# Narrowband signal detection techniques in

## shallow ocean by acoustic vector sensor array

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### Abstract

This paper presents the formulation and performance analysis of four techniques for detection of a narrowband acoustic source in a shallow range-independent ocean using an acoustic vector sensor (AVS) array. The array signal vector is not known due to the unknown location of the source. Hence all detectors are based on a generalized likelihood ratio test (GLRT) which involves estimation of the array signal vector. One non-parametric and three parametric (model-based) signal estimators are presented. It is shown that there is a strong correlation between the detector performance and the mean-square signal estimation error. Theoretical expressions for probability of false alarm and probability of detection are derived for all the detectors, and the theoretical predictions are compared with simulation results. It is shown that the detection performance of an AVS array with a certain number of sensors is equal to or slightly better than that of a conventional acoustic pressure sensor array with thrice as many sensors.<sup>1</sup>

Keywords: Acoustic vector sensor, generalized likelihood ratio test, shallow ocean, subspace detection

### 1. Introduction

Acoustic vector sensors (AVS) are different from traditional acoustic pressure sensors (APS) or hydrophones in that they measure the components of particle velocity in addition to the acoustic pressure [1]. AVS technology with co-located pressure and velocity sensors has grown rapidly in recent years due to the established superiority of AVS over APS in signal processing applications. This superiority stems from the fact that particle velocity measurements provide additional information about the acoustic field, viz., the direction of the impinging acoustic waves. While an APS array can extract directional information by measuring the propagation delay between sensors, a single AVS can obtain this information directly from the particle velocity measurements. These measurements also enable unambiguous bearing estimation of underwater sources using shorter and/or sparser AVS arrays, in contrast to APS arrays which suffer from several ambiguities [1–4].

Velocity sensors per se are not new. They have been available, implemented and studied for quite some time. However, the importance of simultaneous measurement of acoustic pressure and particle velocity was first demonstrated experimentally by D'Spain et al [4] in 1991, sparking interest in the development of acoustic vector sensors. Since then, several experimental setups such as the DIFAR array [5] and the recently conducted Makai experiment [6] have demonstrated the effectiveness of AVS. The impressive performance of vector sensors has been demonstrated in numerous signal processing applications such as source direction-finding and localization [3,7–26], tracking [27–29], communication [30–32], geo-acoustic inversion [33] and source detection [34,35]. The compactness and improved performance of AVS arrays make them ideal for use in fields such as underwater acoustic surveillance for port security [36] and underwater imaging [37].

<sup>&</sup>lt;sup>1</sup> Abbreviations: APS: acoustic pressure sensor, ASFD: approximate signal form detector, AVS: acoustic vector sensor, ED: energy detector, GLRT: generalized likelihood ratio test, HLA: horizontal linear array, MFD: matched filter detector, NMSE: normalized mean square signal estimation error, PDF: probability density function, SD: subspace detector, TSD: truncated subspace detector, VLA: vertical linear array

The initial theoretical framework for signal processing applications using an AVS array was developed by Nehorai and Paldi [1] who presented the AVS array measurement model for plane waves propagating in a homogeneous medium. Several theoretical treatments of the AVS followed, and numerous algorithms for directionfinding and localization of sources employing AVS have been developed. These include modifications of existing direction-of-arrival estimation (DOA) algorithms such as the Capon algorithm [3], MUSIC [10,38], ROOT-MUSIC [11], ESPRIT [8,9,12,13,16,17,38], MLE [18,19], and SIM [20,22] as well as other [23–26] algorithms. Meanwhile, in a series of papers, Hawkes and Nehorai dealt with theoretical aspects of AVS signal processing, such as characterization of the AVS array performance in terms of error measures and the Cramer-Rao bound (CRB) [2,39], and characterization of intra-sensor and inter-sensor correlation of noise in an AVS array [40]. The performance bounds on direction finding using AVS have also been investigated by Tam and Wong [41]. Hawkes and Nehorai also illustrated how a distributed AVS array can function as a sensor network for localization [14]. In this setup, each AVS can function as a separate node of the network. Zha and Qiu [16] illustrated how the AVS could be employed for bearing estimation when the environment is contaminated by impulsive noise. Tichavsky et al [13] demonstrated that a single AVS can be used for direction-finding of an acoustic source in far-field or near-field. Wu and Wong [21] have presented the near-field array manifold of the AVS, thus opening the doors for near-field AVS array signal processing applications. Later, they presented a method of near-field source localization using one triad of velocity sensors and one APS [23]. The problem of tracking source azimuth using an AVS was studied by Felisberto et al [27]. Zhong et al [28] explored the possibility of two-dimensional DOA tracking of acoustic sources with AVS using particle filtering. The feasibility of using AVS in underwater acoustic communications was demonstrated by Song et al [30,31] and Abdi et al [32]. The AVS may also be employed as an effective measurement system for the estimation of geo-acoustic parameters [33].

Work on the development of algorithms for signal detection in shallow ocean using an AVS array is of recent origin [34,35]. In [34] and [35], the authors have proposed several detection strategies such as the energy detector (ED), subspace detector (SD), truncated subspace detector (TSD) and approximate signal form detector (ASFD), for narrowband detection of a source in a range-independent ocean with Gaussian ambient noise. All detectors are based on a generalized likelihood ratio test (GLRT) [42] since the signal vector is unknown due to the unknown location of the source. The ED is formulated assuming that the signal vector is completely unknown. The SD, which is formulated using the matched subspace detection approach [43], exploits the fact that the signal vector belongs to a low-dimensional modal subspace. Subspace detection is a well-discussed problem in the literature [44–46], starting with the works of Scharf and Lytle [44] who applied the theory of invariance in hypothesis testing to the problem of CFAR signal detection. The classic paper by Scharf and Friedlander [43] on matched subspace detection generalized the class of problems for detecting a subspace signal and showed that the GLRT yields a maximal invariant, uniformly most powerful detector. This work has also been extended to the problem of adaptive subspace detection [47–49]. Subspace detection has also been studied for signals in spherically invariant random vector noise [50] and generalized Gaussian noise with interference [51].

In the context of underwater source detection, the SD provides a better performance than the ED. But the SD requires the knowledge of the modal wavenumbers of the underwater channel while the ED does not require any prior information. A modification of the SD, namely the TSD, seeks to optimize the detector by truncating the dimension of the modal subspace. The ED, SD and TSD methods can be employed with either AVS or APS arrays; but AVS arrays provide a significantly better detection performance than APS arrays. The ASFD, which is applicable to AVS arrays only, exploits the fact that a simple approximate relation exists between the components of measurement at each AVS. Neither the ED nor the ASFD assume any knowledge of the channel, but the ASFD provides a better performance than that of the ED. An extension of the above-mentioned detection strategies to the problem of detection in non-Gaussian noise has been explored recently in [52].

