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Robust Resolution of Velocity Ambiguity for Multifrequency Pulse-to-Pulse Coherent Doppler Sonars

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Abstract—Pulse-to-pulse coherent Doppler sonars are widely used to explore boundary layer characteristics of oceans. Multifrequency pulse-topulse coherent Doppler sonars have been proposed in the literature to tackle the velocity ambiguity problem faced by single-frequency systems. A robust algorithm for resolving the velocity ambiguity is crucial in such systems. This paper proposes a method based on the robust Chinese remainder theorem to resolve the velocity ambiguity for multifrequency pulse-to-pulse coherent Doppler sonars. We evaluate the proposed method for resolving the velocity ambiguity in terms of the trial fail rate. The simulations show that the proposed method achieves 1/8 trial failure rate as the reference method. A theoretical criterion is also derived to support the observation that in most cases our proposed method is more robust to estimation errors than the reference method.

Index Terms—Current measurement, Doppler estimation, multifrequency pulse-to-pulse coherent Doppler sonar, robust Chinese remainder theorem (CRT), velocity ambiguity.

I. INTRODUCTION

OPPLER sonars [1]-[5], also referred to as acoustic Doppler current profilers in some cases [2]-[4], have been widely used to observe an ocean. They can be categorized into three types: narrowband (incoherent), broadband, and pulse-to-pulse coherent [2]. Both narrowband and broadband Doppler sonars are usually applied for long-range measurements, where the maximum range is from 30 m to more than 1000 m [2], [5]-[7]. The working frequency of narrowband and broadband Doppler sonars varies from 30 kHz to 1 MHz. Pulse-to-pulse coherent Doppler sonars are only suitable for short-range measurements, where the maximum range is on the order of 1 m [3], [8]–[10]. Since the coherent processing requires that these pulse-to-pulse systems listen to the echo from only one pulse at a time, the maximum range of these systems is determined by the pulse interval [2], [3]. To obtain high accuracy, the pulse interval needs to be much shorter than the decorrelation time. For example, the pulse interval used in the pulse-to-pulse sonar in [3] was 975 μ s, and the maximum range

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was $975 \times 10^{-6} \times 1500/2 \approx 0.73$ m, with a sound speed of 1500 m/s. The working frequency of pulse-to-pulse coherent Doppler sonars is generally over 1 MHz [3], [8]–[19]. The three different types of Doppler sonars have different applications, and among these, pulse-to-pulse coherent ones have the best accuracy and least range [2]. This paper focuses on pulse-to-pulse coherent Doppler sonars. The narrowband and broadband types are not discussed in this paper.

1

Pulse-to-pulse coherent Doppler sonars have been commonly applied to explore boundary layer characteristics, such as turbulence [9]–[13] and sediment transport [14], [15]. For example, a pulse-to-pulse sonar in [10] was placed at a depth of 40–50 cm from the mean water level on the pier at Scripps Institution of Oceanography, to measure waves and turbulence near the surface. In [19], the researchers employed a pulse-to-pulse sonar to observe the wave bottom boundary layer with a thickness on the order of 10 cm. Pulse-to-pulse coherent Doppler sonars transmit a series of short single-pulse pings, and usually estimate current velocities by extracting phase changes from ping to ping at each range cell. Single-frequency pulse-to-pulse coherent sonars are subject to velocity ambiguity, caused by the phase ambiguity [16]–[18]. When pulse-to-pulse coherent sonars extract the phase change from ping to ping, if the phase change is out of the range $(-\pi, \pi)$, a phase ambiguity arises.

Multifrequency pulse-to-pulse coherent Doppler sonars have been proposed in the literature to measure velocities beyond the velocity ambiguity limitation of a single-frequency system [15]–[21]. In multifrequency pulse-to-pulse coherent Doppler sonars, multifrequency pulses are transmitted simultaneously, which means that there is no reduction in the data rate. These systems are able to resolve velocity ambiguities inherent to coherent Doppler in real time, without degrading velocity measurements. It should be noted that measurement data are usually corrupted with estimation errors. The methods of resolving velocity ambiguity employed in multifrequency pulse-to-pulse coherent Doppler sonars must be robust to these estimation errors. This paper focuses on how to robustly resolve velocity ambiguity for multifrequency pulse-to-pulse coherent Doppler sonars.

One possible way to resolve the velocity ambiguity in pulse-to-pulse coherent Doppler sonars would be to employ different pulse repetition intervals of the transmitted pulses, which implies that multiple pulse delays are employed. However, the use of multiple pulse delays would lead to a decrease in the data rate, because the measurement system would spend more time for the additional pulses. More importantly, as pointed out in [8] and [16], in the presence of a boundary, the pulse delay parameter cannot be freely chosen; rather, it must be adapted to avoid interference from multiple boundary returns.

Zedel *et al.* [16] proposed a method to resolve velocity ambiguity for multifrequency pulse-to-pulse coherent Doppler sonars. This method has been tested in [15] and [19]–[21] and is shown to have a good

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performance. The performance of multifrequency pulse-to-pulse coherent Doppler sonars can be improved, if a method of resolving velocity ambiguity can be developed, which is more robust to estimation errors than the method in [16]. In our research, we choose the method in [16] as the reference method for benchmarking.

The Chinese remainder theorem (CRT) [22]–[28] provides a method to recover integers and real numbers from their remainders with a reconstruction formula. It has been applied to many fields, such as channel coding [29], computational neuroscience [31], coprime array processing [32], and wireless sensor networks [33]. Unfortunately, the traditional CRT is not robust to the remainder errors [23]–[26], [30]. Hence, the robust CRT has been recently proposed to resist the remainder errors [23]–[28]. Different algorithms for realizing the robust CRT, such as searching-based [23], [24], closed-form [25], multistage [26], and maximum-likelihood estimation based [27] methods, have been provided. Due to its robustness, the robust CRT has been applied to many fields, such as distance estimation [34], frequency estimation from undersampled waveforms [24], radar signal processing [35]–[37], and multiwavelength optical measurement [38].

