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# Utilizing Orthogonal Coprime Signals for Improving Broadband Acoustic Doppler Current Profilers

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Abstract—Broadband acoustic Doppler current profilers (BBADCPs) are instruments that are widely used for measuring ocean currents. The ambiguity velocity of the conventional method used in BBADCPs must accommodate all possible measurement velocities. Unfortunately, allowing a high-ambiguity velocity results in a high measurement deviation in conventional BBADCPs. We propose a method to break through the limitation imposed by the ambiguity velocity to improve BBADCPs. Our proposed method involves designing an orthogonal coprime signal to replace the conventional transmit signal in BBADCPs. The proposed orthogonal coprime signal consists of two orthogonal subsignals, whose ambiguity velocities are designed to be low and coprime. Utilizing the coprime property, we then employ the robust Chinese remainder theorem to resolve the velocity ambiguity from the two independent measurements made via the two orthogonal subsignals. Simulations show that our proposed method decreases the standard deviation of measurement velocity by nearly three times, compared to the conventional method used in BBADCPs. Our simulations also show that the proposed method can yield a 12-dB improvement of signal-to-noise ratio over the conventional method. This can help increase the profiling range significantly.

*Index Terms*—Broadband acoustic Doppler current profiler (BBADCP), Doppler estimation, orthogonal coprime signal, robust Chinese remainder theorem (CRT), velocity ambiguity.

#### I. INTRODUCTION

COUSTIC Doppler current profilers (ADCPs) have been widely applied to measure ocean currents [1]–[16]. ADCPs can be divided into three categories: narrowband (incoherent) [1], pulse-topulse coherent [2]–[6], and broadband [7]–[16]. Broadband ADCPs (BBADCPs) are able to achieve a combination of the good features of the narrowband and pulse-to-pulse ADCPs: the large range of the former and the high accuracy of the latter [7]. Thus, BBADCPs are more commonly used for long range measurements.

The covariance method, also referred to as the pulse-pair method [15], [16], is a popular Doppler estimation technique used in BBADCPs due to its computational simplicity and high performance. In the covariance method, the transmit signal of BBADCPs consists of several coded

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Fig. 1. Schematic of the covariance method for processing the echoes of BBADCPs. (a) Transmit signal consists of a series of coded pulses. (b) Received signal can be represented as a superposition of backscattered signals from many scatterers. (c) We segment the received signal with the window length  $PT_0$ . (d) Autocorrelation function of the segmented signal. The phase of the peak at lag  $T_0$  is considered for Doppler estimation.

pulses, as shown in Fig. 1. Generally, the transmit signal of BBADCPs is realized by repeating a single coded pulse [8]. The Doppler shift is estimated by extracting the phase of the autocorrelation function of the received complex baseband signal at a lag corresponding to the length of the coded pulse [7], [8]. The radial velocity (beam axis) is obtained based on the estimated Doppler shift. It should be noted that

0364-9059 © 2019 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications\_standards/publications/rights/index.html for more information. the extracted phase can only fall within the range of  $-\pi$  to  $\pi$ . When extracting phases, any other phase (over  $\pi$  or below  $-\pi$ ), which can arise because of a high current velocity being observed, wrap around and fall within the  $(-\pi, \pi)$  limit, which is referred to as the phase ambiguity. The phase ambiguity in turn leads to a velocity ambiguity.

To avoid the velocity ambiguity, conventional BBADCPs require the ambiguity velocity to be configured in advance of system deployment [9]. Because the ambiguity velocity must accommodate all possible velocities we wish to measure, the ambiguity velocity should be set relatively high in such systems. The ambiguity velocity is determined by the length of the single coded pulse. Having a higher ambiguity velocity requires the coded pulse to be shorter. However, setting a high-ambiguity velocity worsens the accuracy of the conventional covariance method [8], [9]. The two tables in [9] illustrate two typical configurations of BBADCPs, which can demonstrate the limit of ambiguity velocity. The different ambiguity velocities are obtained by selecting different lengths of the single coded pulse (53 or 107 code elements) [9]. For the ambiguity velocities 1.74 m/s and 85.9 cm/s, the standard deviations of velocity measurement are 3.46 and 2.43 cm/s. Obviously, a higher ambiguity velocity is accompanied by a higher standard deviation. The theory in [8] can be used to explain the limitation of the ambiguity velocity, which will be shown in Section II. Because the ambiguity velocity limits the performance of conventional BBADCPs, when designing conventional BBADCPs, the tradeoffs between the ambiguity velocity and accuracy must be considered.

From the above, we can see that when fixing the length of the transmit signal, setting a low-ambiguity velocity can decrease the measurement deviation, but it leads to a velocity ambiguity if dealing with large measurement velocities. When using such a transmit signal with a low-ambiguity velocity, if the velocity ambiguity can be resolved robustly, the performance of BBADCPs can be improved. We propose a method based on an orthogonal coprime signal and the robust Chinese remainder theorem (CRT) to relax the limit of the ambiguity velocity of conventional BBADCPs [11]. We design the orthogonal coprime signal to replace the conventional transmit signals in BBADCPs. The designed signal consists of two orthogonal subsignals. The ambiguity velocities of the two orthogonal subsignals are coprime and low. Design of the orthogonal coprime signals will be discussed in Section IV. The "orthogonality" of the two subsignals in our proposed method allows us to obtain two independent simultaneous measurements, which are wrapped-around versions of the true velocity if it is greater than the ambiguity velocities of these subsignals. Using the robust CRT to resolve the velocity ambiguity requires the ambiguity velocities of the two orthogonal signals to be "coprime." The simulations show that when compared to the conventional method in BBADCPs, our proposed method can decrease the standard deviation of measurement by nearly three times and obtain a signal-to-noise ratio (SNR) improvement of 12 dB, which can help increase the profiling range of BBADCPs.