This paper presents improved versions of methods advanced in [35] for narrowband detection of an acoustic source using an AVS array in a range-independent ocean with Gaussian environmental noise. We undertake a detailed theoretical investigation of these methods, that is missing in [35]. The performance of the detectors is analyzed based on several performance measures and simulation studies. We also discuss and illustrate the performance improvement obtained by these detection methods by exploiting the advantages of the AVS array in comparison to the APS array. The paper is organized as follows. The data models for a horizontal linear array (HLA)

and a vertical linear array (VLA) of AVS are presented in Section 2. A detailed formulation of various detection strategies proposed in [34,35] is presented in Section 3. The issue of optimization of the TSD is also discussed in this section. In Section 4, the theoretical performance analysis of the detectors is presented for the case of finite data as well as the case of asymptotically large data records, and the relative merits and demerits of the different detectors are discussed. Simulation results in support of the theoretical predictions are presented in Section 5. A summary and conclusions are presented in Section 6.

### 2. Data model

We consider the problem of using an *N*-element AVS array for the detection of a narrowband point source with a center frequency *f*, using *T* snapshots of data. The ocean is modeled as a range-independent channel, having a water column of density  $\rho_w$  and sound speed  $c_w$  over sediment of density  $\rho_b$  and sound speed  $c_b$ . The source is located at azimuth  $\phi$ , depth  $z_s$  and range *r* with respect to the reference element of the array. The sensor array is assumed to be in the far-field region with respect to the source, i.e.  $2\pi(f/c_w)r >> 1$ . We consider the uniform HLA and uniform VLA configurations for detection. In the case of an HLA, the sensor closest to the source is considered to be the reference sensor whereas in the case of a VLA the topmost sensor is considered to be the reference. The elements of the array are separated by a uniform distance of *d* meters. The source azimuth angle is measured with respect to the end fire direction of the HLA and an arbitrary direction for the VLA.

Each AVS in the array has three outputs, viz., the acoustic pressure and two orthogonal horizontal components of particle velocity. The vertical component of particle velocity is not considered since the inclusion of this additional measurement is found to increase the complexity of the detection algorithm without yielding any significant additional improvement in performance. We shall scale the velocity components by the factor  $\sqrt{2} \rho_w c_w$  to maintain dimensional uniformity of the measured quantities and also to ensure that all the noise components have equal variance [20]. Let  $\mathbf{x}(t)$ ,  $\mathbf{s}(t)$  and  $\mathbf{w}(t)$  denote, respectively, the  $t^{\text{th}}$  snapshot of the 3N x 1 data vector, signal vector and noise vector at the N-sensor AVS array. The signal vector  $\mathbf{s}(t)$  can be represented as

$$\mathbf{s}(t) = [s_1(t), \dots, s_{3N}(t)]^{\mathrm{T}}, \ s_{3n-2}(t) = p_n(t), \\ s_{3n-1}(t) = \sqrt{2}\rho_w c_w v_{xn}(t), \\ s_{3n}(t) = \sqrt{2}\rho_w c_w v_{yn}(t), \\ n = 1, \dots N$$
(1)

where  $p_n(t)$  denotes the complex amplitude of acoustic pressure at the  $n^{th}$  sensor, and  $(v_{xn}(t), v_{yn}(t))$  denote the corresponding complex amplitudes of the horizontal (x, y) components of particle velocity. According to the normal mode theory of sound propagation, the acoustic field in the channel can be expressed as the sum of a discrete spectrum of propagating normal modes and a continuous spectrum of evanescent normal modes [53]. For a source in the far field region, the contribution of the continuous spectrum can be neglected and the expression for the acoustic pressure  $p_n(t)$  can be written as

$$s_{3n-2}(t) = p_n(t) = \sum_{m=1}^{M} p_{mn}(t),$$
(2)

where  $p_{mn}(t)$  is the contribution of the  $m^{th}$  mode to the acoustic pressure at the  $n^{th}$  sensor, given by

$$p_{mn}(t) = B(t)\Omega_{mn}(\phi)\psi_m(z_n)\psi_m(z_s)\exp(ik_mr - \zeta_mr)/\sqrt{k_mr},$$
(3)  

$$\Omega_{mn}(\phi) = \begin{cases} \exp(i(n-1)k_md\cos(\phi)) \text{ for HLA, and} \\ 1 & \text{ for VLA} \end{cases}$$
(4)

where  $k_m$  and  $\zeta_m$  are the modal wave number (real part of the eigen value) and attenuation coefficient (imaginary part of the eigenvalue) of the  $m^{\text{th}}$  normal mode at frequency f,  $\psi_m(z)$  is the normalized eigen function of the  $m^{\text{th}}$  normal mode,  $z_n$  is the depth of the  $n^{\text{th}}$  sensor and B(t) is a slowly varying complex quantity whose root-mean square magnitude is proportional to the strength of the source. For a given channel, the number of propagating modes M is, in general, an increasing function of the signal frequency. The relation between acoustic pressure *p* and particle velocity *v* at a point r = (x, y, z) at time *t* is governed by the law of conservation of momentum

$$\rho_w \frac{dv}{dt} + \nabla p = 0.$$
<sup>(5)</sup>

In most applications of practical interest, the source is located in the far-field, satisfying the condition kr >>1, where  $k = 2\pi f/c_w$ . Using (2), (4) and (5), and invoking the far-field approximation ( $k_m r >>1$  for all m), we can write

$$s_{3n-1}(t) = \sqrt{2}\rho_w c_w v_{xn}(t) = \sqrt{2}\cos(\phi) \sum_{m=1}^{M} (k_m/k) p_{mn}(t), \qquad (6)$$

$$s_{3n}(t) = \sqrt{2}\rho_w c_w v_{yn}(t) = \sqrt{2}\sin(\phi) \sum_{m=1}^{M} (k_m/k) p_{mn}(t).$$
(7)