This paper proposes to apply the robust CRT to resolve velocity ambiguity for multifrequency pulse-to-pulse coherent sonars. We show via the theoretical analysis and simulations that in most cases, the proposed method performs better than the reference method. The trial fail rate (TFR) [25]–[27] is chosen as a metric to evaluate our proposed method and the reference method. Simulations indicate that the TFR of our proposed method is nearly eight times lower than that of the reference method. We also provide a theoretical criterion to demonstrate that in most cases, our proposed method is more robust to the estimated errors than the reference method.

The rest of this paper is organized as follows. Section II outlines the basic theory describing the operation of multifrequency pulse-to-pulse coherent Doppler sonars. Section III introduces the method presented by Zedel and Hay [16] for resolving the velocity ambiguity of multifrequency pulse-to-pulse coherent Doppler sonar, taken as the reference method in our research. Our proposed method based on the robust CRT is described in Section IV. Section V analyzes the theoretical error of the proposed method and the reference one, and describes a theoretical criterion to evaluate the proposed method. Section VI presents simulation results to evaluate the performances of the proposed method and the reference one. Finally, Section VII concludes this paper.

II. BASIC THEORY

This section introduces the basic theory of multifrequency pulse-topulse coherent Doppler sonar. The velocity estimation and ambiguity without considering measured errors are presented first. Next, the analysis takes into account the estimation errors.

Pulse-to-pulse coherent Doppler sonars transmit multiple pulses separated by a pulse delay T_0 . Without considering the estimation errors, the radial velocity component (along the acoustic beam) can be expressed as follows [1], [16]:

$$v = \frac{c}{4\pi f_0 T_0} \Delta \phi \tag{1}$$

where c is the speed of sound in water, f_0 is the carrier frequency, and $\Delta \phi$ is the phase change from pulse to pulse, calculated at each range cell. While using the phase change to determine velocity, a velocity ambiguity arises when the phase change overshoots the range $(-\pi, \pi)$, due to the phase wraparound. At the same time, in addition to tackling the range ambiguity, the pulse-to-pulse coherent Doppler sonar should also consider the maximum range it can measure, which is expressed

as follows [16]:

$$r_{\rm max} = \frac{cT_0}{2}.$$
 (2)

To introduce multifrequency pulse-to-pulse coherent Doppler sonars, we consider a system with two different carrier frequencies f_1 and f_2 in its transmit signal, similar to that in [16]. We also assume without loss of generality that $f_1 < f_2$. Let $\Delta \phi_1$ and $\Delta \phi_2$ denote the actual absolute phase changes obtained due to the Doppler shifts at the frequencies f_1 and f_2 . Without considering the phase wraparounds, the phase changes $\Delta \phi_1$ and $\Delta \phi_2$ for a given velocity v can be expressed using (1) as

 $\Delta \phi_1 = \frac{4\pi f_1 T_0}{c} v$

and

$$\Delta\phi_2 = \frac{4\pi f_2 T_0}{v}.\tag{3b}$$

(3a)

When the phase ambiguity arises, let $\Delta \phi'_1$ and $\Delta \phi'_2$ denote the ambiguous phase changes obtained from the frequencies f_1 and f_2 . The actual absolute phase changes $\Delta \phi_1$ and $\Delta \phi_2$ can be written as

$$\Delta \phi_1 = \Delta \phi'_1 + 2n_1 \pi, \quad -\pi < \Delta \phi'_1 < \pi \tag{4a}$$

and

$$\Delta\phi_2 = \Delta\phi'_2 + 2n_2\pi, \quad -\pi < \Delta\phi'_2 < \pi \tag{4b}$$

where n_1 and n_2 are the folding integers. To avoid the ambiguity, we would require that $-\pi < \Delta \phi_1, \Delta \phi_2 < \pi$. Considering (3a) and (3b), the ambiguity velocities for f_1 and f_2 , which are the maximum unambiguously measurable velocities, are given, respectively, by

$$V_{am1} = \frac{c}{4f_1T_0} \tag{5a}$$

and

$$V_{am2} = \frac{c}{4f_2T_0}.$$
 (5b)

When the phase ambiguities arise in the measurement, the velocities obtained from $\Delta \phi'_1$ and $\Delta \phi'_2$ are expressed, respectively, as follows:

$$v_1 = \frac{c}{4\pi f_1 T_0} \Delta \phi_1' \tag{6a}$$

and

$$v_2 = \frac{c}{4\pi f_2 T_0} \Delta \phi_2'. \tag{6b}$$

Considering (4a), (4b), (5a), (5b), (6a), and (6b), the true velocity v can be expressed in terms of the measurements at the two frequencies f_1 and f_2 as follows

$$\begin{cases} v = v_1 + n_1 \left(2V_{\rm am1} \right) \\ v = v_2 + n_2 \left(2V_{\rm am2} \right). \end{cases}$$
(7)

Equation (7) explains how the velocity ambiguity leads to different velocities v_1 and v_2 being measured at each frequency when probing a true velocity v. Fig. 1 illustrates the velocity ambiguity of the pulse-to-pulse coherent Doppler sonar using the two frequencies f_1 and f_2 .

Generally, the covariance method [1], [2], [16], [17] is used to estimate the phase changes in multifrequency pulse-to-pulse coherent Doppler sonars. When estimating the phase changes, estimation errors due to noise are unavoidable. Let ε_1 and ε_2 denote the estimation phase errors in the measurements using f_1 and f_2 , respectively. The observed phase changes $\delta\phi_1$ and $\delta\phi_2$ can be expressed as follows:

$$\delta\phi_1 = \Delta\phi_1' + \varepsilon_1 \tag{8a}$$



Fig. 1. Illustration of the velocity ambiguity for the pulse-to-pulse coherent Doppler sonar using signals at the two frequencies f_1 and f_2 . The relationship between phase change and true velocity is sawtoothlike and constrained within the limit $(-\pi, \pi)$. The slope of the linearity depends on the expressions in (6a) and (6b), which are different at different frequencies. In the current example, when using two frequencies f_1 and f_2 ($f_1 < f_2$) to measure a true velocity v, where $V_{am2} < v < V_{am1}$, the phase change observed at the frequency f_1 is a linear function of v, whereas that observed at f_2 is wrapped around.

and

$$\delta\phi_2 = \Delta\phi_2' + \varepsilon_2. \tag{8b}$$

The observed velocities \hat{v}_1 and \hat{v}_2 using f_1 and f_2 , respectively, are expressed as follows:

$$\hat{v}_1 = v_1 + e_1$$
 (9a)

and

$$\hat{v}_2 = v_2 + e_2 \tag{9b}$$

where e_1 and e_2 are the errors in the velocity estimates obtained using f_1 and f_2 , respectively.

