Preprocessor Based on Suprathreshold Stochastic Resonance for Improved Bearing Estimation in Shallow Ocean

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Abstract- **Localization of acoustic sources in the ocean is a problem of tremendous interest in underwater acoustics. One of the many factors that limit the performance of processors used for underwater acoustic source localization is the low signal - to - noise ratio (SNR) in the ocean. Preprocessors based on wavelet denoising and suprathreshold stochastic resonance (SSR) have been proposed in the literature for enhancing SNR and thereby improving the performance of processors used for bearing estimation [1,2]. Denoising techniques based on SSR exploit the fact that the environmental noise in shallow ocean has a heavy - tailed non- Gaussian distribution [3]. In this paper, a method for designing an SSR based preprocessor is presented. It is shown that the use of this preprocessor leads to a significant improvement in the bearing - estimation performance of Bartlett, Multiple Signal Classification (MUSIC) and Subspace Intersection Method (SIM) [4] processors at low SNR. The improved performance appears in the form of a sharper peak in the ambiguity function, lower bias and lower RMS error in bearing estimation, and better resolution of closely spaced sources.**

I. INTRODUCTION

Underwater source localization is one of the most commonly encountered problems in underwater acoustic signal processing. Systems used for underwater source localization have to operate in a challenging environment. The primary concern is that of the low SNR encountered in an ocean channel, owing to the high level of ambient noise. The performance of processors used for bearing estimation using standard methods is found to degrade rapidly as the SNR reduces. For example, in the case of plane wave direction-of-arrival estimation by MUSIC processor, the mean square error (MSE) is inversely proportional to the SNR [5]. In a real environment, the SNR encountered is often too low for these methods to provide reliable bearing estimates. One way to extend the usability of such techniques is to use pre-processors to ensure better estimates or improve the range of parameters (e.g. lower SNR, fewer snapshots) over which the methods are usable. Some examples are pre-processors based on SSR [2] and wavelet denoising [1], which have been proposed previously in the literature, and have been shown to aid in bearing estimation.

Stochastic resonance (SR) is a non-linear phenomenon encountered in non-linear devices such as quantizers [6]. An SR system exhibits non-monotonic variation of its performance measures such as output SNR, SNR gain, Fisher information or mutual information with respect to the input noise variance. It has been shown that adding a small amount of noise at the input of the quantizer along with the signal that is in general smaller than the quantizer threshold, tends to aid the performance of the system. When an array of quantizers is used and independent and identically distributed (iid) noise is added to each quantizer along with the signal that may be larger than the threshold, the performance is found to be better than that obtained using a single quantizer. This phenomenon is referred to as SSR [7,8]. The SNR gain provided by an SSR system is greater than unity if the input noise is non-Gaussian, and a significant SNR enhancement can be achieved if the noise is highly leptokurtic [9]. Denoising techniques based on SSR can be designed to exploit the heavy-tailed nature of environmental noise in ocean. Noise sources in ocean include impulsive sources of biological origin, such as the snapping shrimp [10]. These impulsive contributions cause the noise distribution to be heavy-tailed. Hence, SSR based denoising may be employed as a pre-processor to a bearing estimation processor in shallow ocean.

In this paper, we address the design of an SSR-based pre-processor to improve the performance of bearing estimation in shallow ocean. The rest of the paper is organized as follows. In Section II, we lay out the mathematical framework relevant to the problem of bearing estimation in shallow ocean. In Section III, the SSR denoiser is described and its design is discussed in detail. In Section IV, we present some results on bearing estimation using the SSR denoiser. Conclusions are presented in Section V.

II. BEARING ESTIMATION IN SHALLOW OCEAN

Consider the problem of bearing estimation in shallow ocean using a uniform horizontal linear array (HLA) of M sensors. The sensor array receives signals from J mutually uncorrelated narrowband sources (J<M), with center frequency f_0. Let the bearing angle of the j^{th} source with respect to the endfire direction of the array be θ_j, and let it be located at a depth z_j and range r_j with respect to the reference sensor of the HLA. The HLA is considered to be located at a depth z and to have an inter-element spacing of d meters, where d is chosen to be less than or equal to half-wavelength of the signal in order to satisfy Nyquist's sampling criterion. The geometry of the setup

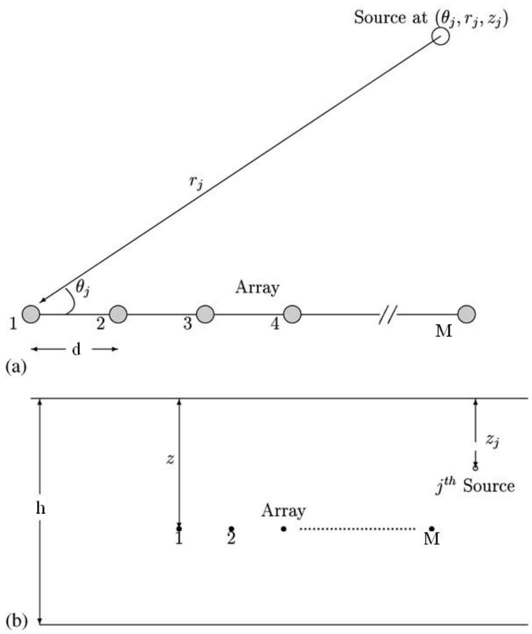

Fig. 1: Geometry of the source and receiver array, a) top view b) side view

is shown in Fig. 1.