It is well known that the CRT can recover integers and real numbers [19], [20], [24], [25] from their remainders with a reconstruction formula. The CRT has been used in many fields, such as channel coding [24], wireless sensor networks [26], [27], computational neuroscience [28], and coprime arrays [29], [30]. However, the traditional CRT is not robust to the remainder errors [18]–[22]. Recently, the robust CRT has been proposed to resist the errors in the remainders. It has been successfully applied to radar signal processing [31], [32], distance [33] and frequency [19] estimation, multiwavelength optical measurement [34], and other fields. Our proposed method employs the robust CRT to resolve the velocity ambiguity in the measurements using the orthogonal coprime signal.

The rest of this paper is organized as follows. Section II describes the covariance method used by BBADCPs and analyzes the limitation of ambiguity velocity theoretically. Section III introduces the robust CRT. Our proposed method is described in Section IV. An example is given in Section V to further demonstrate how to design the orthogonal coprime signals for BBADCPs. Section VI presents the simulations of evaluating the performances of the proposed and the conventional methods. Finally, Section VII concludes this paper.

#### II. THEORY

The covariance method is introduced in this section. The transmit signal conventionally employed in BBADCPs and the corresponding ambiguity velocity are analyzed. The limit of the ambiguity velocity of conventional BBADCPs is described.

#### A. Covariance Method

The covariance (or "pulse-pair") method [7]–[9], [16] is usually employed in BBADCPs. The transmit signal of BBADCPs consists of a series of the coded pulse. A single coded pulse with length  $T_0$  can be expressed as

$$e(t) = a(t)e^{j2\pi f_0 t} \tag{1}$$

where a(t) is the baseband coded pulse and  $f_0$  is the carrier frequency. Currently, acoustical systems [44]–[46] are able to transmit diverse waveforms. Hence, diverse waveforms are being employed in BBADCPs as well. Both phase-coded (Barker [8], m-sequence [9], [43], and multiple phase [10]) and frequency-modulated [47] pulses have been considered for use in BBADCPs. As reported in [47], the linear frequency-modulated (LFM) (or chirp) pulses have been used in the commercial system AD2CP. The reason frequency-modulated pulses are used in BBADCPs is that they allow more flexible tuning of transmit signal parameters, such as length and bandwidth [47]. Our proprosed orthogonal coprime signals also employ LFM pulses.

When a(t) is phase coded, it can be expressed as

$$a(t) = \sum_{l=1}^{L} \operatorname{rect}\left(\frac{t - l\tau_0}{\tau_0}\right) e^{j\varphi_l}$$
(2)

where rect(x) denotes a rectangular window function, which is unity when its argument  $x \in (0, 1)$ , L is the number of code elements,  $\tau_0$  is the duration of each code element, and  $\varphi_l$  is the phase of the *l*th code element. An LFM pulse [42], [48], [49] can be expressed as

$$a(t) = e^{j2\pi \left(\frac{\mu}{2}t^2\right)}, \quad -T_0/2 \le t \le T_0/2 \tag{3}$$

where  $\mu$  is the sweep rate, which is defined by

$$/T_0$$
 (4)

where  $B_0$  is the bandwidth. The transmit signal of BBADCPs is generally realized by repeating a(t) and can be expressed as

 $\mu = B_0$ 

$$s(t) = \left\{ \sum_{n=0}^{P-1} \left[ a \left( t - nT_0 \right) \right] \right\} e^{j2\pi f_0 t}$$
(5)

where P is the number of the coded pulses a(t) used in s(t). The length of the transmit signal s(t) is

$$T_s = PT_0. (6)$$

When designing a BBADCP,  $T_s$  is generally set to be comparable to the length of the depth cell required, as pointed out in [8] and [9].

Denote the demodulated complex received signal of the BBADCP as  $g_1(t)$ . The first step in processing the received signal is to segment  $g_1(t)$  with a rectangular window to measure the current velocity of the depth cell. In BBADCPs, the segmenting window length  $T_r$ , which is

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Fig. 2. Illustration of the difference in velocity ambiguities under the different lengths  $T_0$  of the individual coded pulses in the transmit signal, with the total length of the transmit signal fixed at  $PT_0$ . The variation of phase measured via (9) against the current velocity encountered. In case (b), the ambiguity velocity is smaller than that by case (a). Thus, when a velocity v is measured, the phase measurement in case (a) wraps around  $\pi$  or  $-\pi$  and yields a different phase as compared to that by case (b).

the depth cell size required, is usually chosen to be equal to the length of the transmit signal s(t) [8], [9]. The segment of the received signal denoted by  $g_2(t)$  can be expressed as

$$g_2(t) = \operatorname{rect}\left(\frac{t - t_0}{PT_0}\right)g_1(t) \tag{7}$$

where  $t_0$  is the start time of the segment window. The Doppler shift estimated by using the covariance method is given by [7]-[10]

$$\hat{f}_d = \frac{1}{2\pi T_0} \Delta \phi \tag{8}$$

where  $\Delta \phi$  is the phase of the autocorrelation function of  $r_2(t)$  at a lag  $T_0$ . The phase  $\Delta \phi$  is extracted as

$$\Delta \phi = \arg \left[ \mathbf{R} \left( \mathbf{T}_0 \right) \right] \tag{9}$$

where  $R(T_0)$  is expressed as

$$R(T_0) = \int_{t_0}^{t_0+T_s} r_2(t) r_2^*(t-\tau) dt|_{\tau=T_0}$$
(10)

where "\*" represents the "complex conjugate" operation. The velocity estimate based on  $\hat{f}_d$  is expressed as

$$\hat{v} = \frac{c}{2f_0}\hat{f}_d \tag{11}$$

where c is the sound speed in water. Using (8), (11) can be written as

$$\hat{v} = \frac{c}{4\pi f_0 T_0} \Delta \phi. \tag{12}$$

Equation (12) is used for estimating the velocity in a conventional BBADCP. The covariance method for BBADCPs is summarized in Fig. 1.