For the multifrequency pulse-to-pulse coherent Doppler sonar, resolution of the velocity ambiguity involves accurately reconstructing n_1 and n_2 from the observed phase changes $\delta\phi_1$ and $\delta\phi_2$, or equivalently from the observed velocities \hat{v}_1 and \hat{v}_2 . The reference method is one such way to reconstruct n_1 and n_2 from $\delta\phi_1$ and $\delta\phi_2$, and is described in Section III. Our proposed method in Section IV reconstructs n_1 and n_2 from \hat{v}_1 and \hat{v}_2 .

III. REFERENCE METHOD

In [16], a method of resolving the velocity ambiguity from the observed phase changes is proposed for multifrequency pulse-to-pulse coherent Doppler sonars. This method is robust to the estimation errors to a certain degree. We choose this method as the reference method to evaluate our proposed method.

The observed values of phase changes $\Delta \phi_1$ and $\Delta \phi_2$ that contain estimation errors along with phase wraps are expressed, respectively, as follows [16]:

$$\delta\phi_1 = \Delta\phi_1 + \varepsilon_1 + 2n'_1\pi = \|\Delta\phi_1 + \varepsilon_1\| \tag{10a}$$

and

$$\delta\phi_2 = \Delta\phi_2 + \varepsilon_2 + 2n'_2\pi = \|\Delta\phi_2 + \varepsilon_2\| \tag{10b}$$

where ε_1 and ε_2 are the estimation errors of the phase changes, n'_1 and n'_2 take on the values necessary to constrain $\delta\phi_1$ and $\delta\phi_2$ within the limit $-\pi < \delta\phi_1, \delta\phi_2 < \pi$, and the operator || || denotes the value of an argument constrained between $\pm \pi$ by appropriate additions or subtractions of multiples of 2π . It should be noted that the signs of n'_1 and n'_2 in (10a) and (10b) are opposite to the signs of n_1 and n_2 in (4a) and (4b), or in other words

$$n_1' = -n_1$$
 (11a)

and

$$n_2' = -n_2.$$
 (11b)

The difference $\delta \phi_{12}$ between $\delta \phi_1$ and $\delta \phi_2$ is

$$\delta\phi_{12} = \delta\phi_2 - \delta\phi_1$$

= $\Delta\phi_{12} + \varepsilon_2 - \varepsilon_1 + 2\pi(n'_2 - n'_1)$ (12)

where

$$\Delta \phi_{12} = \Delta \phi_2 - \Delta \phi_1. \tag{13}$$

Using the operator ||||| on (12), we obtain [16]

$$\|\delta\phi_{12}\| = \Delta\phi_{12} + \varepsilon_2 - \varepsilon_1|_{|\Delta\phi_{12} + \varepsilon_2 - \varepsilon_1| < \pi}.$$
 (14)

The phase change $\Delta \phi_{p \text{ est}}$ for some frequency f_p (p = 1 or p = 2) can be estimated using the dual-frequency measurements as follows:

$$\Delta \phi_{p \text{ est}} = \|\delta \phi_{12}\| \frac{f_p}{(f_2 - f_1)}.$$
(15)

The difference between the observed value of $\delta\phi_p$ and the estimated value of $\Delta\phi_p$ is expressed as

$$\delta\phi_p - \Delta\phi_{p \text{ est}} = \Delta\phi_p + \varepsilon_p + 2n'_p\pi - (\Delta\phi_{12} + \varepsilon_2 - \varepsilon_1)\frac{f_p}{(f_2 - f_1)}.$$
(16)

From (3a) and (3b), we have

$$\frac{\Delta\phi_1}{f_1} = \frac{\Delta\phi_2}{f_2}.$$
(17)

From (13) and (17), $\Delta \phi_p$ can be expressed as

$$\Delta \phi_p = \Delta \phi_{12} \frac{f_p}{f_2 - f_1}.\tag{18}$$

Considering (18), (16) can be rewritten as

$$\delta\phi_p - \Delta\phi_{p \text{ est}} = \varepsilon_p + 2n'_p \pi - (\varepsilon_2 - \varepsilon_1) \frac{f_p}{(f_2 - f_1)}.$$
 (19)

If all the terms of the right-hand side of (19) except for $2n'_p\pi$ are small compared to π , the reconstructed integer \hat{n}'_p can be calculated by [16]

$$\hat{n}_p' = \left[\frac{\delta\phi_p - \Delta\phi_{p \text{ est}}}{2\pi}\right] \tag{20}$$

where [.] represents rounding off to the nearest integer. Obtaining \hat{n}'_p accurately translates to successfully resolving the velocity ambiguity for the multifrequency pulse-to-pulse coherent Doppler sonar. Here, it should be noted that the reference method has the limitation that all the error terms on the right-hand side of (19) should be smaller than π . The limit on tolerable variances of the reference method is presented in Section V, and compared against that of our proposed method.

IV. PROPOSED METHOD

Our proposed method based on the robust CRT [22]–[27] is presented in this section. The robust CRT is introduced first. Then, we show how the robust CRT can be utilized to resolve the velocity ambiguity for multifrequency pulse-to-pulse coherent Doppler sonars.

