-1}] \\ f_{\Upsilon,k|k-1}(\eta_k) = & f_{\Upsilon,k-1}(\eta_k^{1:k-1}) f_p\left(\eta_k^k \mid \eta_k^{k-1}\right) \end{aligned}$$

then we obtain the desired form

$$G_{k|k-1}[g,h] = f_{\Upsilon,k|k-1}[g(\tilde{\eta}_k)G_{\Xi,k|k-1}[h \,|\, \tilde{\eta}_k]].$$

The factorization of $G_{\Xi,k|k-1}[h | \eta_k]$ into a Poissoncomponent and MeMBer-component, as well as the details of these components can be directly translated from the MTT formulation of [13]. Consequently, we refer to Appendix A in [13] for further details.

B. DERIVATION OF THE UPDATE STEP

A key tenet of FISST is that Bayes' rule is valid for multiobject densities, just as it is valid for ordinary pdf's and probabilities. In the p.g.fl. domain, the measurement update is represented by a bivariate p.g.fl. which takes one test-function defined on the measurement space \mathbb{R}^s and one test-function defined on the target space \mathbb{R}^d . The updated target-p.g.fl. is then obtained as a the functional derivative of this p.g.fl. with regard to the observed measurement set.

For single-cluster filtering, Bayes' rule can be written

$$f_{\mathbb{X},k}(\eta_k, X_k) = f_{\Sigma}(Z_k \mid \eta_k, X_k) f_{\mathbb{X},k|k-1}(\eta_k, X_k) / \int \int f_{\Sigma}(Z_k \mid \eta_k, X_k) f_{\mathbb{X},k|k-1}(\eta_k, X_k) \mathrm{d}\eta_k \delta X_k.$$

Our aim is to obtain the posterior joint density $f_{\mathbb{X},k}(\eta_k, X_k)$ or its corresponding p.g.fl.-representation. By factorizing the posterior into a parent component and a daughter component according to $f_{\mathbb{X},k}(\eta_k, X_k) = f_{\Upsilon,k}(\eta_k)f_{\Xi,k}(X_k|\eta_k)$ we can rewrite this as

$$f_{\Upsilon,k}(\eta_k)f_{\Xi,k}(X_k|\eta_k) = \frac{f_{\Sigma}(Z_k|\eta_k, X_k)f_{\Upsilon,k|k-1}(\eta_k)f_{\Xi,k|k-1}(X_k|\eta_k)}{\int \int f_{\Sigma}(Z_k|\eta_k, X_k)f_{\mathbb{X},k|k-1}(\eta_k, X_k)\mathrm{d}\eta_k\delta X_k}$$

In order to transform this to the p.g.fl. domain, we generalize Mahler's bivariate p.g.fl. to a trivariate p.g.fl. F[l, g, h] whose test-functions are defined on the measurement space \mathbb{R}^s , parent space \mathbb{R}^{τ} and daughter space \mathbb{R}^d , respectively. We define this entity according to

$$F[l,g,h] = f_{\Upsilon,k|k-1} \left[g(\tilde{\eta}_k) \int h^{X_k} G_{\Sigma}[l \mid X_k, \tilde{\eta}_k] \right]$$
$$f_{\Xi,k|k-1}(X_k|\tilde{\eta}_k) \, \delta X_k$$

where Σ is the full measurement set comprising both target measurements and clutter measurements, and $G_{\Sigma}[l | X_k, \eta_k]$ is the corresponding p.g.fl. It can be shown that its functional derivative with respect to Z_k is

$$\frac{\delta F}{\delta Z_k}[0,g,h] = f_{\Upsilon,k|k-1} \bigg[g(\tilde{\eta}_k) \\ \times \int h^{X_k} f_{\Sigma}(Z_k \mid X_k, \tilde{\eta}_k) f_{\Xi,k|k-1}(X_k \mid \tilde{\eta}_k) \delta X_k \bigg].$$
(23)

Furthermore, it can be shown that

$$G_{k}[g,h] = f_{\Upsilon,k}[g(\tilde{\eta}_{k})G_{\Xi,k}[h \mid \tilde{\eta}_{k}]] = \frac{\frac{\delta F}{\delta Z_{k}}[0,g,h]}{\frac{\delta F}{\delta Z_{k}}[0,1,1]}.$$
 (24)

Thus, we can find the posterior p.g.fl. $G_k[g, h]$ by differentiating the trivariate functional F[l, g, h]. For the sake of brevity, we do not include detailed proofs of (23) and (24) here. Both (23) and (24) can be proven by steps which are analogous to the steps taken in Appendix G.25 of [9].