 $-\pi \leq \Delta \phi \leq \pi.$ Thus, if the phase of  $R(T_0)$  is larger than  $\pi$  or lower than  $-\pi$ , the phase

The extracted phase  $\Delta \phi$  is limited within the range

 $\Delta \phi_R$  extracted via (9) is not equal to the phase of  $R(T_0)$ , giving rise to a phase ambiguity. In this case, the phase extracted from  $R(T_0)$  can be expressed as

$$\Delta\phi_R = \Delta\phi + 2n\pi \tag{14}$$

where n is a folding integer. The range of the folding integer n is generally determined by the maximum current velocity that the BBADCP needs to measure. The phase ambiguity results in a velocity ambiguity, which is also illustrated in Fig. 2. According to (12) and (13), the maximum measurable velocity without wrap-around, which is called the ambiguity velocity, is expressed as

$$V_{\rm am} = \frac{c}{4f_0 T_0}.$$
 (15)

Conventional BBADCPs require the maximum measurable velocity to be configured in advance of system deployment. To avoid the velocity ambiguity, the maximum measurable velocity of conventional BBADCPs should be lower than or equal to the ambiguity velocity  $V_{\rm am}$  [9]. Thus, the ambiguity velocity must accommodate all possible velocities the system needs to measure. For conventional BBADCPs, a somewhat conservative setting is advisable, which means that the ambiguity velocity should be set high. Fig. 2 and (15) show that the ambiguity velocity is determined by the length of the single coded pulse  $T_0$ . However, a high-ambiguity velocity is accompanied by a high measurement deviation. How the ambiguity velocity influences the measurement deviation is analyzed in the following section.

(13)

# B. Limitation of the Ambiguity Velocity

We emphasize that when discussing the limitation of the ambiguity velocity, the total  $T_s$  length of the transmit signal is fixed. Changing the ambiguity velocity is realized by changing the length of the single coded pulse  $T_0$ .

In [9], Wanis *et al.* qualitatively analyze the influence of the ambiguity velocity. As the ambiguity velocity is increased by decreasing  $T_0$ , the system's sensitivity to signal is reduced, whereas the system variance increases in proportion. Thus, increasing the ambiguity velocity increases the measurement deviation. Two typical configurations of a 300-kHz BBADCP are given in [9]. Both use depth cells of size 4 m. When the ambiguity velocities of 1.74 m/s and 85.90 cm/s are set in the systems, the standard deviations of velocity measurement are observed to be 3.46 and 2.43 cm/s, respectively. The comparison between these two configurations demonstrates that the choice of ambiguity velocity limits the performance of BBADCPs.

The theory in [8] can be used to quantitatively analyze the limitation of the ambiguity velocity. When an ideal phase-coded sequence with L code elements is used, the theoretical deviation [8] under the assumption of high SNR can be expressed as

$$\sigma_v^2 = \left(\frac{c}{4\pi f_0}\right)^2 \frac{1}{LT_{\rm ovl}T_{\rm avg}} \left(1 + \frac{T_{\rm avg}}{2T_{\rm ovl}}\right) \tag{16}$$

where  $T_{\rm ovl}$  corresponds to the overlapped common range and  $T_{\rm avg}$  corresponds to the averaged range for Doppler estimation.  $T_{\rm ovl}$  is determined by the length of the transmit signal  $T_s$  and the length of single coded pulse  $T_0$  expressed as

$$T_{\rm ovl} = T_s - T_0. \tag{17}$$

 $T_{\text{avg}}$  is determined by the length of  $r_2(t)$  denoted by  $T_r$  and the length of single coded pulse  $T_0$  expressed as

$$T_{\rm avg} = T_r - T_0. \tag{18}$$

As analyzed in [8] and [9] and Section III-A,  $T_r$  is usually set to be equal to  $T_s$  [8], [9], which means

$$T_{\rm ovl} = T_{\rm avg}.$$
 (19)

When using the phase-coded pulses described in (2), the length of phase-coded pulse  $T_0$  is

$$T_0 = L\tau_0. \tag{20}$$

The signal bandwidth *B* can be approximated as

$$B = 1/\tau_0. \tag{21}$$

Considering (15), (19), and (20), the analytical expression for the variance of (16) at high SNRs can be expressed as

$$\sigma_v^2 = \left(\frac{c}{4\pi^2 f_0}\right) \left(\frac{V_{\rm am}}{B}\right) \left(\frac{1}{T_{\rm ovl}T_{\rm avg}}\right) \left(1 + \frac{T_{\rm avg}}{2T_{\rm ovl}}\right).$$
(22)

Even though (22) is derived from the transmit signal consisting of phase-coded pulses, (22) can also be used to approximately evaluate the theoretical deviation of other broadband signals such as LFM. Equation (22) shows that the theoretical deviation of BBADCPs is proportional to the ambiguity velocity, when using the conventional signals for BBADCPs.

## III. ROBUST CRT

The CRT is a useful tool for resolving number ambiguity, both integer and real. Recently, the robust CRT has been proposed to robustly resolve number ambiguity. This marks a significant development in tackling this problem. This paper employs the robust CRT to improve BBADCPs. The robust CRT is introduced briefly in this section.