The considerations regarding the acoustic environment in a shallow ocean are numerous and challenging. In general, the ocean is an inhomogeneous and time-varying acoustic medium. The presence of reflecting boundaries at the top and bottom of the ocean channel lead to multi-path propagation of waves due to multiple reflections from these boundaries. For the present study, we use a relatively simple model of the ocean called the Pekeris model, which comprises a homogeneous water layer of constant depth h over a fluid half-space of sediment. It is assumed that the variation of acoustic properties of the ocean in the horizontal direction is negligible in the range of interest.

The array data model can be described as in [4]. The complex amplitude of the signal at any frequency at the m^{th} element of the array due to the j^{th} source can be represented by

$$s_{jm} = p_{jm}\,\eta_j, \tag{1}$$

j = 1, ... , J ; m = 1, ... , M;

where η_j is a complex Gaussian random variable with zero mean, and the variance of η_j given by

$$\sigma^2{}_j = E[|\eta_j|^2] \tag{2}$$

is a measure of the strength of the source. The term p_{jm} in (1) can be written as the sum of the discrete normal modes of the channel [11].

$$p_{jm} = \sum_{n=1}^{N} b_{jn}\, e^{i(m-1)k_n d cos\theta_j}, \tag{3}$$

where N is the number of modes, and

$$b_{jn} = \left(\frac{2\pi}{k_m r_j}\right)^{1/2} \psi_m(z_j)\psi_m(z) e^{-\alpha_n r_j - i(k_n r_j - \pi/4)} \tag{4}$$

is the complex amplitude of the n^{th} normal mode at the first element of the array due to the j^{th} source, the function $\psi_m(z)$ is the eigenfunction of the n^{th} normal mode of the oceanic waveguide, and the quantities k_n and α_n are the corresponding wavenumber and attenuation coefficient, respectively. The output of the array of narrowband sensors can be expressed as the vector

$$\begin{aligned} \mathbf{y} &= [\mathbf{y}_1 \cdots \mathbf{y}_M]^T \\ &= \mathbf{P}(\mathbf{X})\eta + \mathbf{n}, \end{aligned} \tag{5}$$

where $\eta = [\eta_1 \, . \, . \, .\eta_J]^T$ is the source signal vector,

$\mathbf{n} = [\mathbf{n}_1 \, . \, . \, .\mathbf{n}_M]^T$ is the array noise vector,

$$\mathbf{X} = [\mathbf{x}_1 ... \mathbf{x}_J]^T, \tag{6}$$

$$\mathbf{x}_j = [\theta_j \;\; r_j \;\; z_j]^T, \quad j = 1, ..., J \tag{7}$$

is the (unknown) position vector of the j^{th} source, and

$$\mathbf{P} = \mathbf{P}(\mathbf{X}) = [\mathbf{p}(\mathbf{x}_1)...\mathbf{p}(\mathbf{x}_J)]^T \tag{8}$$

is an M × J matrix whose columns

$$\mathbf{p}(\mathbf{x}_j) = [p_{j1} \, . \, . \, . \, p_{jM}]^T, \quad j = 1, ..., J \tag{9}$$

are the array signal amplitude vectors. The vectors $\mathbf{p}(\mathbf{x}_j)$ can be expressed as

$$\mathbf{p}(\mathbf{x}_j) = \mathbf{A}(\theta_j)\mathbf{b}(r_j, z_j), \quad j = 1, ..., J \tag{10}$$

where

$$\mathbf{b}(r_j, z_j) = [b_{j1} ... b_{jN}]^T, \quad j = 1, ..., J \tag{11}$$

are the mode amplitude vectors whose elements b_{jn} are defined by (4), and

$$\mathbf{A}(\theta) = [\mathbf{a}(k_1 \cos(\theta), \; ... \; , \mathbf{a}(k_N \cos(\theta)]^T \tag{12}$$

is an M×N matrix whose columns are the steering vectors defined as

$$\mathbf{a}(k_n \cos\theta) = [1 \; e^{ik_n d\cos\theta}, \; ... \; , e^{i(M-1)k_n d\cos\theta}]^T, \quad n = 1, ..., N. \tag{13}$$

The environmental noise in the ocean must be modeled by probability density functions (pdf) that can represent the heavy-tailed nature of noise usually found in such an environment. One such model that may be used is the generalized Gaussian noise (GGN). This model is a parameterized noise model that can describe well the impulsive nature of ocean noise, and includes the Gaussian distribution as a special case. The pdf of a GGN random variable X with variance σ^2 is given by

$$f_X(x) = \frac{p}{2\,A(p)\,\Gamma(1/p)} \;\exp\left\{-\left[\frac{|x|}{A(p)}\right]^p\right\}, \; p > 0, \tag{14}$$

where $A(p) = \left[\sigma^2 \frac{\Gamma(1/p)}{\Gamma(3/p)}\right]^{1/2}$, $\Gamma(.)$ is the gamma function.

This pdf reduces to a Gaussian distribution for p=2 and is leptokurtic for p<2. Fig. 2 shows several examples of the GGN pdf at different values of parameters p.

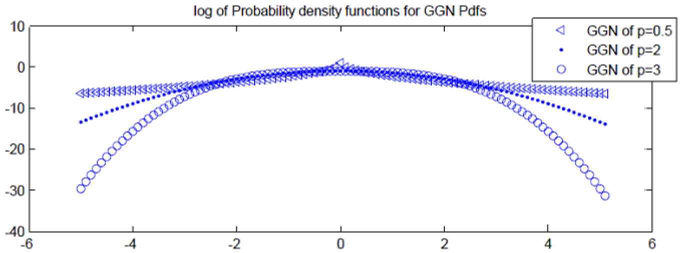

Fig. 2: log of Probability density functions (zero mean unit variance) for GGN pdf for different values of p

The problem of source localization in shallow ocean consists of range, depth, and bearing estimation of a source. For the estimation of source bearing, several processors have been proposed in the literature, notable ones being the Bartlett processor [12], Minimum Variance distortionless beamformer (Capon beamformer) [12], MUSIC [12], ESPRIT [13], min-norm [14] and SIM [4]. We consider three processors, namely the Bartlett, MUSIC and SIM processors, to study the effect on bearing estimation in an ocean.