Hence the  $t^{th}$  snapshot of the signal vector can be expressed as

$$\mathbf{s}(t) = \mathbf{A}(\phi) \, \mathbf{b}(t), \tag{8}$$

where  $\mathbf{A}(\phi)$  is a  $3N \times M$  modal steering matrix and  $\mathbf{b}(t)$  is the mode amplitude vector. In the case of an HLA (denoted by the subscript H),  $\mathbf{b}(t)$  and  $\mathbf{A}(\phi)$  are defined as

$$\boldsymbol{b}_{H}(t) = B(t) \Big[ \psi_{1}(z_{a}) \psi_{1}(z_{s}) \exp(ik_{1}r - \zeta_{1}r) / \sqrt{k_{1}r} \dots \psi_{M}(z_{a}) \psi_{M}(z_{s}) \exp(ik_{M}r - \zeta_{M}r) / \sqrt{k_{M}r} \Big]^{T},$$
(9)  
$$\boldsymbol{A}_{H}(\phi) = [\boldsymbol{a}_{1,H}(\phi) \dots \boldsymbol{a}_{M,H}(\phi)],$$
(10)

where  $z_a$  is the array depth and  $\{a_{m,H}(\phi); m = 1, ..., M\}$  are the modal steering vectors for the HLA, defined as

$$\boldsymbol{a}_{m,H}(\boldsymbol{\phi}) = \boldsymbol{d}_{m}(\boldsymbol{\phi}) \otimes \boldsymbol{g}_{m}(\boldsymbol{\phi}), \tag{11}$$

$$\boldsymbol{d}_{m}(\boldsymbol{\phi}) = \begin{bmatrix} 1 & \exp(ik_{m}d\cos(\boldsymbol{\phi})) & \dots \exp(i(N-1)k_{m}d\cos(\boldsymbol{\phi})) \end{bmatrix}^{T},$$
(12)

$$\boldsymbol{g}_{m}(\boldsymbol{\phi}) = \begin{bmatrix} 1 & \sqrt{2}\cos(\boldsymbol{\phi})k_{m}/k & \sqrt{2}\sin(\boldsymbol{\phi})k_{m}/k \end{bmatrix}^{T},$$
(13)

where the symbol  $\otimes$  denotes the Kronecker product. In the case of a VLA (denoted by the subscript V), b(t) and  $A(\phi)$  are defined as

$$\boldsymbol{b}_{V}(t) = B(t) \left[ \psi_{1}(z_{s}) \exp(ik_{1}r - \zeta_{1}r) / \sqrt{k_{1}r} \dots \psi_{M}(z_{s}) \exp(ik_{M}r - \zeta_{M}r) / \sqrt{k_{M}r} \right]^{T},$$
(14)

$$\mathbf{A}_{V}(\phi) = [\boldsymbol{a}_{1,V}(\phi) \dots \boldsymbol{a}_{M,V}(\phi)], \tag{15}$$



Fig. 1: (a) Vertical spatial correlation function  $r(z_1, z_1 + \Delta)$  at f = 350 Hz. (b) Variation of  $r(z_1, z_1 + \lambda_w)$  with f

and the modal steering vectors for a VLA are defined as

$$\boldsymbol{a}_{m,V}(\boldsymbol{\phi}) = \left[\boldsymbol{\psi}_m(\boldsymbol{z}_1)\boldsymbol{g}_m^T(\boldsymbol{\phi}) \dots \boldsymbol{\psi}_m(\boldsymbol{z}_N)\boldsymbol{g}_m^T(\boldsymbol{\phi})\right]^T.$$
(16)

The signal is assumed to be contaminated by additive temporally white Gaussian noise. Let the noise vector at the  $n^{th}$  sensor of the AVS array be denoted by  $[w_{3n-2}(t) w_{3n-1}(t) w_{3n}(t)]^{T}$ . For a given time *t*, the three components of noise at an AVS are uncorrelated and identically distributed [20]. But the noise is spatially correlated and it has a depth-dependent variance. Let the correlation matrix of the noise vector w(t) be denoted by

$$\mathbf{R}_{w} = \mathbf{E}[\boldsymbol{w}(t) \ \boldsymbol{w}^{\mathrm{H}}(t)] = \sigma^{2} \mathbf{R}_{0}, \tag{17}$$

where  $\sigma^2$  is the variance of each component of noise at the reference depth  $z_1$  and  $\mathbf{R}_0$  is a  $3N \times 3N$  matrix. The spatial correlation decays rapidly in the horizontal direction, and it can be ignored when the distance is  $\lambda_w/2$  (half-wavelength) or larger. But the vertical spatial correlation between similar components of noise remains strong even at large distances. Hence, for an HLA at depth  $z_1$ , the matrix  $\mathbf{R}_0$  is given by

$$\mathbf{R}_{0,H} = \mathbf{I}_{3N},\tag{18}$$

where  $\mathbf{I}_{3N}$  is the  $3N \times 3N$  identity matrix, and for a VLA whose elements are located at depths  $z_1, \dots, z_N$ , the matrix  $\mathbf{R}_0$  is given by [35]

$$\mathbf{R}_{0,V} = \begin{bmatrix} r(z_1, z_1) & \dots & r(z_1, z_N) \\ . & & \\ r(z_N, z_1) & \dots & r(z_N, z_N) \end{bmatrix} \otimes \mathbf{I}_3,$$
(19)

where

$$r(z_{k}, z_{n}) = \frac{\sum_{m=1}^{M} sin(m\gamma z_{k}) sin(m\gamma z_{n})}{\sum_{m=1}^{M} sin^{2}(m\gamma z_{1})}.$$
(20)

The vertical spatial correlation function  $r(z_1, z_1 + \Delta)$  is shown in Fig. 1 for a shallow iso-velocity ocean whose parameters are listed in Section 5. Figure 1(a) shows a plot of  $r(z_1, z_1 + \Delta)$  versus  $\Delta$  for  $z_1 = 10$  m and frequency f =350 Hz. Figure 1(b) shows the variation of the correlation with frequency for  $z_1 = 10$  m and  $\Delta = \lambda_w$ , where  $\lambda_w = c_w/f$  is the wavelength in water. It is seen from these figures that the correlation is small for  $\Delta \ge \lambda_w$ , and exhibits small irregular fluctuations with frequency. Hence, for a VLA, it is desirable to choose inter-sensor spacing  $d \cong \lambda_w$  to minimize the degradation of detector performance due to inter-sensor noise correlation. In the following, we shall assume that the matrix  $\mathbf{R}_0$  is known. The variance  $\sigma^2$  is generally not known.