We restate here that the problem at hand is to recover the *true current velocity* from the measurements. We have prior knowledge of the ambiguous velocities which are *moduli* around which our measurements are wrapped, thus yielding measured velocities that are *remainders* of this wrap-around, corrupted with errors due to other influences such as ambient noise or self-noise [7], [10]. To demonstrate how the robust CRT can be used to solve this problem, for simplicity, we first consider its use to recover an integer number *N*. This method can be easily generalized to work with real numbers.

Let N denote a positive integer and L moduli be denoted by  $M_1, M_2, \ldots, M_L$ , where  $0 < M_1 < M_2 < \cdots < M_L$  without loss of generality. Let  $r_1, r_2, \ldots, r_L$  denote the L remainders of N, which can be expressed as

$$N = n_i M_i + r_i \text{ or } N \equiv r_i \mod M_i \begin{pmatrix} 0 \le r_i < M_i \\ 1 \le i \le L \end{pmatrix}$$
(23)

where  $n_i$  is an unknown integer (called folding integer). If and only if

$$0 \le N < \operatorname{lcm} \{M_1, M_2, \dots, M_L\}$$
(24)

where "lcm" stands for the least common multiple, then N can be uniquely reconstructed from the L remainders  $r_i$ . It should be noted that determining the folding integers accurately is the key to reconstructing N.

In many applications, such as phase unwrapping in radar imaging [17], [31], [32] and frequency [19] and range [33] estimation, the "true" number required to be estimated from measurements and the remainders  $r_i$  are real numbers. Usually, the remainders are also corrupted with errors that can arise due to some influences. We denote the erroneous remainders by  $\hat{r}_i$ . Our proposed method for BBADCPs also needs the reconstruction of real numbers. This can be achieved by noting that in our method, the true current velocity we need to obtain corresponds to N in (23). The ambiguity velocities in the first terms of the right side of (38) and (39) correspond to the moduli in (23). The current velocities estimated from the phase measurements in (38) and (39) correspond to the robust CRT. Based on this equivalence, we can utilize the robust CRT to reconstruct the true current velocities.

Assume that all the moduli  $M_i$  have the greatest common divisor M. Let

$$\Gamma_i = M_i / M \quad (1 \le i \le L) \,. \tag{25}$$

All  $\Gamma_i$  are coprime. The traditional CRT [20], [24], [25] can reconstruct N with a simple formula. However, the traditional CRT is not robust, which means a small error from any remainders may result in a large reconstruction error [19]–[23].

The robust CRT [17]–[23] is proposed to robustly recover the real number N from its erroneous remainders  $\hat{r}_i$ , which are also real numbers. The expression of  $\hat{r}_i$  should be

$$\hat{r}_i = r_i + \Delta r_i \quad (1 \le i \le L) \tag{26}$$

where  $\Delta r_i s$  represent the errors and are assumed to be independent of each other. To resist errors, a special remainder redundancy should be considered. The robust CRT [17]–[23] guarantees that the real number N can be uniquely reconstructed from the erroneous remainders  $\hat{r}_i$ , when for every  $1 \leq i \leq L$ 

$$|\Delta r_i| < M/4. \tag{27}$$

This means that under the condition  $|\Delta r_i| < M/4$ , the folding integers  $n_i$  can be accurately determined.

To date, several algorithms for implementing the robust CRT, such as search-based [18], [19], closed-form [20], multistage [21], and maximum-likelihood estimation based [22], have been proposed. The robustness in all the algorithms of the robust CRT [17]–[23] owes itself mainly to the differential process that does not exist in the traditional CRT. For simplicity, the closed-form robust CRT algorithm is employed in this paper.

The closed-form robust CRT algorithm can be described by the four steps as follows.

Step 1: Calculate  $\hat{q}_{i,1}$  expressed as

$$\hat{q}_{i,1} = \left[\frac{\hat{r}_i - \hat{r}_1}{M}\right], \ 2 \le i \le L \tag{28}$$

where [.] stands for the rounding integer.

Step 2: Calculate the remainder of  $\hat{q}_{i,1}\bar{\Gamma}_{i,1}$  modulo  $\Gamma_i$  expressed as

$$\hat{\xi}_{i,1} = \hat{q}_{i,1}\bar{\Gamma}_{i,1} \mod \Gamma_i \tag{29}$$

where  $\overline{\Gamma}_{i,1}$  is the modular multiplicative inverse of  $\Gamma_1$  modulo  $\Gamma_i$ , and can be calculated in advance.

*Step 3:* Calculate  $\hat{n}_1$  by

$$\hat{n}_{1} = \sum_{i=2}^{L} \hat{\xi}_{i,1} b_{i,1} \frac{\gamma_{1}}{\Gamma_{i}} \mod \gamma_{1}$$
(30)

where  $b_{i, 1}$  is the modular multiplicative inverse of  $\gamma_1/\Gamma_i$  modular  $\Gamma_i$  and  $\gamma_1$  is

$$\gamma_1 = \Gamma_1 \cdots \Gamma_L / \Gamma_1 = \Gamma_2 \cdots \Gamma_L. \tag{31}$$

Step 4: Calculate  $\hat{n}_i$  for  $2 \le i \le L$  by

$$\hat{n}_{i} = \frac{\hat{n}_{1}\Gamma_{1} - \hat{q}_{i,1}}{\Gamma_{i}}.$$
(32)

# IV. PROPOSED METHOD

To improve the accuracy of BBADCPs, we propose a method whereby we transmit an orthogonal coprime signal and employ the robust CRT to resolve the velocity ambiguity. The orthogonal coprime signal consists of two orthogonal subsignals. The ambiguity velocities of the two orthogonal signals are coprime. The details of our proposed method are given in this section.