The Bartlett processor [12], also known as the delay-and-sum beamformer, is one of the oldest conventional methods of direction-of-arrival estimation of plane waves. It involves maximizing the ambiguity function

$$\mathbf{A}_{bart}(\theta) = \mathbf{s}^H(\theta)\, \mathbf{R}\, \mathbf{s}(\theta), \tag{15}$$

which is equivalent to finding the direction θ that maximizes the average correlation between the received data vector and the steering vector $\mathbf{s}(\theta)$. In (16), R denotes the array data covariance matrix.

In the case of a shallow ocean, $\mathbf{s}(\theta)$ has to be replaced by the steering vector $\mathbf{s}(\theta, r, z)$ corresponding to bearing θ, range *r* and depth z, and $\mathbf{A}_{bart}(\theta, r, z)$ has to be maximized with respect to all its arguments. If r and z are known, bearing can be estimated by maximizing the ambiguity function with respect to θ.

MUSIC [12] is a popular method of multiple source bearing estimation, based on the eigen decomposition of the data covariance matrix **R**. In MUSIC, the estimation of J sources is done by searching for the J highest peaks of the MUSIC ambiguity function

$$\mathbf{A}_{MUSIC}(\theta) = \frac{1}{\mathbf{s}^H(\theta)\, \mathbf{U}\, \mathbf{U}^H\, \mathbf{s}(\theta)}, \tag{16}$$

where **U** is the matrix of noise eigenvectors, which correspond to the M-J smallest eigen values obtained from the eigen decomposition of **R**.

The Subspace Intersection Method (SIM) of bearing estimation proposed by Lakshmipathi and Anand [4] is a method that allows bearing estimation in ocean without prior knowledge of the range or depth of the source. SIM is based on determining the intersection of two subspaces, namely the modal subspace $M(\theta)$ spanned by the modal steering vectors, and the signal subspace $S(\theta_j)$, *j=1, 2, . . . J*, spanned by the signal eigenvectors of array data covariance matrix **R**. It is known that these subspaces intersect only at bearing angles that correspond to source positions, i.e, at $\theta = \theta_j$. This fact is exploited in order to estimate the source positions. Recently, an enhanced version of SIM, namely the constrained least squares SIM has been proposed by Pang, Lin, Zhang and Huang [15].

In the next section, we describe the design of the SSR denoiser, which will be used as a preprocessor to the above mentioned bearing estimation methods to improve their performance.

III. SSR DENOISER

The SSR denoiser is based on the phenomenon of SSR, which is encountered in non-linear systems such as quantizers. It consists of an array of Q one-bit quantizers with a common input x(t), t = 0, 1, ... ,L -1, where L is the number of data samples. The input noisy signal x(t) consists of a pure signal As(t) and an environmental noise w(t), i.e,

$$x(t) = A\, s(t) + w(t),\ t = 0,1,\dots L-1, \tag{17}$$

Both the signal and noise *w(t)* are scaled by the same factor so that noise w(t) has unit variance. We further assume that

$$\frac{1}{L}\sum_{t=0}^{L-1} s^2(t) = 1 \quad , \tag{18}$$

so that the parameter A^2 denotes the signal power. Independent and identically distributed white noises $a_1(t)$, $a_2(t)$,.. . $a_Q(t)$ that are independent of w(t) are added separately to the quantizer inputs. We will refer to these collectively as SSR noise. The quantizer outputs {y_q (t), q =1, 2, ..., Q} are averaged to obtain the denoised signal z(t) at the output of the quantizer array. Thus, the output of the q^{th} quantizer is

$y_q(t) = \text{sgn}[\, x(t) + a_q(t)\,] = \text{sgn}[\, x(t) + \sigma \upsilon_q(t)\,]$, q=1,2,...,Q, (19)

where *sgn(.)* denotes the signum function, and $\{\upsilon_q(t), q = 1, \cdots$ are stationary independent and identically distributed noises with zero mean and unit variance. The output of the SSR denoiser is

$$z(t) = \frac{1}{Q}\sum_{q=1}^{Q} y_q(t). \tag{20}$$

A schematic diagram of the above system is shown in Fig. 3. It can be shown that [9]

$$E[z(t)] = 2\int_{-\infty}^{\infty} F_{\upsilon}(u/\sigma)\, f_w(u - As(t))\, du - 1, \tag{21}$$

$$E[z^2(t)] = 1 - (4/Q)(Q-1)\int_{-\infty}^{\infty} [1 - F_{\upsilon}(u/\sigma)]\, F_{\upsilon}(u/\sigma) f_w(u - As(t))\, du \tag{22}$$