Let us look more closely at the details of F[l, g, h]. Invoking the assumptions inherent in the predicted p.g.fl. and the multiobject likelihood from Section 6 yields

$$F[l,g,h] = f_{\Upsilon,k|k-1}[g(\tilde{\eta}_k)F^*[l,h|\tilde{\eta}_k]]$$

where

$$F^{*}[l, h | \eta_{k}] = G_{\Xi, k | k-1}[h(\tilde{x}_{k})G_{\Theta}[l | \tilde{x}_{k}, \eta_{k}] | \eta_{k}]G_{K}[l]$$
(25)

is the parent-conditional part of F[l, g, h]. The fact that $f_{\Upsilon,k|k-1}[g]$ is a linear functional implies that⁵

$$\frac{\delta F}{\delta Z_k}[l,g,h] = f_{\Upsilon,k|k-1} \left[g(\tilde{\eta}_k) \frac{\delta F^*}{\delta Z_k}[l,h \,| \tilde{\eta}_k] \right].$$
(26)

In order to proceed, we want to establish a concrete expression for $(\delta F^*/\delta Z_k)[l, h | \eta_k]$. For this, let us first recall that the predicted daughter p.g.fl. $G_{\Xi,k|k-1}[h|\eta_k]$ is a sum over W-hypotheses in the set \mathcal{A}_{k-1} . Therefore, $F^*[l, h | \eta_k]$ is also a sum over W-hypotheses in the set \mathcal{A}_{k-1} . Furthermore, let us recall the structures of the Poisson- and MeMBercomponents of $G_{\Xi,k|k-1}[h|\eta_k]$, and let us recall that Θ as it appears in (25) is a Bernoulli RFS defined for any currently existing target, and that K is a Poisson RFS. Combining all this yields

$$\frac{\delta F^*}{\delta Z_k}[l,h \mid \eta_k] = \sum_{a \in \mathcal{A}_{k-1}} \frac{\delta}{\delta Z_k} \left(F_0[l,h \mid \eta_k] \right)$$
$$\prod_{i \in \mathcal{T}_{k-1}} F_a^i[l,h \mid \eta_k] \right)$$
(27)

where

$$\begin{split} F_{0}[l,h \mid \eta_{k}] &= \exp \left(\lambda[l] + v_{k|k-1}[h(\tilde{x}_{k})(1-P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k}) \\ &+ P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})f_{z}[l \mid \tilde{x}_{k},\eta_{k}^{k}]) \mid \eta_{k}] \right) \\ F_{a}^{i}[l,h \mid \eta_{k}] &= w_{k|k-1}^{i,a^{i}}(\eta_{k}) \left(1 - r_{k|k-1}^{i,a^{i}}(\eta_{k}) \\ &+ r_{k|k-1}^{i,a^{i}}(\eta_{k})f_{k|k-1}^{i,a^{i}}[h(\tilde{x}_{k})(1-P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k}) \\ &+ P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})f_{z}[l \mid \tilde{x}_{k},\eta_{k}^{k}]) \mid \eta_{k}] \right). \end{split}$$

Thus, for each $a \in A^{k-1}$ we must evaluate the functional derivative of a product containing $n_{k-1} + 1 = |\mathcal{T}_{k-1}| + 1$ factors. This is done by means of Mahler's general product rule [9 p. 395], which in our case reads

$$\frac{\delta}{\delta Z_k} (F_0[l,h|\eta_k]F_a^1[l,h|\eta_k]\cdots F_a^{n_{k-1}}[l,h|\eta_k])$$
$$= \sum_{W_0 \uplus \dots \uplus W_{n_{k-1}}=Z_k} \frac{\delta F_0}{\delta W_0}[l,h|\eta_k]\cdots \frac{\delta F_a^{n_{k-1}}}{\delta W_{n_{k-1}}}[l,h|\eta_k].$$

The derivatives on the right-hand side of this product rule are of three distinct forms:

$$\frac{\delta F_{0}}{\delta Z_{k}}[l,h]\Big|_{l=0} = \exp(v_{k|k-1}[h(\tilde{x}_{k})(1-P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})) \mid \eta_{k}])) \\ \prod_{z \in Z_{k}} \left(\lambda(z) + v_{k|k-1}[h(\tilde{x}_{k})P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})f_{z}(z|\tilde{x}_{k},\eta_{k}^{k}) \mid \eta_{k}]\right) \\ \frac{\delta F_{a}^{i}}{\delta \emptyset}[l,h]\Big|_{l=0} = w_{k|k-1}^{i,a^{i}}(\eta_{k}) \left(1 - r_{k|k-1}^{i,a^{i}}(\eta_{k}) + r_{k|k-1}^{i,a^{i}}(\eta_{k})f_{k|k-1}^{i,a^{i}}[h(\tilde{x}_{k})(1-P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})) \mid \eta_{k}]\right) \\ \frac{\delta F_{a}^{i}}{\delta z}[l,h]\Big|_{l=0} = w_{k|k-1}^{i,a^{i}}(\eta_{k})r_{k|k-1}^{i,a^{i}}(\eta_{k}) \\ f_{k|k-1}^{i,a^{i}}[h(\tilde{x})P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})f_{z}(z\mid\tilde{x}_{k},\eta_{k}^{k}) \mid \eta_{k}]. \tag{28}$$

Notice that all higher-order derivatives of the Bernoullicomponents $F_a^i[l, h | \eta_k]$ are zero. This is because the $F_a^i[l, h | \eta_k]$ is linear in the test function l, implying that its first-order derivative does not depend on l, leading to zero higher-order derivatives. The Poisson-component $F_0[l, h | \eta_k]$ on the other hand, can be differentiated an unlimited number of times.