#### A. Orthogonality of Subsignals

For conventional BBADCPs, it is important to select a highambiguity velocity that accommodates all the anticipated velocities to be measured. As analyzed in Section II, the measurement standard deviation is inversely proportional to the ambiguity velocity. Thus, the high-ambiguity velocity accommodating all the anticipated velocities in conventional BBADCPs cannot guarantee the lowest measurement deviation. Instead, if a low-ambiguity velocity is selected and the velocity ambiguity, which is caused by a high current velocity, can be robustly resolved, the measurement standard deviation will be decreased significantly.

We propose an orthogonal coprime transmit signal for BBADCPs. The proposed signal consists of two orthogonal subsignals with lowambiguity velocities, shown in Fig. 3. The "orthogonality" of the subsignals guarantees that the system obtains two independent simultaneous measurements with low deviations. Our signal proposed for BBADCPs can be written as

$$s(t) = s_1(t) + s_2(t) \tag{33}$$

where  $s_1(t)$  and  $s_2(t)$  are the two orthogonal subsignals. Their carrier frequencies are  $f_1$  and  $f_2$ . Assume that  $a_1(t)$  and  $a_2(t)$  are the single



Fig. 3. Diagram depicting the two subsignals used in our proposed orthogonal coprime signal for improving BBADCPs.

coded pulses employed in  $s_1(t)$  and  $s_2(t)$ , respectively. The lengths of  $a_1(t)$  and  $a_2(t)$  are  $T_1$  and  $T_2$ . Referring to (5), for the application of BBADCPs,  $s_1(t)$  can be written as

$$s_1(t) = \left\{ \sum_{n=0}^{P_1 - 1} \left[ a_1 \left( t - nT_1 \right) \right] \right\} e^{j2\pi f_1 t}$$
(34)

where  $P_1$  is the number of  $a_1(t)$  in  $s_1(t)$ .  $s_2(t)$  can be written as

$$s_2(t) = \left\{ \sum_{n=0}^{P_2 - 1} \left[ a_2 \left( t - nT_2 \right) \right] \right\} e^{j2\pi f_2 t}$$
(35)

where  $P_2$  is the number of  $a_2(t)$  in  $s_2(t)$ . According to (15), the ambiguity velocity of  $s_1(t)$  can be expressed as

$$V_{\rm am1} = \frac{c}{4f_1T_1}.$$
 (36)

The ambiguity velocity of  $s_2(t)$  can be expressed as

$$V_{\rm am2} = \frac{c}{4f_2 T_2}.$$
 (37)

The true estimated velocity  $\hat{v}'$  obtained from  $s_1(t)$  can be expressed as

$$\hat{v}' = n_1(2V_{\rm am1}) + \hat{v}_1 \tag{38}$$

where  $\hat{v}_1$  is the velocity estimated from the phase measurement via  $s_1(t)$ and  $n_1$  is a folding integer. The true estimated velocity  $\hat{v}''$  obtained from  $s_2(t)$  can be expressed as

$$\hat{v}'' = n_2(2V_{\rm am2}) + \hat{v}_2 \tag{39}$$

where  $\hat{v}_2$  is the velocity estimated from the phase measurement via  $s_2(t)$  and  $n_2$  is a folding integer.

Although orthogonal signals have been widely used in communication [35]–[37] and radar [38], [39] systems, they have not been used to design transmit signals for BBADCPs. This paper proposes to design two orthogonal subsignals for BBADCPs. We divide the total bandwidth available to the BBADCP into two subbands for each subsignal, to realize the orthogonality of  $s_1(t)$  and  $s_2(t)$ .

#### B. Coprime Property of Subsignals

We denote the ratio of the ambiguity velocities of the two orthogonal subsignals  $s_1(t)$  and  $s_2(t)$  by

$$\frac{V_{\rm am1}}{V_{\rm am2}} = \frac{\Gamma_1}{\Gamma_2} \tag{40}$$

where  $\Gamma_1$  and  $\Gamma_2$  are two integers.  $\Gamma_1$  and  $\Gamma_2$  are coprime, which is indicated in the nomenclature of our proposed signal. According to (40) and the theory of the robust CRT [18]–[23], the maximum measurable



Fig. 4. Schematic of the proposed method based on the orthogonal coprime signals and the robust CRT.

velocity of our proposed signal should be

$$V_{\rm max} = V_{\rm am1} \Gamma_2 = V_{\rm am2} \Gamma_1. \tag{41}$$

When analyzing the robust CRT in Section III, to resolve ambiguity robustly, there must exist a redundancy, which requires that all the error bounds of the remainders should be lower than M/4. In our orthogonal coprime signals, M is given by

$$M = \frac{2V_{\rm am1}}{\Gamma_1} = \frac{2V_{\rm am2}}{\Gamma_2}.$$
 (42)

Based on (36) and (37), the ratio of the ambiguity velocities  $V_{\rm am1}$  and  $V_{\rm am2}$  can be written as

$$\frac{V_{\rm am1}}{V_{\rm am2}} = \frac{f_1 T_1}{f_2 T_2}.$$
(43)

When designing the orthogonal coprime signals, we can ensure that the ambiguity velocities are coprime by tuning the values of  $f_1$ ,  $T_1$ ,  $f_2$ , and  $T_2$ .

When realizing our designed signal in practical systems, the ambiguity velocities of the two subsignals may be approximately coprime due to some limitations, such as the accuracy of carrier frequencies in hardware systems. The method of resolving velocity ambiguity used in this paper is robust to the approximation error. Thus, even though the designed signal in practical systems is approximately coprime, the true velocity can still be accurately reconstructed.