A. Robust CRT

The CRT is a useful tool for resolving number (integer or real) ambiguity. Recently, the robust CRT has been proposed to robustly resolve number ambiguity, which is an important progress in solving practical versions of this problem. The robust CRT is introduced briefly in this section.

Let N denote a positive integer, and L moduli be denoted as M_1, M_2, \ldots, M_L , where $0 < M_1 < M_2 < \cdots < M_L$. Furthermore, let r_1, r_2, \ldots, r_L denote the L remainders of N with these L moduli. This relation can be mathematically expressed as follows:

$$N = n_i M_i + r_i \text{ or } N \equiv r_i \mod M_i \begin{pmatrix} 0 \le r_i < M_i \\ 1 \le i \le L \end{pmatrix}$$
(21)

where n_i are unknown integers (called folding integers). If and only if

$$0 \le N < \operatorname{lcm}\left(M_1, M_2, \dots, M_L\right) \tag{22}$$

where "lcm()" stands for the least common multiple of its arguments, N can be uniquely reconstructed from the L remainders r_i , $i = 1, 2, \ldots, L$. Determining the folding integers n_i accurately is the key to reconstructing N.

In some applications, such as frequency estimation [24], distance estimation [34], and phase unwrapping in radar imaging [35]–[37], N and the remainders r_i are real numbers. This paper also involves the reconstruction of real numbers. Assume that all the moduli M_i have the greatest common divisor M. Let

$$\Gamma_i = M_i/M, \quad 1 \le i \le L. \tag{23}$$

All Γ_i are coprime, or in other words, M is the greatest common divisor of all the moduli M_i . When all the moduli Γ_i are coprime, the traditional CRT [25], [29], [30] gives a procedure to reconstruct N with a simple formula. However, the traditional CRT is not robust, which means that a small error from any remainders may result in a large reconstruction error [22]–[27].

The robust CRT [22]–[27] was proposed to robustly recover the real number N from its erroneous remainders \hat{r}_i . The expression for \hat{r}_i is expressed as follows:

$$\hat{r}_i = r_i + \Delta r_i, \quad 1 \le i \le L \tag{24}$$

where Δr_i represents the remainder errors in the *i*th remainder measurement, and all Δr_i are independent of each other. To resist the remainder errors, information from the remainder redundancy should be considered. The robust CRT guarantees that the real number N can be uniquely reconstructed from the erroneous remainders \hat{r}_i , when for every $1 \le i \le L$

$$\left|\Delta r_{i}\right| < M/4. \tag{25}$$

This means that under the condition $|\Delta r_i| < M/4$, for every $1 \le i \le L$, the folding integers n_i can be accurately determined.

To date, several algorithms of realizing the robust CRT based recovery, such as searching-based [22]–[24], closed-form [25], multistage [26], and maximum-likelihood estimation [27], have been proposed. The robustness in all the algorithms of realizing the robust CRT [22]– [27] is mainly due to a differential processing with the remainders that does not exist in the traditional CRT. The closed-form robust CRT algorithm [25] is employed in this paper.

The closed-form robust CRT algorithm [25] can be described by the four steps given as follows.

Step 1: Calculate $\hat{q}_{i,1}$ expressed as

$$\hat{q}_{i,1} = \left[\frac{\hat{r}_i - \hat{r}_1}{M}\right], \quad 2 \le i \le L.$$
(26)

Step 2: Calculate the remainder of $\hat{q}_{i,1}\bar{\Gamma}_{i,1}$ modulo Γ_i expressed as

$$\hat{\xi}_{i,1} = \hat{q}_{i,1}\bar{\Gamma}_{i,1} \mod \Gamma_i \tag{27}$$

where $\overline{\Gamma}_{i,1}$ is the modular multiplicative inverse of Γ_1 modulo Γ_i and can be calculated in advance by the formula [42]

$$\bar{\Gamma}_{i,1}\Gamma_1 = 1 \mod \Gamma_i. \tag{28}$$

Step 3: Calculate \hat{n}_1 , the reconstructed version of n_1 by

$$\hat{n}_1 = \sum_{i=2}^{L} \hat{\xi}_{i,1} b_{i,1} \frac{\gamma_1}{\Gamma_i} \mod \gamma_1$$
 (29)

where $b_{i,\ 1}$ is the modular multiplicative inverse of γ_1/Γ_i modular, and γ_1 is

$$\gamma_1 = \Gamma_1 \cdots \Gamma_L / \Gamma_1 = \Gamma_2 \cdots \Gamma_L. \tag{30}$$

Step 4: Calculate \hat{n}_i , the reconstructed version of n_i for $2 \le i \le L$ by

$$\hat{n}_i = \frac{\hat{n}_1 \Gamma_1 - \hat{q}_{i,1}}{\Gamma_i}.$$
(31)

B. Resolving the Velocity Ambiguity

Noticing the similarity between (7) and (21), we aim to use the robust CRT to resolve the velocity ambiguity from the observed velocities \hat{v}_1 and \hat{v}_2 . To employ the robust CRT, the ambiguity velocities $V_{\rm am1}$ and $V_{\rm am2}$ of the signals at f_1 and f_2 , respectively, should hold a coprime relationship. Let M be the normalization factor, which plays a role similar to that of (23) in the robust CRT, to guarantee the error redundancy. Comparing (7) with (21), we have

$$M_1 = 2V_{\rm am1} \tag{32a}$$

and

$$M_2 = 2V_{\rm am1}.\tag{32b}$$

Considering (23), (32a), and (32b), we obtain

$$2V_{\rm am1} = M\Gamma_1$$

$$2V_{\rm am2} = M\Gamma_2$$
(33)

where Γ_1 and Γ_2 are coprime. Equation (33) can be rewritten as

$$M = \frac{2V_{\rm am1}}{\Gamma_1} = \frac{2V_{\rm am2}}{\Gamma_2}.$$
 (34)

When designing a pulse-to-pulse coherent Doppler sonar, according to (5a) and (5b), we can set the values of f_1 and f_2 to realize a coprime relationship between V_{am1} and V_{am2} in (33) and (34). For example, for $f_1 = 1.40$ MHz and $f_2 = 1.75$ MHz, we have $\Gamma_1 = 5$ and $\Gamma_2 = 4$, which are coprime. Likewise, for Doppler sonars employing three or more frequencies, the robust CRT can be used to resolve the velocity ambiguity by designing the frequencies such that every pair within the set of frequencies employed meet the coprime requirement, similar to the criterion described in (34).