For any old hypothesis $a \in \mathcal{A}_{k-1}$, the derivative in (27) can be expressed as a sum of $(n_{k-1}!m_k!/(2n_{k-1} - m_k)!) \sum_{c=0}^{m_k} 1/(m_k - c)!$ terms. Each of these terms, when combined with its parent hypothesis a, constitutes a new W-hypothesis. The first of the three derivatives in (28) represents the Poisson-component of the parent-conditional posterior (the exponential factor), as well the new tracks in the MeMBer-component (the product over $z \in Z_k$). The third derivative represents the hypothesis (conditional on the parent hypothesis a) that the target of track number i is misdetected. This includes the possibility that this target may not exist. The fourth derivative represents the hypothesis (conditional

 $^{^{5}}$ This follows from linearity of the integral. It can also be shown using the generalized chain rule proposed in [35], noting that only the single-subset partition in [35 eq. (10)] will yield any contribution.

on the parent hypothesis a) that the target of track number i is detected.

We denote the set of new W-hypotheses by \mathcal{A}_k . For any $a \in \mathcal{A}_k$ we can then partition the track indices $i \in \{1, \ldots, n_k\}$ into the 4 sets N(a), B(a), M(a) and D(a) defined in (10). The set N(a) contains the track indices of tracks that under the hypothesis *a* represent non-existing targets. The set B(a) represents newborn targets and corresponds to the product term in the derivative $(\delta F_0/\delta Z_k)[l, h|\eta_k]$. The set M(a) represents misdetections and corresponds to the derivative $(\delta F_a^i/\delta \emptyset)[l, h|\eta_k]$. The set D(a) represents detected and previously existing targets and corresponds to the derivative $(\delta F_a^i/\delta z)[l, h|\eta_k]$. Based on this we can write

$$\frac{\delta F^*}{\delta Z_k} [l, h \mid \eta_k] \bigg|_{l=0} \propto H[h \mid \eta_k]$$
(29)

where the functional $H[h \mid \eta_k]$ is given by

$$\begin{split} H[h | \eta_k] &= \sum_{a \in \mathcal{A}_k} \exp(v_{k|k-1}[h(\tilde{x}_k)(1 - P_{\mathrm{D}}(\tilde{x}_k, \eta_k^k)) | \eta_k])) \\ &\times \prod_{i \in B(a)} \left(\lambda(z_k^{a_k^i}) \\ &+ v_{k|k-1}[h(\tilde{x})P_{\mathrm{D}}(\tilde{x}_k, \eta_k^k) f_z(z_k^{a_k^i} | \tilde{x}_k, \eta_k^k) | \eta_k] \right) \\ &\times \prod_{i \in M(a)} w_{k|k-1}^{i,a_{1:k-1}^i}(\eta_k) \left(1 - r_{k|k-1}^{i,a_{1:k-1}^i}(\eta_k) \\ &+ r_{k|k-1}^{i,a_{1:k-1}^i}(\eta_k) f_{k|k-1}^{i,a_{1:k-1}^i}[h(\tilde{x}_k)(1 - P_{\mathrm{D}}(\tilde{x}_k, \eta_k^k)) | \eta_k] \right) \\ &\times \prod_{i \in D(a)} \left(w_{k|k-1}^{i,a_{1:k-1}^i}(\eta_k) r_{k|k-1}^{i,a_{1:k-1}^i}(\eta_k) \right) \end{split}$$

$$\times f_{k|k-1}^{i,a_{1:k-1}^{i}}[h(\tilde{x}_{k})P_{\mathrm{D}}(\tilde{x}_{k},\eta_{k}^{k})f_{z}(z_{k}^{a_{k}^{i}} \,|\, \tilde{x}_{k},\eta_{k}^{k}) \,|\, \eta_{k}] \bigg).$$

This is identical to the functional that was described in (8) and subsequent equations. We refer to Appendix B in [13] for further details. It should be noted that (29) defines $H[h | \eta_k]$. That is, $H[h | \eta_k]$ is defined as a functional which is proportional to the derivative in (29) as a function of η_k . This is in contrast to the desired daughter p.g.fl. $G_{\Xi,k}[h | \eta_k]$, which is normalized conditional on η_k , and therefore is not proportional to this derivative. However, $H[h | \eta_k]$ and $G_{\Xi,k}[h | \eta_k]$ are proportional in the test function h when η_k is fixed, and thus $G_{\Xi,k}[h | \eta_k]$ is given by $H[h | \eta_k]$.

As for the parent update, the desired formula (12) follows from comparing (11) with (6), or more precisely from comparing their functional derivatives with regard to η_k . Furthermore, (11) follows from comparing (26) with (24), and inserting (29) into (24).

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