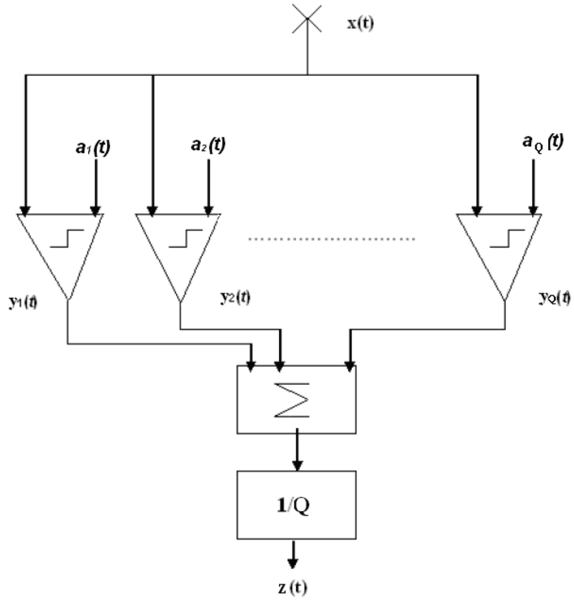

Fig. 3: Schematic of an SSR denoiser system

where $f_W(u)$ is the pdf of $w(t)$ and $F_\upsilon(u)$ is the cumulative distribution function of $\upsilon(t)$. The standard deviation σ of the SSR noise is to be chosen so as to maximize the correlation gain *G* defined in (26).

It may seem counter-intuitive that the purposeful addition of noises $a_q(t)$), q =1, 2, ..., Q at the quantizers to the existing environmental noise w(t) in the noisy signal should aid in reducing distortion due to environmental noise. However, according to the phenomenon of SSR, it is known [7][8] that for maximizing the SNR gain, the standard deviation σ of the SSR noise may not be zero. In general, an SSR denoiser can be optimized in two ways: by tuning system parameters (such as the value of threshold) for the best performance or by optimizing the SSR noise added at the input of the quantizers. In this paper, the latter approach is used to optimize performance.

For the purpose of bearing estimation, all the afore-mentioned methods require the knowledge of the array covariance matrix **R**. Hence, the performance of bearing estimation depends on the accuracy of estimation of **R,** which is estimated using a finite number of snapshots of the array data vector as

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K} y_k y_k^{\,H}, \tag{23}$$

where y_k denotes the k[th] snapshot. Due to the presence of environmental noise and finite number of snapshots, the estimate $\hat{\mathbf{R}}$ has errors. This estimation error is due to the imperfect correlation between the noisy signal and the clean signal.

We define the noisy signal vector as **x** = $[x(0) \ldots x(L-1)]^T$ and the clean signal vector A**s** = $[As(0)\ldots As(L-1)]^T$ The correlation between the noisy signal vector **x** and the clean signal vector A**s** is defined as

$$C_{sx} = E[A\mathbf{s}^T\mathbf{x}] / \{E[A^2\mathbf{s}^T\mathbf{s}\ \mathbf{x}^T\mathbf{x}]\}^{1/2} \quad , \tag{24}$$

where E[.] is the expectation operator. After denoising, the correlation of the denoiser output vector **z** = $[z(0)\ldots z(L-1)]^T$ with the clean signal vector A**s** is

$$C_{sz} = E[A\mathbf{s}^T\mathbf{z}] / \{E[A^2\mathbf{s}^T\mathbf{s}\ \mathbf{z}^T\mathbf{z}]\}^{1/2} \quad . \tag{25}$$

The correlation gain is given by

$$G = C_{sz} / C_{sx} \quad . \tag{26}$$

The design objective of the SSR preprocessor is to choose the value of standard deviation of SSR noise, pdf of SSR noise, and the number of quantizers in the SSR preprocessor to maximize the value of G.

Let the value of σ that maximizes G be referred to as σ_{opt}. In general, the value of σ_{opt} depends on several factors, such as the pdf of the environmental noise w(t), SSR noise a(t) and input SNR. In practice, it is not possible to find σ_{opt} since SNR at different sensors is unknown. To address this problem, we redefine the noisy signal vector **x** for the entire array as the MxL dimensional vector $\mathbf{x} = [\mathbf{x}_1^T \cdots$, where $\mathbf{x}_m$ is the data vector at the m[th] sensor. The clean signal vector *As* and denoised signal vector **z** are similarly redefined. Now A^2 is the signal power averaged over all sensors, and A is the input RMS amplitude.

In order to select the optimal value of σ, it is of interest to study the variation of G as well as σ_{opt} with the input SNR (taken over all sensors). Consider a 10-element uniform HLA of sensors in a shallow ocean channel modeled by a Pekeris model with the following parameters. The ocean channel has a constant depth of 100 m, the sound speed in water is 1500 m/s, sound speed in sediment layer is 1700 m/s, sediment density is 1500 kg/m^3 and the attenuation in the sediment layer is 0.2 dB/wavelength. The HLA is placed at a depth of 10 m in the ocean. A narrowband transmitting source is located at a range of 3000 m from the first sensor element of the HLA, and at a depth of 30 m. The transmitting frequency of the source is 50 Hz, and the bearing of the source with respect to the array axis is 45 degrees. The elements of the HLA are placed half-wavelength distance apart. The received signals are denoised by an SSR denoiser, with Gaussian SSR noise. Fig. 4 shows the variation of σ_{opt} with the input RMS amplitude A. The corresponding variation of gain G of the SSR denoiser with the input SNR is shown in Fig. 5. The plots were obtained by averaging over 400 Monte Carlo trials. At each SNR, the value of σ_{opt} is obtained as the σ that maximizes the value of G. The number of quantizers used in the SSR denoiser is 200. The

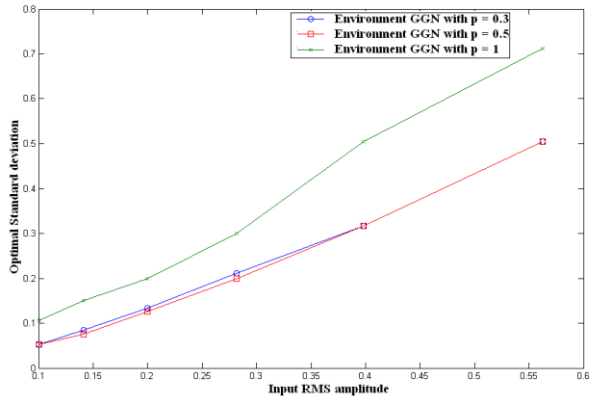

Fig. 4: σ $_{opt}$ vs input RMS amplitude A for different GGN environmental noises