The two observed velocities \hat{v}_1 and \hat{v}_2 for f_1 and f_2 , respectively, can be equated to the variables \hat{r}_1 and \hat{r}_2 in (24). To employ the closed-form robust CRT algorithm [25], similar to (26), we calculate the difference between \hat{v}_1 and \hat{v}_2 , by the formula

$$\hat{q}_{2,1} = \left[\frac{\hat{v}_2 - \hat{v}_1}{M}\right].$$
 (35)

The folding integers n_1 and n_2 in (7) can be calculated by three steps similar to those shown by (27)–(31). As shown in (9a) and (9b), e_1 and e_2 are the estimation errors of \hat{v}_1 and \hat{v}_2 . The robust CRT guarantees that if

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$$x_1 < \frac{M}{4} \tag{36a}$$

and

$$e_2 < \frac{M}{4} \tag{36b}$$

 n_1 and n_2 can be uniquely determined. Equations (36a) and (36b) represent the limits on tolerable deviations for using the robust CRT to resolve the velocity ambiguity.

Algorithm I: The Modified Algorithm of the Closed-Form Robust CRT Algorithm used by us to Resolve the Velocity Ambiguity.

Parameters:

Moduli 1: Γ_1 Moduli 2: Γ_2 Modular multiplicative inverse of Γ_1 modulo Γ_2 : $\overline{\Gamma}_{2,1}$ Modular multiplicative inverse of γ_1/Γ_2 modulo Γ_2 : $b_{2,1} = 1$ $(\gamma_1 = \Gamma_1\Gamma_2/\Gamma_1 = \Gamma_2)$

Algorithm:

1: Calculate $\hat{q}_{2,1} = \left[\frac{\hat{v}_2 - \hat{v}_1}{M}\right]$. 2: Calculate $\hat{\xi}_{2,1} = \hat{q}_{2,1} \overline{\Gamma}_{2,1} \mod \Gamma_2$. If $\hat{\xi}_{2,1} > \Gamma_2/2$, let $\hat{\xi}_{2,1} = \hat{\xi}_{2,1} - \Gamma_2$. If $\hat{\xi}_{2,1} < -\Gamma_2/2$, let $\hat{\xi}_{2,1} = \hat{\xi}_{2,1} + \Gamma_2$. 3: Calculate $\hat{n}_1 = \hat{\xi}_{2,1} b_{2,1} \frac{\gamma_1}{\Gamma_2} \mod \gamma_1$. If $\hat{n}_1 > \Gamma_1/2$, let $\hat{n}_1 = \hat{n}_1 - \Gamma_1$. If $\hat{n}_1 < -\Gamma_1/2$, let $\hat{n}_1 = \hat{n}_1 + \Gamma_1$. 4: Calculate $\hat{n}_2 = \frac{\hat{n}_1 \Gamma_1 - \hat{q}_{2,1}}{\Gamma_2}$. If $\hat{n}_2 > \Gamma_2/2$, let $\hat{n}_2 = \hat{n}_2 - \Gamma_2$. If $\hat{n}_2 < -\Gamma_2/2$, let $\hat{n}_2 = \hat{n}_2 + \Gamma_2$.

The analyses of the robust CRT in [23]–[27] generally consider positive real number values of N. The Steps 1–4 of the closed-form robust CRT algorithm [25] shown in Section IV-A also consider a positive real number. However, for resolving the velocity ambiguity of multifrequency pulse-to-pulse coherent Doppler sonars, the true velocity v can be negative. Thus, for our application, the closed-form robust CRT algorithm needs to be modified slightly to take into account the possibility of negative folding integers. The modified algorithm of the closed-form robust CRT algorithm used by us is shown in Algorithm I.

To retain consistency with the discussion of the reference method in [16] and to facilitate a fair comparison, the proposed method based on the robust CRT is presented by using two frequencies: f_1 and f_2 . As analyzed in [25] and Section IV-A, the closed-form robust CRT can be used to solve the ambiguity problem with more than two moduli as well, provided the coprime requirement is met for all frequencies employed. Thus, our proposed method can also be extended in a straightforward way for pulse-to-pulse coherent Doppler sonars with three or more frequencies.

V. THEORETICAL EVALUATION

The limits on tolerable variances in measurement values of the proposed and reference methods are analyzed in this section.

A. Reference Method

As shown in Section III, the reference method resolves the velocity ambiguity from the observed phase changes $\delta\phi_1$ and $\delta\phi_2$. For the reference method, the error term (p = 1 or p = 2) in (19) is given by the expression

$$\gamma = \|\delta\phi_p - \Delta\phi_{p \text{ est}}\|$$
$$= \varepsilon_p - (\varepsilon_2 - \varepsilon_1) \frac{f_p}{(f_2 - f_1)}.$$
(37)

In [16], $\sigma_{\varepsilon_1}^2$ and $\sigma_{\varepsilon_2}^2$ are denoted as the variances of the estimation errors ε_1 and ε_2 . $\sigma_{\varepsilon_1}^2$ and $\sigma_{\varepsilon_2}^2$ are assumed to be equal in [16], i.e.,

$$\sigma_{\varepsilon_1}^2 = \sigma_{\varepsilon_2}^2 = \sigma_{\varepsilon}^2 \tag{38}$$

6

where σ_{ε}^2 is the variance used for analyzing the tolerable limit for the reference method [16]. For $f_p = f_1$, (37) can be rewritten as

$$\gamma = \varepsilon_1 \left(1 + G \right) - \varepsilon_2 G \tag{39}$$

where G is a quantity referred to as the dual-frequency gain in [16] expressed as

$$G = \frac{f_1}{f_2 - f_1}.$$
 (40)