### 3. Formulation of detectors

In this section, we shall present several alternative methods for detecting a narrowband source. The detection problem can be cast in the form of the following binary hypothesis testing problem:

$$H_{0}: \tilde{\boldsymbol{x}}(t) = \tilde{\boldsymbol{w}}(t)$$
  

$$H_{1}: \tilde{\boldsymbol{x}}(t) = \tilde{\boldsymbol{s}}(t) + \tilde{\boldsymbol{w}}(t), \ t = 1,...T , \qquad (21)$$

where  $H_i$  denotes the hypothesis that j sources are present, and

$$\tilde{\boldsymbol{x}}(t) = \mathbf{R}_0^{-1/2} \boldsymbol{x}(t), \tilde{\boldsymbol{s}}(t) = \mathbf{R}_0^{-1/2} \boldsymbol{s}(t), \tilde{\boldsymbol{w}}(t) = \mathbf{R}_0^{-1/2} \boldsymbol{w}(t),$$
(22)

are the pre-whitened versions of the data vector, signal vector, and noise vector respectively. It is assumed that the noise correlation matrix  $\mathbf{R}_0$ , defined in (18) for HLA and in (19) for VLA, is known. The joint likelihood functions of *T* snapshots of the data vector under hypotheses  $H_0$  and  $H_1$  are given by

$$f\left(\boldsymbol{X} \mid \boldsymbol{\sigma}^{2}; \boldsymbol{H}_{0}\right) = \frac{1}{\left(\pi\boldsymbol{\sigma}^{2}\right)^{3NT}} \exp\left[-\frac{1}{\boldsymbol{\sigma}^{2}} \sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}\left(t\right) \tilde{\boldsymbol{x}}\left(t\right)\right],$$
(23)

$$f\left(\boldsymbol{X} \mid \boldsymbol{\sigma}^{2}, \boldsymbol{S}; \boldsymbol{H}_{1}\right) = \frac{1}{\left(\pi\boldsymbol{\sigma}^{2}\right)^{3NT}} \exp\left[-\frac{1}{\boldsymbol{\sigma}^{2}} \sum_{t=1}^{T} \left(\tilde{\boldsymbol{x}}\left(t\right) - \tilde{\boldsymbol{s}}\left(t\right)\right)^{H} \left(\tilde{\boldsymbol{x}}\left(t\right) - \tilde{\boldsymbol{s}}\left(t\right)\right)\right], \tag{24}$$

where  $\boldsymbol{X} = [\tilde{\boldsymbol{x}}^T(1), ..., \tilde{\boldsymbol{x}}^T(T)]^T$ ,  $\boldsymbol{S} = [\tilde{\boldsymbol{s}}^T(1), ..., \tilde{\boldsymbol{s}}^T(T)]^T$ . The logarithm of the likelihood ratio (LR) is given by

$$\log L(\boldsymbol{X}) = \frac{1}{\sigma^2} \sum_{t=1}^{T} \left[ 2\operatorname{Re}\left(\tilde{\boldsymbol{x}}^H(t)\tilde{\boldsymbol{s}}(t)\right) - \tilde{\boldsymbol{s}}^H(t)\tilde{\boldsymbol{s}}(t) \right].$$
(25)

The primary hurdle faced in the detection of a signal in the ocean is that the signal vectors  $\{s(t), t = 1, ..., T\}$  at the sensor array are unknown due to lack of knowledge of the source location and other uncertainties in source, receiver or environment parameters. Hence we perform a generalized likelihood ratio test after replacing each vector  $\tilde{s}(t)$  in (24) by its maximum likelihood (ML) estimate  $\hat{s}(t)$ . The ML estimate may use any prior information that may be available. The different detectors presented in this paper use different types of prior information. The noise variance  $\sigma^2$  is also not known in general. Sometimes, it may be possible to obtain a prior estimate of  $\sigma^2$  from noise-only data. If a reliable a priori estimate of  $\sigma^2$  is not available, it has to be treated as an additional unknown parameter. Hence we shall consider the detection problem for two different cases, viz. (a) the noise variance  $\sigma^2$  is known, and (b)  $\sigma^2$  is unknown.

For case (a), the logarithm of the generalized likelihood ratio (GLR) is obtained from (25) on replacing  $\tilde{s}(t)$  by  $\hat{s}(t)$ . Hence the test statistic for this case can be written as

$$\gamma_{Case(a)}\left(\boldsymbol{X}\right) = \sum_{t=1}^{T} \left[ 2\operatorname{Re}\left(\tilde{\boldsymbol{x}}^{H}\left(t\right)\hat{\boldsymbol{s}}\left(t\right)\right) - \hat{\boldsymbol{s}}^{H}\left(t\right)\hat{\boldsymbol{s}}\left(t\right) \right],$$
(26)

where the signal-vector estimates  $\hat{s}(t)$  are different for different detectors, and Re(.) denotes the real part of (.).

For case (b), the following estimates of  $\sigma^2$  under  $H_0$  and  $H_1$  are readily obtained by maximizing the likelihood functions in (23) and (24):

$$\hat{\sigma}_{0}^{2} = \frac{1}{3NT} \sum_{t=1}^{T} \tilde{\mathbf{x}}^{H}(t) \tilde{\mathbf{x}}(t),$$

$$\hat{\sigma}_{1}^{2} = \frac{1}{3NT} \sum_{t=1}^{T} \left[ \tilde{\mathbf{x}}(t) - \hat{\mathbf{s}}(t) \right]^{H} \left[ \tilde{\mathbf{x}}(t) - \hat{\mathbf{s}}(t) \right].$$
(27)

On substituting (27) into (23) and (24) respectively, we get the following expression for GLR for case (b):

$$L_{G,Case(b)}(\boldsymbol{X}) = \left[\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2}\right]^{3NT},$$
(28)

and further simplification yields the following expression for the test statistic:

$$\gamma_{Case(b)}\left(\boldsymbol{X}\right) = \frac{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \tilde{\boldsymbol{x}}(t)}{\sum_{t=1}^{T} \left[ \left( \tilde{\boldsymbol{x}}(t) - \hat{\boldsymbol{s}}(t) \right)^{H} \left( \tilde{\boldsymbol{x}}(t) - \hat{\boldsymbol{s}}(t) \right) \right]}.$$
(29)

We shall now introduce the different detectors and discuss their performance in detail.