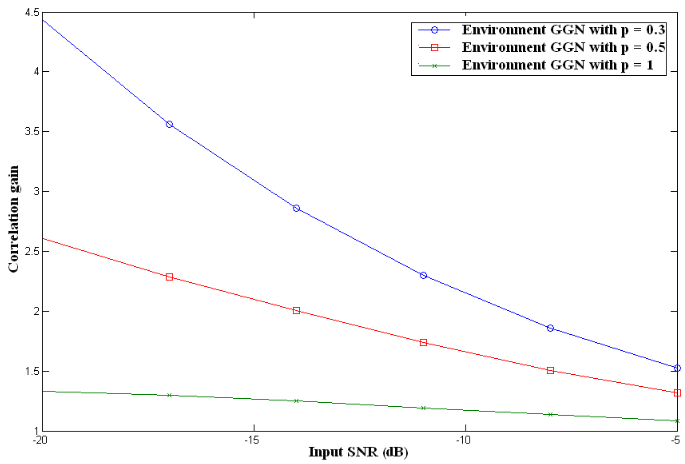

Fig. 5: Correlation gain VS input SNR for different GGN environmental noises for $\sigma = \sigma_{opt}$

environmental noise is modeled as GGN, with different values of parameter p.

From Fig 4, we can see that the value of σ $_{opt}$ is a monotonically increasing function of A, and is nearly linear in its variation. That is, we need to add more SSR noise at higher SNRs and a lower amount of SSR noise at lower SNRs. From Fig. 5, we see the SSR preprocessor provides a correlation gain greater than one when the environmental noise is leptokurtic, and the gain provided increases at lower values of SNR. The gain is higher when the value of parameter p is lower, i.e, when the noise is more leptokurtic the preprocessor provides higher denoising performance.

We now observe the variation of performance of the SSR denoiser at different values of SSR standard deviation σ . Fig. 6 shows the variation of correlation gain *G* plotted as a function of input SNR for different values of σ . The environmental noise is GGN with parameter p=0.5. It is seen that at low SNR, G decreases rapidly as σ is increased; while at high SNR, G decreases slowly as σ is reduced. Depending on prior knowledge regarding the input SNR, moderate values of σ (e.g. σ =0.32) may be selected, such that they provide a moderately

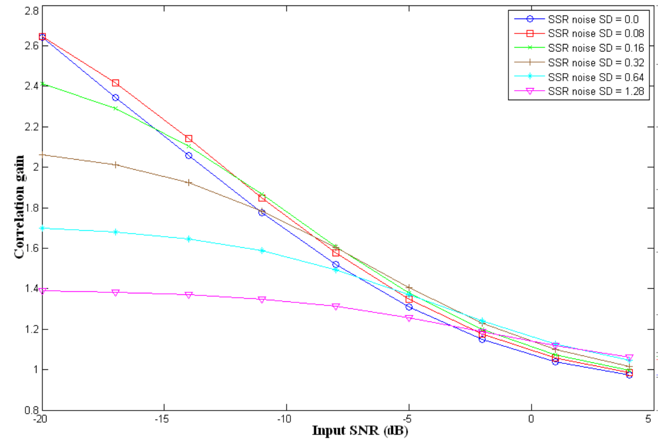

Fig. 6: Correlation gain vs SNR for different values of σ for 10-element array in GGN (p = 0.5) noise

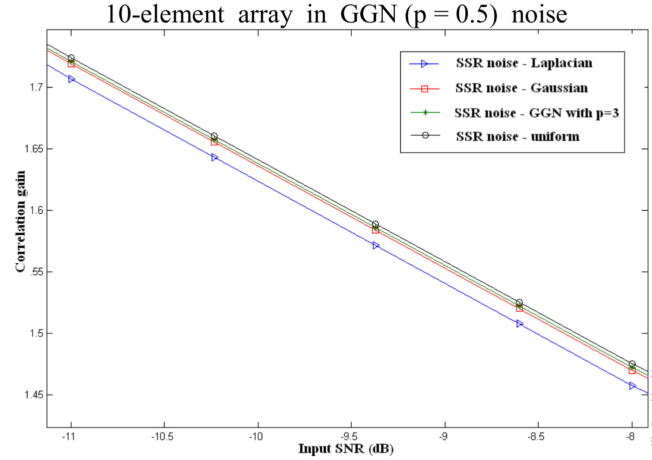

Fig. 7: Correlation gain vs input SNR for different SSR noise pdfs for 10-element array in GGN (p = 0.5) noise for $\sigma = \sigma_{opt}$

high value of G and a robust performance over a wide range of SNRs.

The pdf of the SSR noise used is another factor that affects the performance of the preprocessor. Fig. 7 shows the variation of G with input SNR when SSR noise with different pdfs is added at the optimal variance. The environment is GGN with p=0.5. The SSR noises added are the Laplacian (GGN with p=1), Gaussian (p=2), GGN with p=3, and uniform noise (p = infinity). It can be seen that as the kurtosis of the added noise decreases, the SSR denoiser seems to offer slightly higher performance in terms of G. However, the performance difference is found to be very marginal. In general, uniform SSR noise is found to give the best performance.