#### C. Resolving Velocity Ambiguity

When using the proposed orthogonal coprime signal, two measured velocities are obtained. The key to resolving the velocity ambiguity in our proposed method is to uniquely determine the folding integers  $n_1$  and  $n_2$  in (38) and (39) from the measured velocities  $\hat{v}_1$  and  $\hat{v}_2$ . We propose to employ the robust CRT [18]–[23] to resolve the velocity ambiguity. The differential process in the robust CRT is crucial for realizing the robustness [20]. In a way similar to (28), the differential

process in our proposed method based on the robust CRT can be expressed as

$$\hat{q}_{2,1} = \left[\frac{\hat{v}_2 - \hat{v}_1}{M}\right].$$
 (44)

Using Steps 1–4 of the closed-form CRT algorithm in Section III, we can reconstruct the true estimated velocities  $\hat{v}'$  and  $\hat{v}''$ . The final estimated velocity of our proposed method should be the average of  $\hat{v}'$  and  $\hat{v}''$  given by

$$\hat{v} = \frac{\hat{v}' + \hat{v}''}{2}.$$
(45)

Let  $e_1$  and  $e_2$  be the errors of  $\hat{v}_1$  and  $\hat{v}_2$ . When using the robust CRT, to recover  $n_1$  and  $n_2$  uniquely, we need to guarantee that

$$\begin{cases} e_1 < \frac{M}{4} \\ e_2 < \frac{M}{4}. \end{cases}$$

$$\tag{46}$$

The design example in Section V and the simulations in Section VI point out that for the application of BBADCPs, the error requirements are easy to be satisfied.

#### D. Summary

Fig. 4 shows the schematic of our proposed method. "Orthogonalilty" in our proposed signal is for obtaining the independent wrapped-around measurements with low deviations. The "coprime" nature of subsignals in our proposed signal is designed for employing the robust CRT to resolve velocity ambiguity. An example is given in Section V to further illustrate the design and use of our proposed orthogonal coprime transmit signal.

In the proposed method, only two subsignals are employed. In theory, it is also possible to consider more subsignals in our proposed orthogonal coprime signal. However, more subsignals lead to an increase in the estimation error using each subsignal. Employing the robust CRT needs the estimation error using each subsignal to be under the error bound of (46). Thus, currently, we limit our analysis to two orthogonal subsignals. In our future work, we will consider the problem of optimizing the number of subsignals to achieve the best performance.

#### V. DESIGN EXAMPLE OF THE ORTHOGONAL COPRIME SIGNALS

In this section, a design example is given to explain how we design orthogonal coprime signals for improving BBADCPs. The error redundancy for using the robust CRT is also analyzed.

#### A. Orthogonal Coprime Signal

We consider a BBADCP with the center frequency  $f_0 = 500$  kHz and bandwidth  $B_0 = 100$  kHz. Considering settings similar to typical commercial BBADCPs [40], [41] of center frequency 500 kHz, we set the length of the depth cell to be 4.50 m. Generally, for design considerations, the length of the transmit signals of BBADCPs is set to a value comparable to that of the depth cell [9]. Similar to [40] and [41], we consider that the maximum velocity to be measured is 3.75 m/s. Based on the ambiguity velocity expression of (15), when using the conventional signals, the length  $T_0$  of the single coded pulse should be 0.20 ms. In this design example, we choose to employ LFM pulses.

We briefly run through the design considerations and selected parameters for the signals used in our simulations. For the purpose of comparison, the coded pulses  $a_0(t)$  used in the conventional transmit signal are also LFM. In the BBADCP, the desired length of the depth cell is 4.50 m. To achieve this, considering the sound speed of 1500 m/s, the duration of the conventional transmit signals should be  $4.50 \times 2.00/1500.00 = 6.00 \times 10^{-3}$  s. To ensure that the duration of the transmit signal yields a length that is comparable to that of the depth cell, for  $T_0 = 0.20$  ms, the number of the LFM pulses used in the conventional transmit signal should be 6.00/0.20 = 30. The bandwidth of  $a_0(t)$  is 100 kHz.

In our proposed orthogonal coprime signal, two subsignals  $s_1(t)$  and  $s_2(t)$  are needed. We set the center frequencies  $f_1$  and  $f_2$  of  $s_1(t)$  and  $s_2(t)$  to be 475 and 525 kHz, respectively. Both the bandwidths  $B_1$  and  $B_2$  of  $a_1(t)$  and  $a_2(t)$  used in  $s_1(t)$  and  $s_2(t)$  are 50 kHz.  $s_1(t)$  and  $s_2(t)$  are in two separate subbands and hence they are orthogonal. These setup parameters of the two subsignals are chosen so as to guarantee that our proposed orthogonal coprime signal has the same central frequency and bandwidth as the conventional signal, and thus they are comparable.