#### 3.1. Matched Filter Detector (MFD)

It is well-known that, if {s(t), t = 1,..., T} and  $\sigma^2$  are known, the optimal detector in the Neyman-Pearson sense is the replica-correlator which may be implemented as a matched filter detector (MFD) [42]. Even though this case is of little practical interest, we shall consider it solely for the purpose of comparison since the performance of the MFD provides an upper bound on the performance of other realizable detectors. In this case, the log-likelihood ratio in (25) leads to the following likelihood ratio test (LRT): Decide  $H_1$  if  $\gamma_{MFD}(X) > \eta$ , where  $\eta$  is the detection threshold, and

$$\gamma_{MFD}\left(\boldsymbol{X}\right) = \sum_{t=1}^{T} \operatorname{Re}\left(\tilde{\boldsymbol{x}}^{H}\left(t\right)\tilde{\boldsymbol{s}}\left(t\right)\right)$$
(30)

is the test statistic of the MFD.

#### 3.2. Energy Detector (ED)

If {*s*(*t*), *t* = 1,..., *T*} are totally unknown, the ML estimate of  $\tilde{s}(t)$  is given by

$$\hat{\boldsymbol{s}}_{ED}(t) = \tilde{\boldsymbol{x}}(t) \text{ for all } t, \tag{31}$$

If the noise variance  $\sigma^2$  is known, the test statistic is given by

$$\gamma_{ED}\left(\boldsymbol{X}\right) = \sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}\left(t\right) \tilde{\boldsymbol{x}}\left(t\right)$$
(32)

This is the well-known energy detector (ED) [42], and it is presented here only for the sake of comparison. We note for later reference that the normalized mean-square signal estimation error (NMSE), defined as

$$\varepsilon_{ED} \triangleq \frac{\sum_{t=1}^{T} E[\left(\hat{s}_{ED}(t) - \tilde{s}(t)\right)^{H} \left(\hat{s}_{ED}(t) - \tilde{s}(t)\right)]}{\sum_{t=1}^{T} E[\tilde{s}^{H}(t)\tilde{s}(t)]}, \text{ is given by}$$

$$\varepsilon_{ED} = \frac{3NT}{\lambda},$$

$$\lambda = \frac{E_{s}}{\sigma^{2}}, E_{s} = \sum_{t=1}^{T} E[\tilde{s}^{H}(t)\tilde{s}(t)],$$
(34)

where  $E_s$  is the total energy of the modified signal vector  $\tilde{s}(t)$  summed over all the snapshots, and  $\lambda$  is the signal energy to noise power ratio (ENR).

The GLRT does not provide a meaningful solution if the signal is completely unknown and the noise variance is also unknown.

#### 3.3. Subspace Detector (SD)

The ED described in Section 3.2 assumes no prior information about the structure of the signal vector. It is possible to achieve better performance by using any prior information that may be available. One such detector is the subspace detector (SD) [34,35]. This detector is based on the fact that the 3N - dimensional signal vector belongs to an *M*-dimensional modal subspace. We shall present here a simpler formulation of the subspace detector and its theoretical performance analysis.

We know that the pre-whitened signal vector can be expressed as  $\tilde{s}(t) = \mathbf{R}_0^{-1/2} \mathbf{s}(t) = \mathbf{R}_0^{-1/2} \mathbf{A}(\phi) \mathbf{b}(t)$ , where  $\mathbf{A}(\phi) = [\mathbf{a}_1(\phi)...\mathbf{a}_M(\phi)]$  is the  $3N \times M$  modal steering matrix and  $\{\mathbf{a}_m(\phi); m = 1,...M\}$  are modal steering vectors defined in (10) and (11) or (15) and (16) respectively. Here and in the following, we omit the subscripts *H* and *V* for the sake of brevity. The columns of  $\mathbf{A}(\phi)$  are linearly independent if 3N > M. Hence the 3N-dimensional signal vector  $\tilde{s}(t)$  belongs to the *M*-dimensional modal subspace  $V(\phi)$  spanned by the linearly independent columns of  $\mathbf{R}_0^{-1/2} \mathbf{A}(\phi)$ , i.e.

$$V(\phi) = span\{\mathbf{R}_{0}^{-1/2}\boldsymbol{a}_{1}(\phi)...\mathbf{R}_{0}^{-1/2}\boldsymbol{a}_{M}(\phi)\} = span\{\boldsymbol{u}_{1}(\phi)...\boldsymbol{u}_{M}(\phi)\},$$
(35)

where  $\{u_1(\phi) \dots u_M(\phi)\}$  is the orthonormal basis of  $V(\phi)$  obtained through a Gram-Schmidt transformation process. We note that  $\tilde{s}(t)$  depends on  $\phi$  though this dependence is generally suppressed for the sake of brevity. The signal vector  $\tilde{s}(t)$  may therefore be expressed as

$$\tilde{\boldsymbol{s}}(t;\boldsymbol{\phi}) = \mathbf{U}(\boldsymbol{\phi})\boldsymbol{\beta}(t), \mathbf{U}(\boldsymbol{\phi}) = [\boldsymbol{u}_1(\boldsymbol{\phi})...\boldsymbol{u}_M(\boldsymbol{\phi})].$$
(36)

The *M*-dimensional vector  $\boldsymbol{\beta}(t)$  and the bearing  $\phi$  are unknown. For a given  $\phi$ , the subspace  $V(\phi)$  is known if the modal wavenumbers  $\{k_m; m = 1, ..., M\}$  are known in the case of an HLA, and if the modal wavenumbers  $\{k_m; m = 1, ..., M\}$  as well as the mode functions  $\{\psi_m(z); m = 1, ..., M\}$  are known in the case of a VLA. Assuming that this information is available, the problem of estimating the 3*N*-dimensional signal vector  $\tilde{\boldsymbol{s}}(t)$  is reduced to the problem of estimating the *M*-dimensional vector  $\boldsymbol{\beta}(t)$  and the bearing  $\phi$ . The conditional ML estimator of  $\boldsymbol{\beta}(t)$  is given by

$$\hat{\boldsymbol{\beta}}(t \mid \phi) = [\mathbf{U}^{H}(\phi)\mathbf{U}(\phi)]^{-1}\mathbf{U}^{H}(\phi)\tilde{\boldsymbol{x}}(t) = \mathbf{U}^{H}(\phi)\tilde{\boldsymbol{x}}(t).$$
(37)

Hence, for a given  $\phi$ , the estimate of  $\tilde{s}(t;\phi)$  can be written as

.

$$\hat{\boldsymbol{s}}_{SD}(t \mid \boldsymbol{\phi}) = \mathbf{U}(\boldsymbol{\phi})\hat{\boldsymbol{\beta}}(t \mid \boldsymbol{\phi}) = \mathbf{D}(\boldsymbol{\phi})\tilde{\boldsymbol{x}}(t),$$
(38)

where

$$\mathbf{D}(\phi) = \mathbf{U}(\phi)\mathbf{U}^{H}(\phi) \,. \tag{39}$$

(40)