The performance of an SSR preprocessor is also directly affected by the number of quantizers Q. To study this variation, in Fig. 8 we plot G versus Q at an input SNR of −10 dB. The environmental noise is GGN with p=0.5. It is observed that the gain increases monotonically with the number of quantizers used, but seems to saturate at value of around Q = 100 quantizers. Using a higher number of quantizers than this does not seem to offer significant performance improvement in terms of gain G. Hence, for the rest of the simulations, we will

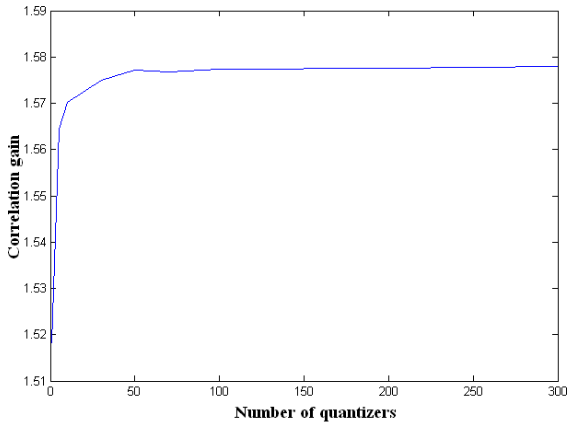

Fig. 8: Correlation gain vs number of quantizers, at input SNR −10 dB for 10-element array in GGN (p = 0.5) noise for $\sigma = \sigma_{opt}$

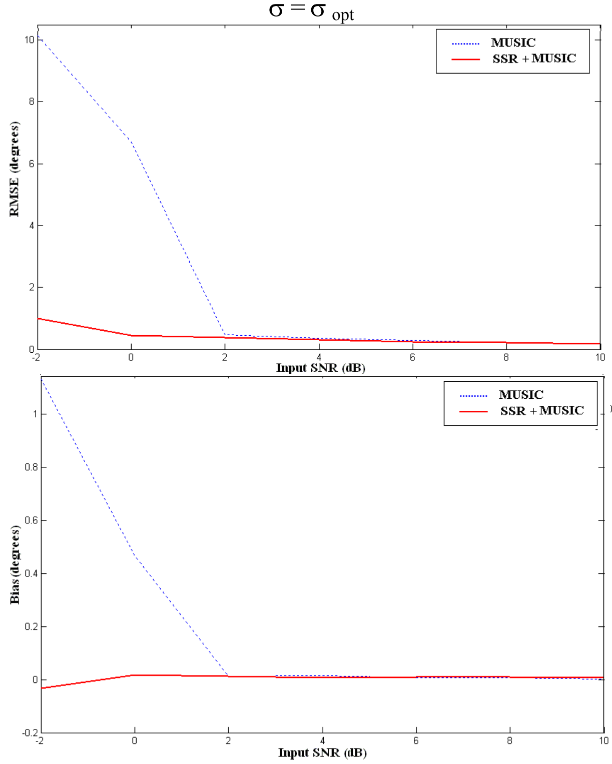

Fig. 9: a) RMSE and b) Bias of MUSIC estimator vs input SNR (in dB)

use 100 quantizers in the SSR preprocessor for bearing estimation.

The various aspects of design of the SSR preprocessor have hence been discussed. In the next section, we present some results to show the performance improvement offered by an SSR preprocessor for the problem of bearing estimation in shallow ocean.

IV. RESULTS

In this section, we present results to show the performance improvement in bearing estimates by the MUSIC, Bartlett and

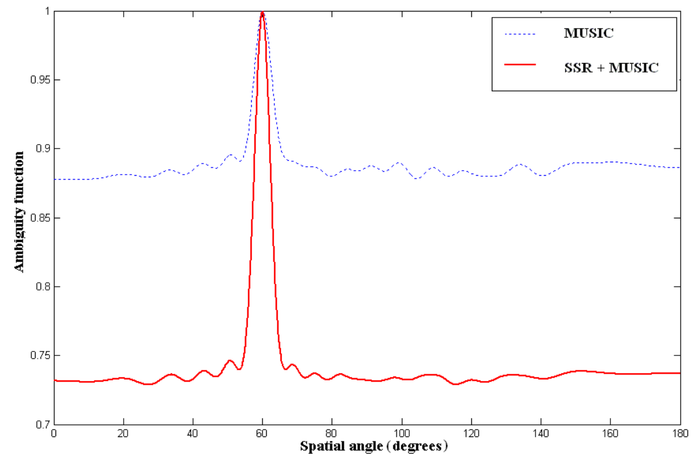

Fig. 10: Ambiguity function of MUSIC estimator at input SNR −10 dB

SIM processors, when preceded by a SSR preprocessor. The bias and RMS error of the bearing estimators, and the ambiguity function plots of the processors will be the performance measures considered. We consider the bearing estimation of sources in a Pekeris channel with the following parameters unless otherwise specified. The ocean channel has a constant depth of 100 m, the sound speed in water is 1500 m/s, sound speed in sediment layer is 1700 m/s, sediment density is 1500 kg/m^3 and the attenuation in the sediment layer is 0.5 dB/wavelength. A 20-element HLA is placed at a depth of 10 m in the ocean. A narrowband transmitting source is located at a range of 5000 m from the first sensor element of the HLA, and at a depth of 30 m. The transmitting frequency of the source is 50 Hz, and the bearing of the source with respect to the array axis is 60 degrees. The elements of the HLA are placed half-wavelength distance apart. The environmental noise is GGN with parameter p=0.5. The SSR denoiser is assumed to have 100 quantizers, and the added SSR noise is Gaussian. All plots are obtained by averaging over 500 Monte Carlo trials.