Considering (39), the variance of γ can be written as [16]

$$\sigma_{\gamma}^2 = \sigma_{\varepsilon}^2 \left[\left(1 + G \right)^2 + G^2 \right]. \tag{41}$$

For the reference method, when $|\gamma| < \pi$, the folding integers n'_p can be resolved accurately, and it fails when $|\gamma| > \pi$. Following [16], we assume that ε_1 and ε_2 are random variables with a normal distribution. From this, it follows that γ is also a normally distributed random variable. If the probability of $|\gamma| > \pi$ (i.e., failure rate) is negligibly low (say, on the order of 10^{-5}), the performance of resolving the velocity ambiguity can be considered to be very reliable. For the normally distributed variable γ with a standard deviation σ_{γ} , the probability for $|\gamma| > 4\sigma_{\gamma}$ is 6.3×10^{-5} [40], which is low enough to be considered a negligible probability of occurrence. Thus, we consider $4\sigma_{\gamma}$ as the threshold for defining negligible occurrence of failure of resolving the velocity ambiguity. Based on this, we obtain that when the condition

$$4\sigma_{\gamma} < \pi \tag{42}$$

is met, the folding integers n'_p are determined accurately by the reference method. Combining (41) and (42), the limit on the tolerable variances of measurement for the reference method is given by

$$\sigma_{\varepsilon} < \frac{\pi}{4\sqrt{\left(1+G\right)^2 + G^2}}.$$
(43)

In [16], the condition of $2\sigma_{\gamma} < \pi$ is considered for the reference method. Under the assumption of normal distribution, the probability for $|\gamma| > 2\sigma_{\gamma}$ is 0.05. The limit on the tolerable variances of measurement for the reference method in [16] is

$$\sigma_{\varepsilon} < \frac{\pi}{2\sqrt{\left(1+G\right)^2 + G^2}}.$$
(44)

If we consider the limit in [16], an additional lowpass filter is needed. In this paper, to compare the reference method and the proposed method fairly, we assume that no additional processing is performed on the resolved velocities. In other words, the performance comparison is done on the outputs before any postprocessing. Thus, (43) is employed in this paper to describe the operational limits of the reference method for the comparison.

B. Proposed Method

As shown in (36a) and (36b), when using the robust CRT to resolve the velocity ambiguity in our proposed method, both the errors e_1 and e_2 of the estimated velocities should be lower than M/4. Let $\sigma_{e_1}^2$ and $\sigma_{e_2}^2$ denote the variances of e_1 and e_2 . Similar to the approach followed by us in analysis of the reference method, we assume that the thresholds $4\sigma_{e_1}$ and $4\sigma_{e_2}$ define negligible failure rate for the errors e_1 and e_2 , respectively. Thus, the proposed method would be considered successful with negligible failure rate under the condition

$$|e_1| < 4\sigma_{e_1} \tag{45a}$$

$$|e_2| < 4\sigma_{e_2}.\tag{45b}$$

Combining (34), (36a), (36b), (45a), and (45b), we can express the limits on the tolerable variances of e_1 and e_2 , respectively, as

$$\sigma_{e_1} < \frac{M}{16} = \frac{V_{\text{am1}}}{8\Gamma_1} \tag{46a}$$

and

$$au_{e_2} < \frac{M}{16} = \frac{V_{\rm am2}}{8\Gamma_2}.$$
(46b)

According to (1), the relationships among σ_{e_1} , σ_{e_2} , σ_{ε_1} , and σ_{ε_2} can be expressed as

$$\sigma_{\varepsilon_1} = \frac{4\pi f_1 T_0}{c} \sigma_{e_1} \tag{47a}$$

and

$$\sigma_{\varepsilon_2} = \frac{4\pi f_2 T_0}{c} \sigma_{e_2}.$$
(47b)

Considering (5), (46a), (46b), (47a), and (47b), the limit for the proposed method can be written as

$$\begin{cases} \sigma_{\varepsilon_1} < \frac{\pi}{2\Gamma_1} \\ \sigma_{\varepsilon_2} < \frac{\pi}{2\Gamma_2}. \end{cases}$$
(48)

When $f_1 < f_2$, according to (5a), (5b), and (34), we have that $\Gamma_1 > \Gamma_2$, which means that $\pi/2\Gamma_1 < \pi/2\Gamma_2$. To compare to the reference method, a conservative limit of the proposed method is chosen as

$$\sigma_{\varepsilon} < \frac{\pi}{2\Gamma_1}.\tag{49}$$

C. Comparison

We evaluate the performance of the proposed method against the reference method by comparing the limits described in (43) and (49). From (43) and (49), we can see that if

$$\frac{\pi}{2\Gamma_1} > \frac{\pi}{4\sqrt{(1+G)^2 + G^2}}$$
(50)

the proposed method would be more robust than the reference method. Considering (5a), (5b), and (34), the dual-frequency gain G in (40) can be rewritten as

$$G = \frac{\Gamma_2}{\Gamma_1 - \Gamma_2}.$$
(51)

Substituting (51) into (50), it can be rewritten as

$$\frac{1 + (\Gamma_2 / \Gamma_1)^2}{(\Gamma_1 - \Gamma_2)^2} > \frac{1}{4}.$$
(52)

Equation (52) provides a theoretical criterion defining the scenarios where our proposed method performs better than the reference method for resolving the velocity ambiguity of multifrequency pulse-to-pulse coherent Doppler sonars. If the criterion of (52) is not satisfied, the reference method performs better than our proposed method. The following example shows that the criterion of (52) can be satisfied easily.