On replacing  $\tilde{s}(t)$  by  $\hat{s}_{SD}(t | \phi)$  in (24) and maximizing with respect to  $\phi$ , we get the following estimate of  $\phi$ :  $\hat{\phi}_{SD} = \arg \max[\sum_{i=1}^{T} \tilde{x}^{H}(t) \mathbf{D}(\phi) \tilde{x}(t)].$ 

From (38), the unconditional estimate of  $\tilde{s}(t)$  can be written as

$$\hat{\boldsymbol{s}}_{SD}(t) = \mathbf{D}(\hat{\boldsymbol{\phi}}_{SD})\tilde{\boldsymbol{x}}(t) \,. \tag{41}$$

On substituting (41) into (26) and (29), we obtain the following expressions for the test statistic for case (a) ( $\sigma^2$  known) and case (b) ( $\sigma^2$  unknown)

$$\gamma_{SD,Case(a)}(\boldsymbol{X}) = \sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}(\hat{\boldsymbol{\phi}}_{SD}) \tilde{\boldsymbol{x}}(t), \qquad (42)$$

$$\gamma_{SD,Case(b)}(\boldsymbol{X}) = \frac{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}(\hat{\phi}_{SD}) \tilde{\boldsymbol{x}}(t)}{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \left(\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})\right) \tilde{\boldsymbol{x}}(t)}.$$
(43)

#### 3.4. Truncated Subspace detector (TSD)

The SD method presented in Section 3.3 can be employed only if the columns of  $\mathbf{A}(\phi)$  are linearly independent, i.e. if  $M \leq 3N$ . Since the number of modes M increases as the frequency is increased [53], the applicability of SD is limited by an upper cut-off frequency  $f_c$  that increases with the number of sensors N; a longer array (larger N) is required for detection of signals of higher frequency. Moreover it is observed that, for a given array length N, the performance of SD suffers degradation as the signal frequency f is increased even if  $f < f_c$ . This progressive degradation can be explained by considering the normalized mean-square signal estimation error  $\varepsilon_{SD}$  (NMSE)

$$\varepsilon_{SD} \triangleq \frac{\sum_{t=1}^{T} E\left[\left(\hat{s}_{SD}(t) - \tilde{s}(t)\right)^{H} \left(\hat{s}_{SD}(t) - \tilde{s}(t)\right)\right]}{\sum_{t=1}^{T} E\left[\tilde{s}^{H}(t)\tilde{s}(t)\right]}.$$
 It can be readily shown that  
$$\varepsilon_{SD} = \frac{MT\sigma^{2} + E\left[\sum_{t=1}^{T} \tilde{s}^{H}(t;\phi) \left(\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})\right)\tilde{s}(t;\phi)\right]}{E_{s}}.$$

where  $E_s$  is the total signal energy,  $\hat{\phi}_{SD}$  is defined in (40). The quantity  $\tilde{s}^H(t;\phi) (\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})) \tilde{s}(t;\phi)$  is equal to zero if the estimate  $\hat{\phi}_{SD}$  is equal to the true value  $\phi$ . We may therefore assume that  $E\left[\sum_{t=1}^{T} \tilde{s}^H(t;\phi) (\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})) \tilde{s}(t;\phi)\right]$  is small compared to  $MT\sigma^2$  to arrive at the following result

$$\mathcal{E}_{SD} = \frac{MT}{\lambda},\tag{44}$$

where  $\lambda$  is the ENR defined in (34). The NMSE increases linearly with increasing *M* and hence it increases with increasing frequency.

In order to extend the applicability of the SD to shorter arrays/higher frequencies and also to arrest the degradation associated with increasing frequency, we propose a detector called truncated subspace detector (TSD) which uses a truncated model of the signal vector obtained by projecting  $\tilde{s}(t, \phi)$  onto a truncated modal subspace defined as

$$V'(\phi) = span\{\mathbf{R}_{0}^{-1/2}\boldsymbol{a}_{1}(\phi)...\mathbf{R}_{0}^{-1/2}\boldsymbol{a}_{M'}(\phi)\} = span\{\boldsymbol{u}_{1}(\phi)...\boldsymbol{u}_{M'}(\phi)\}, M' < M.$$
(45)

The set of spanning vectors of  $V'(\phi)$  is a subset of the set of spanning vectors of the full modal subspace  $V(\phi)$  defined in (35). The truncated signal vector  $\tilde{s}'(t, \phi)$  can be written as

$$\tilde{\boldsymbol{s}}'(t,\phi) = \mathbf{R}_0^{-1/2} \mathbf{A}'(\phi) \boldsymbol{b}'(t) = \mathbf{U}'(\phi) \boldsymbol{\beta}'(t), \tag{46}$$

where

$$\mathbf{A}'(\phi) = [a_1(\phi)...a_{M'}(\phi)], \ \mathbf{U}'(\phi) = [u_1(\phi)...u_{M'}(\phi)]$$
(47)

The conditional ML estimator of the *M*'-dimensional vector  $\beta'(t)$  is given by

$$\hat{\boldsymbol{\beta}}'(t \mid \phi) = \mathbf{U}'(\phi)^H \,\tilde{\boldsymbol{x}}(t) \,. \tag{48}$$

Hence, for a given  $\phi$ , the estimate of  $\tilde{s}(t, \phi)$  can be written as

$$\hat{\boldsymbol{s}}_{TSD}(t \mid \boldsymbol{\phi}) = \mathbf{U}'(\boldsymbol{\phi}) \boldsymbol{\beta}'(t \mid \boldsymbol{\phi}) = \mathbf{D}'(\boldsymbol{\phi}) \tilde{\boldsymbol{x}}(t),$$
(49)

where

$$\mathbf{D}'(\phi) = \mathbf{U}'(\phi)\mathbf{U}'(\phi)^{H}.$$
(50)

Expressions for the estimate of  $\phi$  and the unconditional estimate of  $\tilde{s}(t)$  can be obtained using a procedure analogous to that for SD described in Section 3.3. Thus we get

$$\hat{\phi}_{TSD} = \arg \max\left[\sum_{t=1}^{I} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}'(\boldsymbol{\phi}) \tilde{\boldsymbol{x}}(t)\right],\tag{51}$$

$$\hat{\boldsymbol{s}}_{TSD}(t) = \mathbf{D}'(\hat{\boldsymbol{\phi}}_{TSD})\tilde{\boldsymbol{x}}(t).$$
(52)