Fig. 9 shows the plots of RMSE and bias versus input SNR for the MUSIC bearing estimator in the environment mentioned above with and without the aid of SSR denoising. The processor uses 350 data snapshots for bearing estimation. The range and depth are assumed to be known in this case. It may be observed from the plot that the solid line representing the SSR aided MUSIC estimator clearly performs better than the dotted line representing the normal MUSIC estimator in terms of reduced bias and RMSE of bearing estimation. This is noticeable especially at lower SNR. As the input SNR is reduced, the performance of the MUSIC estimator degrades progressively and finally breaks down at an SNR of around 2 dB but with SSR preprocessing the onset of the breakdown has been delayed. The ambiguity function of the MUSIC estimator at an input SNR of −10 dB has been plotted in Fig. 10. It may be observed that the SSR aided MUSIC has a sharper ambiguity function than normal MUSIC, thus showing improvement in bearing estimation in leptokurtic noise.

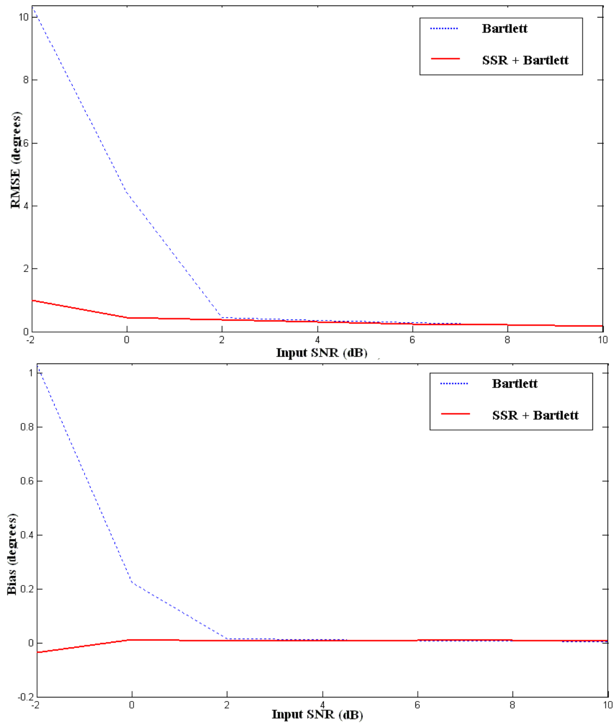

Fig. 11: a) RMSE and b) Bias of Bartlett estimator vs Input SNR (in dB)

The plots in Fig. 11 are similar to those in Fig. 9, but have been plotted for the case of a Bartlett processor. The range and depth of the source are assumed to be known and the bearing is estimated. It may be observed again that the performance of the bearing estimation of the Bartlett processor has been enhanced by adding an SSR preprocessing stage to it and input SNR at the onset of the performance breakdown has been reduced.

Simulations performed with the SIM processor also show that SSR preprocessing aids in enhancing bearing estimation. The SIM processor is seen to have slightly lower performance than the previous two processors, but the former has the advantage of lower computational complexity and does not require prior knowledge of range and depth. Fig. 12 shows the RMSE and bias of a SIM processor. The error is lower when SSR denoising is employed. The SIM ambiguity function plotted in Fig. 13 is less sharp than the MUSIC ambiguity function in Fig. 10. It may be seen that SSR-SIM ambiguity function is still sharper than that of normal SIM and thus the performance is improved by pre-processing.

We now observe the effect of SSR denoising on the resolution of bearing estimation. Resolution is defined as the ability of a bearing estimator to distinguish signals coming from two closely placed sources as separate ones. As the input SNR reduces, two signal sources that are close in terms of angular separation tend to become indistinguishable from one another, leading to lower resolution. Using SSR preprocessing, however, it is possible to improve the resolution of the estimators. We will consider resolution of the MUSIC estimator in an

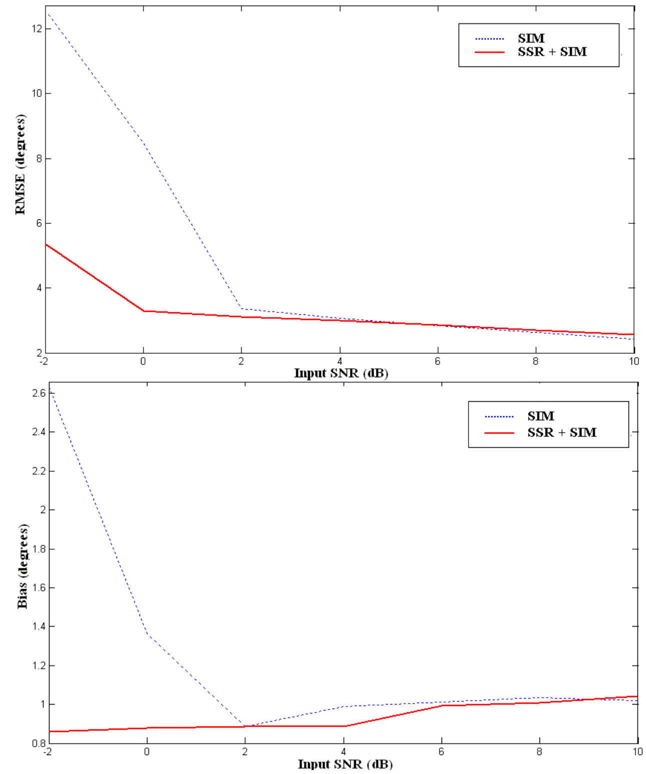

Fig. 12: a) RMSE and b) Bias of SIM estimator vs input SNR (in dB)