In our design, the ambiguity velocities  $V_{am1}$  and  $V_{am2}$  of  $s_1(t)$  and  $s_2(t)$  should be coprime. We set the design parameters as  $V_{am1} =$ 0.50 m/s and  $V_{\rm am1} = 0.80$  m/s, setting the coprime ratio in (40) to be  $\Gamma_1/\Gamma_2 = 5/8$ . After setting the coprime ambiguity velocities of the subsignals, we need to determine the lengths  $T_1$  and  $T_2$  of  $a_1(t)$  and  $a_2(t)$  by using (36) and (37). Considering the values of  $V_{\text{am1}}$ ,  $V_{\text{am2}}$ ,  $f_1$ , and  $f_2$ , according to (36) and (37),  $T_1$  and  $T_2$  are 1.60 ms and 0.89 ms, respectively. The numbers of repetitions of  $a_1(t)$  and  $a_2(t)$  are decided based on the required total lengths of the orthogonal coprime signal  $T_1$ and  $T_2$ . We ensure that the duration of the proposed orthogonal coprime signal is comparable to that of the conventional signal. Note that the length of the conventional signal is 6.00 ms, and also that the lengths, i.e.,  $T_1$  and  $T_2$ , of each pulse in each coprime signal are 1.60 and 0.89 ms. The number of repetitions must obviously be an integer value. Thus, we set the numbers of repetitions of  $a_1(t)$  and  $a_2(t)$  in our designed signal to be integers closest to 6.00/1.60 and 6.00/0.89, which turn out to be 4 and 7, respectively. The waveforms used for  $a_1(t)$  and  $a_2(t)$ in our design are LFM. Considering (43), the maximum measurable velocity of our designed orthogonal coprime signal is 4.00 m/s. The autocorrelation functions of the conventional signal and the designed



Fig. 5. Autocorrelation functions of the conventional signal and the designed orthogonal coprime signal.

orthogonal coprime signal are shown in Fig. 5. It can be seen that the subpeaks of the autocorrelation functions of the designed orthogonal coprime signal are nonuniformly distributed. The position of the first subpeak in Fig. 5(b) is at  $T_2$ . The position of the second subpeak in Fig. 5(b) is at  $T_1$ .

When using our designed orthogonal coprime signal, the received signals of the BBADCPs should be divided into two subbands. Then, our proposed method demodulates the received signals of the two subbands by  $f_1$  and  $f_2$ . After demodulating, the covariance method is used to estimate the two wrapped-around velocities. Finally, the robust CRT is employed to reconstruct the true velocity from the two estimated velocities.

#### B. Requirement for the Robust CRT

When using the designed orthogonal coprime signal, the error bound limit of the robust CRT, given by (46), should be considered. If the errors of the two estimated velocities obtained from each of the two orthogonal subsignals are greater than the error bound, reconstruction of the velocity ambiguity will fail. For  $V_{am1} = 0.50$  m/s and  $V_{am1} =$ 0.80 m/s, the greatest common divisor *M* is 20.00 cm/s, according to (42). Hence, the error bound of the two estimated velocities should be M/4 = 5.00 cm/s. If the error absolute values of the velocity estimates of using  $s_1(t)$  and  $s_2(t)$  are over the error bound of 5.00 cm/s, the resolved velocity would differ from the true value. If just a few of these faults happen, a lowpass filter can be used to remove them.

Equation (22) can be used to approximately evaluate the theoretical standard deviations of  $s_1(t)$  and  $s_2(t)$  in our designed signal. The theoretical standard deviations of  $s_1(t)$  and  $s_2(t)$  are 0.75 and 0.76 cm/s, based on (22). Note that these values are much smaller than the error bound 5.00 cm/s. For the sake of getting a feel of how much these deviations can influence our estimates, let us consider the velocity estimates of  $s_1(t)$  and  $s_2(t)$  to be normally distributed random variables. For a normal distribution with a standard deviation of 0.75 cm/s, the probability that the absolute value of the estimation error is over 5.00 cm/s is lower than  $1.97 \times 10^{-7}$ . This is so low that the resolving errors when using the robust CRT can be neglected.

In Section VI, we conduct a simulation study. The standard deviations of estimates obtained by using  $s_1(t)$  and  $s_2(t)$ , calculated from 3000 velocity estimates are 1.12 and 1.13 cm/s. Over 3000 velocity estimates, no estimation errors are observed to exceed the limit of 5.00 cm/s. Thus, it is reliable to use the robust CRT to recover the true velocity by using the two velocity estimates obtained from using  $s_1(t)$  and  $s_2(t)$ .

#### C. Performance Comparison

As shown in (45), the final estimated velocity  $\hat{v}$  of our proposed method is the average of  $\hat{v}'$  and  $\hat{v}''$ . Let  $\sigma_{\hat{v}}$  denote the standard deviation of  $\hat{v}$ , and  $\sigma_{\hat{v}'}$  and  $\sigma_{\hat{v}''}$  denote the standard deviations of  $\hat{v}'$  and  $\hat{v}''$ , respectively. From (45), the theoretical value for the standard deviation of our proposed method is

$$\sigma_{\hat{v}} = \frac{\sqrt{\sigma_{\hat{v}'}^2 + \sigma_{\hat{v}''}^2}}{2}.$$
(47)

Combining (22) and (47), the calculations for the design example show that the theoretical standard deviation of our proposed method is 2.13 times lower than that of the conventional method.

#### VI. SIMULATION

First, we evaluate the performance of the conventional method at different ambiguity velocities in this section. Then, we compare our proposed method with the conventional method. Here, it should be emphasized that the approach of using the conventional signals of (5) to estimate the Doppler shift is referred to as the conventional method. Our proposed method employs the orthogonal coprime signal and the robust CRT. We perform the comparison via simulations by employing the method used in [10], wherein the echoes of BBADCPs are modeled as the superposition of the backscattered signals from many randomly distributed scatterers. The Doppler shift is realized by the resampling operation.

Let  $\hat{v}_i$  denote the *i*th estimated velocity. The standard deviation  $\sigma_v$  of the current measurement is calculated as

$$\sigma_v = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} (\hat{v}_i - \bar{v})^2}$$
(48)

where M is the number of independent velocity estimates and  $\bar{v}$  is the average velocity given by

$$\bar{v} = \frac{1}{M} \sum_{i=1}^{M} \hat{v}_i. \tag{49}$$

In this paper, each of the standard deviations shown in the figures is obtained by calculating the velocity estimates from 3000 trials (M = 3000) at the same depth.