It can be readily shown that the NMSE of the signal estimator defined in (52) is given by



Fig. 2: Plots of NMSE  $\mathcal{E}_{TSD}$  and its components ( $\mathcal{E}_{TSD}^{(1)}(M')$  and  $\mathcal{E}_{TSD}^{(2)}(M',\phi)$ ) vs. M' for HLA. (a) SNR 0 dB, (b) SNR 10 dB, (c) SNR 30 dB



Fig. 3: Plots of NMSE  $\mathcal{E}_{TSD}$  and its components ( $\mathcal{E}_{TSD}^{(1)}(M')$  and  $\mathcal{E}_{TSD}^{(2)}(M',\phi)$ ) vs. M' for VLA. (a) SNR -10 dB, (b) SNR 0 dB, (c) SNR 10 dB

$$\varepsilon_{TSD}(M',\phi) \triangleq \frac{\sum_{t=1}^{T} E[(\hat{s}_{TSD}(t) - \tilde{s}(t))^{H} (\hat{s}_{TSD}(t) - \tilde{s}(t))]}{\sum_{t=1}^{T} E[\tilde{s}^{H}(t)\tilde{s}(t)]} = \varepsilon_{TSD}^{(1)}(M') + \varepsilon_{TSD}^{(2)}(M',\phi),$$
(53)

where

$$\mathcal{E}_{TSD}^{(1)}(M') = \frac{M'T}{\lambda},$$

$$E\left[\sum_{T}^{T} \tilde{\mathbf{s}}^{H}(t;\phi) \left(\mathbf{I}_{-} - \mathbf{D}'(\hat{\phi}_{-})\right) \tilde{\mathbf{s}}(t;\phi)\right] = E\left[\sum_{T}^{T} \tilde{\mathbf{s}}^{H}(t) \mathbf{D}'(\phi) \tilde{\mathbf{s}}(t)\right]$$
(54)

$$\varepsilon_{TSD}^{(2)}(M',\phi) = \frac{E\left[\sum_{t=1}^{\infty} \tilde{s}^{H}(t;\phi) \left(\mathbf{I}_{3N} - \mathbf{D}'(\tilde{\phi}_{TSD})\right) \tilde{s}(t;\phi)\right]}{E_{s}} \cong 1 - \frac{E\left[\sum_{t=1}^{\infty} \tilde{s}^{H}(t) \mathbf{D}'(\phi) \tilde{s}(t)\right]}{E_{s}} = 1 - \frac{\lambda'}{\lambda},$$
(55)

where

$$\lambda' = \frac{E_{s'}}{\sigma^2}, E_{s'} = \sum_{t=1}^{T} E[\tilde{s}'(t)^H \tilde{s}'(t)].$$
(56)



In (56),  $E_{s'}$  is the total energy of the truncated signal vectors over all snapshots and  $\lambda'$  is the ENR for the truncated signal. The NMSE  $\varepsilon_{TSD}(M',\phi)$  has two components  $\varepsilon_{TSD}^{(1)}(M')$  and  $\varepsilon_{TSD}^{(2)}(M',\phi)$ . The first component  $\varepsilon_{TSD}^{(1)}(M')$  is the error due to noise, and it is analogous to the MSE  $\varepsilon_{sD}$  defined in (44). Equation (54) indicates that  $\varepsilon_{TSD}^{(1)}(M')$  decreases linearly as the number of retained modes M' is reduced. The second component  $\varepsilon_{TSD}^{(2)}(M',\phi)$  is the error due to truncation of the normal mode expansion of the signal vector.

It is seen from (55) that  $\varepsilon_{TSD}^{(2)}(M',\phi) = 0$  when M' = M. As M' is reduced,  $\varepsilon_{TSD}^{(2)}(M',\phi)$  increases, but this increase is very slow. Illustrative plots of the total MSE and its components versus the number of retained modes M' are shown in Fig. 2 for a 6-sensor HLA and three different values of SNR viz. 0, 10 and 30 dB. Similar plots for a 6sensor VLA are shown in Fig. 3 for -10, 0 and 10 dB. The signal frequency is 350 Hz and the number of modes is M= 15. The channel parameters, array parameters, and source position for these figures are the same as those listed in Section 5. It is seen from Figs. 2 and 3 that, as M' is reduced from M to 1, the total MSE  $\varepsilon_{TSD}(M',\phi)$  reduces and reaches a minimum at an optimal value of M' which is quite small (especially in the case of the HLA), and may even be equal to 1. Let the optimal value of M' be denoted by  $M'_{opt}$ . We note that  $\varepsilon_{TSD}(M'_{opt},\phi)$  is significantly smaller than  $\varepsilon_{SD}$ . We can therefore expect the performance of TSD to be significantly better than that of SD. The use of a truncated signal model in TSD also has the additional advantages of (1) reducing the need for channel information to modal wavenumbers of the first M' modes only, and (2) reducing the computational complexity.

The value of  $M'_{opt}$  depends primarily on the signal-to-noise ratio (SNR), the degree of correlation among the modal steering vectors  $\{a_1(\phi)..., a_M(\phi)\}$ , and the total number of modes M, and to a lesser extent on the channel parameters, and the location of the source. In general, the value of  $M'_{opt}$  for the HLA is smaller than that for the VLA since the modal steering vectors of the HLA are more highly correlated than those of the VLA. It is seen from (54) that the rate of reduction of  $\varepsilon_{TSD}^{(1)}(M')$  with decreasing M' becomes higher if  $\lambda$  is reduced. Consequently, the value of  $M'_{opt}$  decreases as  $\lambda$  is reduced. Figures 2 and 3 illustrate the dependence of  $M'_{opt}$  on the more commonly used measure of SNR defined in (104). It is seen that  $M'_{opt}$  has a larger value at higher SNR, and also that the values of  $M'_{opt}$  for an HLA are smaller than those for a VLA.

The error  $\varepsilon_{TSD}^{(2)}(M',\phi)$  due to truncation of the modal expansion is small because (1) the modal vectors  $\{a_1(\phi),\ldots,a_M(\phi)\}$  in the expansion of  $\tilde{s}(t,\phi)$  are highly correlated, and (2) amplitudes of the discarded higher order modes are generally quite small due to faster attenuation of the higher order modes. Let  $P_M a_m(\phi)$  denote the projection of  $a_m(\phi)$  on the truncated modal subspace  $V'(\phi)$  for m > M'. The  $L_2$  norm of the projection error vector is given by

$$E_{M',m}(\phi) = || \boldsymbol{a}_{m}(\phi) - \mathcal{P}_{M} \boldsymbol{a}_{m}(\phi) ||^{2}.$$
(57)



Fig. 5:  $K_{M'}(0)$  versus *M'* for (a) HLA and (b) VLA, f = 350 Hz, M = 15.