Considering an example where $f_1 = 1.40$ MHz, $f_2 = 1.75$ MHz, $\Gamma_1 = 5$, and $\Gamma_2 = 4$, we have that

$$\frac{\pi}{2\Gamma_1} = \frac{\pi}{10} \tag{53}$$

$$\frac{\pi}{4\sqrt{\left(1+G\right)^2+G^2}} \approx \frac{\pi}{26} \tag{54}$$

CHI et al.: ROBUST RESOLUTION OF VELOCITY AMBIGUITY FOR MULTIFREQUENCY PULSE-TO-PULSE COHERENT DOPPLER SONARS



Fig. 2. Geometry of the monostatic pulse-to-pulse coherent Doppler sonar and steady flow simulated in this paper. The parameter values used are defined in Table I.

and

$$\frac{1 + (\Gamma_2 / \Gamma_1)^2}{(\Gamma_1 - \Gamma_2)^2} = 1.64 > \frac{1}{4}.$$
(55)

The analysis in (53)–(55) shows that in this case, our proposed method is more robust.

D. Summary

The theoretical analysis indicates that the limit on tolerable measurement variance of the proposed method is higher than that of the reference method. This means that our proposed method is more robust to the measurement errors, than the reference method, for resolving the velocity ambiguity of multifrequency pulse-to-pulse coherent Doppler sonars.

It should be noted that our proposed method has the requirement that the ambiguity velocities V_{am1} and V_{am2} should be designed to be coprime, as shown in (35). However, the reference method does not have this requirement.

VI. SIMULATION

We use the model proposed in [39] for pulse-to-pulse coherent Doppler sonars to conduct our simulations. This model has been validated with experimental data, and has shown to accurately reproduce the characteristics of pulse-to-pulse coherent Doppler sonar data. Thus, it is a reliable model to generate data for comparing the algorithms presented here, which are used for postprocessing the data. The model simulates a steady flow with a preset velocity profile for pulse-to-pulse coherent Doppler sonars. It involves introducing point scatterers randomly into a three-dimensional (3-D) domain in the model, which surrounds the area sampled by the sonar beams, as shown in Fig. 2. The signal received by a sonar transducer is determined by overlapping the individual signals backscattered from all of the scatterers. The signal backscattered from any particular scatterer is determined as a bandwidth-limited transmitted pulse, which is delayed by the travel time from the source to the scatterer and the target to the receiver, considering the transducer beam patterns, spherical spreading, and other factors [39]. The details about this modeling technique are given in the appendix.

We employ a monostatic pulse-to-pulse coherent Doppler sonar model as illustrated in Fig. 2, to test our proposed method and the reference one. Two carrier frequencies $f_1 = 1.40$ MHz and $f_2 = 1.75$ MHz were used, which are similar to the frequencies employed in the multifrequency Doppler sonar setup considered in [15]. The transmit pulse length was set at 4 μ s. The ping number was set to be 4 and the ping interval was 0.9 ms. The transducer diameter was 2 cm. The

TABLE I Setup Parameters of the Simulation

Parameters	Value
Carrier frequency f_1	1.40 MHz
Carrier frequency f_2	1.75 MHz
Transmit pulse length	4 µs
Ping interval T_0	0.90 ms
Ping number	4
Ambiguity velocity V_{am1} for f_1	0.30 m/s
Ambiguity velocity V_{am2} for f_2	0.24 m/s
Transducer diameter	2 cm
Receiver SNR	20 dB
Magnitude of the flow velocity v_0	1.60 m/s
Velocity direction	$\theta = 45^\circ, \ \beta = 45^\circ$

velocity vector of the steady flow in Fig. 2 is expressed as

$$\mathbf{v}_0 = v_0 \cdot (\sin\theta\cos\beta, \sin\theta\sin\beta, \cos\theta) \tag{56}$$

where v_0 is the magnitude of v_0 , and θ and β are the azimuth and elevation angles as illustrated in Fig. 2. In our simulation, we set the parameter values $v_0 = 1.6$ m/s, $\theta = 45^\circ$, and $\beta = 0^\circ$. Similar to [17], the particle scattering cross section and receiver noise level were adjusted to obtain a signal-to-noise ratio (SNR) of 20 dB at the output of the receiver. The simulation scenario is representative of pulse-to-pulse coherent Doppler sonars. Table I summarizes the setup parameters in our simulation.

The covariance method [1], [2], [16], [17] is used to process the received signal to obtain the observed velocities \hat{v}_1 and \hat{v}_2 , and the observed phase changes $\delta\phi_1$ and $\delta\phi_2$. The relationships among \hat{v}_1 , \hat{v}_2 , $\delta\phi_1$, and $\delta\phi_2$ are

$$\dot{\phi}_1 = \frac{c}{4\pi f_1 T_0} \delta \phi_1 \tag{57a}$$

and

$$\hat{v}_2 = \frac{c}{4\pi f_2 T_0} \delta \phi_2. \tag{57b}$$

Our proposed method reconstructs \hat{n}_1 and \hat{n}_2 from \hat{v}_1 and \hat{v}_2 . The reference method [16] reconstructs \hat{n}'_1 and \hat{n}'_2 from $\delta\phi_1$ and $\delta\phi_2$. When using the proposed method, the estimated velocities for f_1 and f_2 are obtained as follows:

$$\hat{v}' = \hat{v}_1 + \hat{n}_1(2V_{\rm am1}) \tag{58}$$

and

$$\hat{v}'' = \hat{v}_2 + \hat{n}_2(2V_{\rm am2}). \tag{59}$$

When using the reference method, the estimated velocities for f_1 and f_2 are obtained as follows:

$$\hat{v}' = \hat{v}_1 + (-\hat{n}_1') \left(2V_{\rm am1}\right) \tag{60}$$

and

$$\hat{v}'' = \hat{v}_2 + (-\hat{n}_2') (2V_{\rm am2}). \tag{61}$$

The final estimated velocity for the two methods is obtained as the average of these two given by

$$\hat{v} = \frac{\hat{v}' + \hat{v}''}{2}.$$
(62)



Fig. 3. Observed velocities at the two carrier frequencies and the true velocity (v = 1.13 m/s).



Fig. 4. Comparison of (a) resolved velocity estimates obtained by the proposed method and (b) those obtained by the reference method, against the true velocity v = 1.13 m/s.