The simulations in this section use the designed orthogonal coprime signal in Section V. The different ambiguity velocities of the conventional signal are realized by changing the length  $T_0$  of the LFM pulse  $a_0(t)$ . The SNR in our simulations is defined by (34) of [10], and the



Fig. 6. Variation of the standard deviations of velocity estimates against SNR at different ambiguity velocities of the conventional signal: 0.50, 1.50, and 3.75 m/s.

definition is reproduced as follows:

$$SNR = \frac{\left\langle \left(r_0(t)\right)^2 \right\rangle}{\left\langle \left(n(t)\right)^2 \right\rangle}$$
(50)

where  $r_0(t)$  is obtained by the superposition of the backscattered signals from many randomly distributed scatterers, n(t) is the inband component of the additive white Gaussian noise in the simulation, and " $\langle \rangle$ " denotes time averaging. The true velocity is set to be 3.00 m/s.

When using the conventional method, the variation of the standard deviations of velocity estimates with the SNR at the different values of ambiguity velocity, 0.50, 1.50, and 3.75 m/s, is shown in Fig. 6. It can be seen that at high SNRs, the standard deviation at the ambiguity velocity of 3.75 m/s is nearly three times higher than the standard deviation at the ambiguity velocity of 0.50 m/s. Thus, Fig. 6 demonstrates the fact that the ambiguity velocity limits the performance of the conventional method for BBADCPs.

The observed velocities obtained using the conventional signal and the designed orthogonal coprime signal including Signal1 and Signal2 are shown in Fig. 7. The ambiguity velocity of the conventional signal is 3.75 m/s, which is greater than the true velocity of 3 m/s. From Fig. 7, it can be found that the velocity ambiguity arises for Subsignal1 and Subsignal2, but apparently the standard deviations of Subsignal1 and Subsignal2 are lower than that of the conventional signal. Our method recovers the true measured velocity from the two wrapped-around observed velocities, by using the robust CRT. The conventional method does not require the operation of resolving the velocity ambiguity, and the observed velocity is the same as the measured velocity in this case.

The velocity estimates using the conventional method and our proposed method are shown in Fig. 8 for 50 independent simulated measurement samples. From Fig. 8, it can be seen that though the ensemble averages of the velocity estimates using both the conventional method and our proposed method are equal to the true velocity of 3.0 m/s, the deviation of the conventional method is much higher.

Fig. 9 compares the standard deviations of velocity estimates, obtained by the proposed method and the conventional one. It can be found that at high SNRs, the standard deviation of the proposed method is nearly three times lower than that of the conventional method. The theoretical standard deviations for the proposed and conventional methods obtained by using (22) are shown in Fig. 9. The theoretical standard deviations also demonstrate that our proposed method is expected to perform superior to the conventional method. Additionally, in Fig. 9, the standard deviation of our proposed method at an SNR of -7 dB is comparable to that of the conventional method at an SNR of 5 dB. This means that for the simulation parameters considered, an SNR



Fig. 7. Observed velocities at an SNR of 15 dB by using (a) conventional signal and (b) proposed orthogonal coprime signal including two subsignals (Subsignal1 and Subsignal2), with the true velocity set at 3.0 m/s. A velocity ambiguity arises when using Subsignal1 and Subsignal2, leading to the observed velocities having averages of 0 and -0.2 m/s, respectively.



Fig. 8. Velocities estimated by the conventional method and our proposed method over 50 independent measurement samples, when the true velocity is 3 m/s.

gain of 12 dB is achieved by our proposed method, compared to the conventional method. The SNR gain provided by our method can help improve the profiling range of BBADCPs.

In the simulation, we also observed that when the SNR is lower than -7 dB, the fail rate of reconstructing the true velocity from the two observed velocities of the two subsignals increases drastically. This means that the proposed method does not perform well at SNRs lower than a threshold of around -7 dB for the simulation parameters considered. Fortunately, due to the use of the modulated signals, the SNR is usually not the main limiting factor for BBADCPs as discussed in [7]–[9]. This means that the improvement provided by the proposed method could be useful in advancing the performance of BBADCPs in practical scenarios.

Here, it should be mentioned that the cost of using the proposed method is the higher computational complexity, as compared to the



Fig. 9. Standard deviations of measured velocity and the theoretical standard deviations for the proposed method and the conventional one. (Simulation-conventional: the standard deviation of measured velocity for the conventional method; Theoretical-conventoinal: the theoretical standard deviation of measured velocity for the proposed method; and Theoretical-proposed: the theoretical standard deviation of the proposed method.)

conventional method. In the proposed method, the velocity estimation is based on two subsignals instead of a single signal as used in the conventional method. Our proposed method needs to estimate two wrapped-around velocities and uses the robust CRT to resolve the velocity ambiguity to improve the accuracy. As shown in Section V, Steps 1–4 of the robust CRT algorithm involve simple operations. Thus, the overall complexity of our proposed method is around two times of that of the conventional method. With the progress in digital processor technology, the computational complexity caused by our proposed method is easy to handle. When considering the standard deviation reduction of three times, the complexity cost incurred by our proposed method is affordable.

### VII. CONCLUSION

This paper proposed a method based on the orthogonal coprime signals and the robust CRT to improve BBADCPs. The proposed method breaks through the limitation of ambiguity velocity encountered in conventional BBADCPs. Compared to the conventional method, the proposed method can decrease the measurement standard deviation by nearly three times. Additionally, our method yields up to 12 dB of SNR gain over the conventional method before performance breakdown, for the simulation parameters considered. The SNR gain achieved by our proposed method can help increase the profiling range of BBADCPs.

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