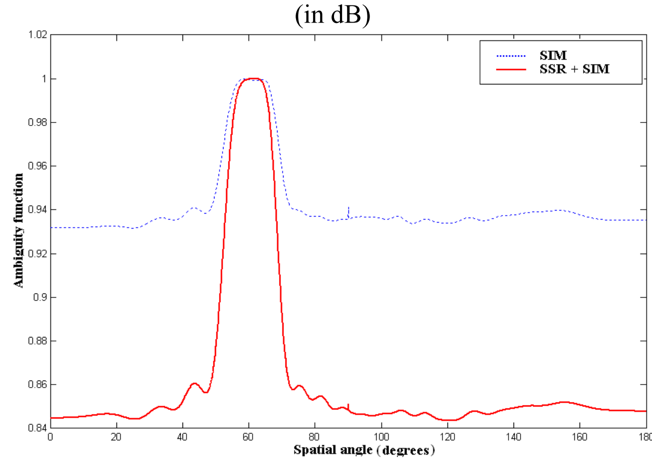

Fig. 13: Ambiguity function of SIM estimator at input SNR −10 dB

environment similar to the one mentioned previously. There are two sources located at 43 and 48 degrees bearing with respect to the array axis. The bearing estimation is done using 500 data snapshots. The rest of the parameters are as specified in the previous simulation. The two sources are considered to be resolved if the processor detects two distinct peaks in the region surrounding the true source angle positions. Fig. 14 shows the probability of resolution of the two sources plotted as a function of the input SNR. It can be seen that the resolution improves as the SNR increases and approaches a value of 1. It is also seen that the probability of resolution of the SSR enhanced MUSIC estimator is always higher than that of the normal MUSIC estimator.

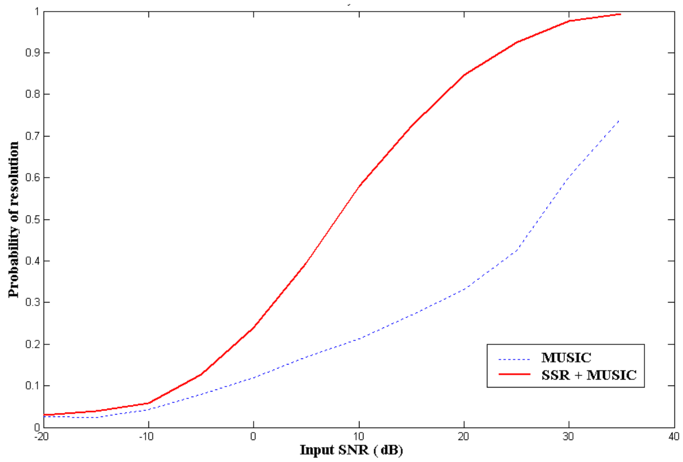

Fig. 14: Probability of resolution of two sources at 43 ° and 48°

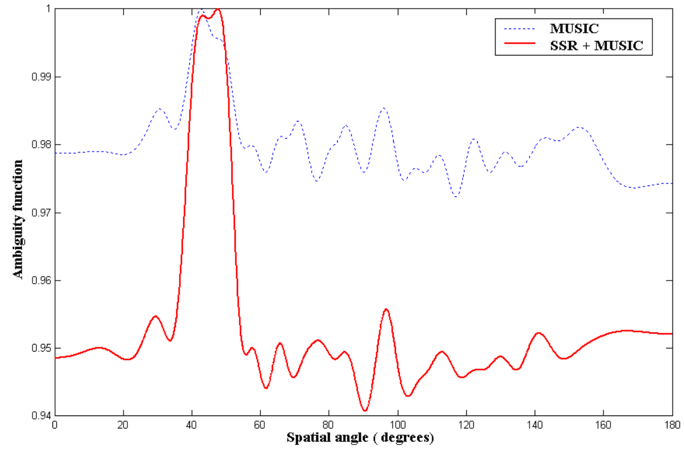

Fig. 15: Ambiguity function of MUSIC estimator at input SNR −17 dB, showing resolution of sources at 43 ° and 48°

In Fig. 15 the ambiguity function of the MUSIC estimator at a low input SNR of -17 dB is shown. It can be seen that at this low SNR, MUSIC fails to resolve the two sources. But SSR enhanced MUSIC is capable of resolving the two sources as can be seen from the two peaks in the ambiguity function. Thus the improvement in resolution offered by using the SSR preprocessor is clearly demonstrated.

V. CONCLUSION

The ability of processors to localize underwater acoustic sources is limited by the low SNR encountered in the ocean. This paper discusses the design of a preprocessor based on the phenomenon of SSR, which can be used to improve the performance of bearing estimation in such environments. The SSR denoiser is a non-linear processor, which uses the fact that noise in underwater acoustic channels is leptokurtic in nature. The performance improvement offered by an SSR denoiser can be optimized by appropriate selection of the SSR noise pdf, standard deviation and the number of quantizers. The performance improves at lower input SNR and as the environmental noise pdf becomes more heavy-tailed in nature. The preprocessor is shown to have a beneficial effect on the performance of several bearing estimators, namely, MUSIC, Bartlett and SIM processors. The performance is found to be better in terms of reduced bias and RMSE of the bearing estimates. The ambiguity functions of the SSR enhanced processors are also observed to be sharper. The effect of the SSR preprocessor on the resolution of two close sources by the bearing estimators is considered. It is found that the resolution is also improved by using the SSR denoiser in a leptokurtic noise environment. It may be concluded that the SSR denoiser can be effectively used as a preprocessor to bearing estimators for application in underwater acoustic channels.

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