Fig. 3 shows the true velocity and the observed velocities at f_1 and f_2 , for the setup $v_0 = 1.60$ m/s. For the monostatic Doppler sonar in Fig. 2, the radial velocity component, which is the true velocity being measured, is 1.13 m/s. From Fig. 3, it can be seen that the observed velocities are at values that are different from the true velocity, because the latter exceeds the ambiguity velocity limits at both the frequencies. We attempt to resolve the velocities, by using our proposed method and the reference one. Fig. 4 presents the results of resolving the velocity

ambiguity from the observed velocities using the two methods in 40 independent trials. We see that while the proposed method fails only once in these trials, the reference method fails 13 times.

Following the footsteps of Wang and Xia [25] and Wang *et al.* [27], we choose the TFR as a metric to evaluate the performances of resolving the velocity ambiguity. The TFR is defined as the ratio of the number of times the resolution of the velocity ambiguity fails, within a given number of estimation attempts in independent trials. We employ 10 000 estimation trials for the performance evaluation. We set $v_0 = 1.60$ m/s for calculating the TFRs. Our simulations show that the TFR of our proposed method is 4.36%, whereas the TFR of the reference method is 33.25%. In other words, for $v_0 = 1.60$ m/s and the simulation parameters considered, the TFR of our proposed method is nearly eight times lower as compared to that of the reference method. The results demonstrate that for resolving the velocity ambiguity of multifrequency pulse-to-pulse coherent sonars, our proposed method is more robust to the measurement errors, than the reference method.

VII. CONCLUSION

We propose a method based on the robust CRT for resolving the velocity ambiguity of multifrequency pulse-to-pulse coherent Doppler sonars. The theoretical analysis shows that the limit on tolerable measurement variance of our proposed method is higher than that of the reference method in most practical cases, which provides a theoretical justification to demonstrate that our proposed method is more robust. A theoretical criterion is given to indicate the operational regime where our proposed method is superior to the reference method. The simulations show that the TFR of our proposed method is nearly eight times lower than that of the reference method, for the parameter setup considered. Even though the proposed method is evaluated by considering a Doppler sonar with two frequencies, it can also be applied to resolve the velocity ambiguity for pulse-to-pulse coherent Doppler sonars with three or more frequencies.

Even though both the theoretical analysis and simulations demonstrate that our proposed method has a better performance against the reference method, our proposed method is limited by the coprime requirement on the ambiguity velocity values, and thus the operational frequencies. Because our proposed method employs the robust CRT, the ambiguity velocities of the multiple frequencies used in the coherent Doppler sonar should be coprime when using the proposed method. However, the reference method has no such requirement. The coprime requirement of the proposed method can be achieved by appropriate design of the system.

APPENDIX

Numerical simulation of steady flow was performed using the model of pulse-to-pulse coherent Doppler sonar proposed in [39]. The model simulates coherent scattering from individual particles moving inside a rectangular region with a preset velocity profile. As particles exit the 3-D model domain shown in Fig. 5, they are reintroduced at the opposite face using a uniform random distribution.

The positions of particles are updated for each backscatter calculation. The position for the *j*th particle is calculated as

$$\mathbf{X}_{j}(t_{i+1}) = \mathbf{X}_{j}(t_{i}) + \mathbf{V}_{\mathbf{j}}[\mathbf{X}_{j}(t_{i})]T_{0} + \Gamma_{\mathbf{v}}[\mathbf{X}_{j}(t_{i})]T_{0}$$
(63)

where $\mathbf{X}_j(t_i)$ is the position vector (in terms of *x*-*y*-*z* coordinates) of the *j*th particle at time t_i , $\mathbf{V}_j[\mathbf{X}_j(t_i)]$ is the particle velocity vector at the position $\mathbf{X}_j(t_i)$, $\Gamma_{\mathbf{v}}[\mathbf{X}_j(t_i)]$ is a random vector with mean zero and standard deviation assigned for the velocity, and $t_{i+1} - t_i = T_0$, which is the period of pulse repetition.



Fig. 5. Model rectangular domain for a bistatic coherent Doppler sonar. The dashed lines represent the sonar beams. The dots indicate the point scatterers.

Inevitably particles will drift out of the sample domain in Fig. 5. These escaped particles are reintroduced back into the domain on the opposite surface from which they escaped so that the total number of particles does not change. Particles are reintroduced by wrapping their positions around so that they appear to drift in on the opposite side of the domain, but their positions in the other two dimensions are randomized [39]. For example, a particle that drifts out onto the negative-x domain face will have its position reassigned as

$$\mathbf{X}_{j}(t_{i+1}) = [x_{j}(t_{i+1}) + (x_{\max} - x_{\min})]\mathbf{i} + [y_{\min} + (y_{\max} - y_{\min})\rho_{y}]\mathbf{j} + [z_{\min} + (z_{\max} - z_{\min})\rho_{z}]\mathbf{k}$$
(64)

where $x_j(t_{i+1}) \leq x_{\min}$ is the *x* position calculated using (63), **i**, **j**, and **k** are the unit vectors in the *x*, *y*, and *z* directions respectively, and the rectangular model domain is defined in the directions of *x*, *y*, and *z* by x_{\max} , x_{\min} , y_{\max} , y_{\min} , z_{\max} , and z_{\min} , and ρ_y and ρ_z are random variables uniformly distributed between 0 and 1. Particle positions are randomized upon wrapping to accurately reproduce an evolving backscatter domain.

The signal backscattered from any one particle is assumed to be an amplitude-scaled, bandwidth-limited version of the transmitted pulse. Thus, the overall return is expressed as

$$r(t) = \sum_{i} a_{i} p\left(t - \frac{r_{si}(t) + r_{ri}(t)}{c}\right) + n(t)$$
 (65)

where a_i is the backscatter amplitude of the *i*th particle, p(t) is the transmitted pulse, *t* is the time starting from the pulse transmission, $r_{si}(t)$ and $r_{si}(t)$ are the source-target and target-receiver distances for the *i*th particle, and n(t) is the added ambient noise.

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