It can be readily shown that  $E_{M',m}(\phi)$  increases as  $|\phi - \pi/2|$  is increased. Hence,  $E_{M',m}(\phi)$  is maximum at  $\phi = 0$ . The proximity between  $V(\phi)$  and  $V'(\phi)$  is illustrated in Fig. 4, which shows plots of  $E_{M',m}(0)$  versus *m* at frequency of 350 Hz for a 6-sensor HLA and four different values of *M'*, viz. M'= 1, 2, 3, 4. The channel parameters, signal frequency, and source range-depth have the same values as in Section 5. It is evident that the difference between the full modal subspace  $V(\phi)$  and the truncated modal subspace  $V'(\phi)$  is negligible for all *m* and for all M'>2. It follows that the signal vector may be modeled using a drastically truncated modal subspace without causing a significant modeling error. This conclusion can be tested by considering the modeling error due to approximation of the signal vector  $\tilde{s}(t, \phi)$  by its truncated version  $\tilde{s}'(t, \phi)$  defined in (46). The  $L_2$  norm of the modeling error vector is given by

$$K_{M'}(\phi) = \|\tilde{s}(\phi) - \tilde{s}'(\phi)\|^2.$$
(58)

In (58), the dependence of the signal vector on *t* is suppressed. Figure 5 shows the plots of  $K_{M'}(0)$  versus *M'* for a 6-sensor HLA and also for a 6-sensor VLA for 3 different values of SNR. All the other parameters have the same values as in Fig. 4. It is seen from Fig. 5 that the signal modeling error due to modal subspace truncation is negligible for  $M' \ge 3$  for the HLA and  $M' \ge 7$  for the VLA. Consequently, the optimal value of M' for minimizing the mean square signal estimation error  $\mathcal{E}_{TSD}(M', \phi)$  is very small for an HLA and somewhat larger for a VLA.

Expressions for the test statistics for TSD can be obtained using a procedure analogous to that for SD described in Section 3.3. Thus we get

$$\gamma_{TSD,Case(a)}(\boldsymbol{X}) = \sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}'(\hat{\phi}_{TSD}) \tilde{\boldsymbol{x}}(t), \qquad (59)$$

$$\gamma_{TSD,Case(b)}(\boldsymbol{X}) = \frac{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}'(\phi_{TSD}) \tilde{\boldsymbol{x}}(t)}{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \left(\mathbf{I}_{3N} - \mathbf{D}'(\hat{\phi}_{TSD})\right) \tilde{\boldsymbol{x}}(t)}.$$
(60)

3.5. Approximate Signal vector Form detector (ASFD)

The SD and TSD seek to achieve better performance than that of the ED by exploiting the knowledge of the modal wavenumbers. If this information is not available, it is still possible to achieve better detection than that of the ED by using the knowledge of the structure of the vector  $g_m(\phi)$  (see (13)) associated with each AVS. Since the modal wavenumbers are subject to fairly tight bounds, viz.  $1 > k_1/k > ... k_M/k > c/c_b$ , where c = sound speed in water,  $c_b =$  sound speed in the ocean bottom, and  $k = 2\pi f/c$ , we may use the approximation  $k_m/k \approx 1$  for all m and hence approximate  $g_m(\phi)$  as

$$\boldsymbol{g}_{m}(\boldsymbol{\phi}) \approx \boldsymbol{g}(\boldsymbol{\phi}) = \begin{bmatrix} 1 & \sqrt{2}\cos(\boldsymbol{\phi}) & \sqrt{2}\sin(\boldsymbol{\phi}) \end{bmatrix}^{T}.$$
(61)

This approximation considerably simplifies the detection problem and leads to a much simpler detection algorithm which we refer to as the approximate signal form detector (ASFD) [35]. The signal vector  $\tilde{s}(t,\phi) = \mathbf{R}_0^{-1/2} s(t,\phi)$  can now be approximated as

$$\tilde{\boldsymbol{s}}^{"}(t,\phi) = \mathbf{R}_{0}^{-1/2} \mathbf{H}(\phi) \boldsymbol{p}(t), \qquad (62)$$

where

$$\mathbf{H}(\phi) = \mathbf{I}_{N} \otimes \mathbf{g}(\phi) = [\mathbf{h}_{1}(\phi)...\mathbf{h}_{N}(\phi)]$$
(63)

is a  $3N \times N$  matrix with linearly independent columns, and  $p(t) = [p_1(t)...p_N(t)]^T$  where  $p_n(t)$  denotes the acoustic pressure at the  $n^{\text{th}}$  sensor. The approximate signal vector  $\tilde{s}''(t,\phi)$  belongs to the *N*-dimensional subspace  $V''(\phi)$  defined as

$$V''(\phi) = span\{\mathbf{R}_{0}^{-1/2}\boldsymbol{h}_{1}(\phi)...\mathbf{R}_{0}^{-1/2}\boldsymbol{h}_{N}(\phi)\} = span\{\boldsymbol{u}''_{1}(\phi)...\boldsymbol{u}''_{N}(\phi)\},$$
(64)

where  $\{\boldsymbol{u}_{1}^{"}(\phi)...\boldsymbol{u}_{N}^{"}(\phi)\}$  is the orthonormal basis of  $V''(\phi)$ . We can therefore rewrite (62) as

$$\widetilde{\boldsymbol{s}}^{"}(t,\phi) = \mathbf{U}^{"}(\phi)\boldsymbol{\beta}^{"}(t), \text{ where } \mathbf{U}^{"}(\phi) = [\boldsymbol{u}^{"}_{1}(\phi)...\boldsymbol{u}^{"}_{N}(\phi)].$$
(65)

The signal vector approximation defined in (62), (63), and (65) is qualitatively different from that defined in (46). The approximation in (46) involves truncation of the normal mode expansion, whereas (62) and (63) involve an approximation of the relation among the components of the signal vector measured by an AVS. In (46), the unknown vector  $\boldsymbol{\beta}'(t)$  is *M*'-dimensional, whereas the unknown vector  $\boldsymbol{\beta}''(t)$  in (65) is *N*-dimensional. The conditional ML estimate of  $\boldsymbol{\beta}''(t)$  and the corresponding estimate of  $\tilde{\boldsymbol{s}}(t, \phi)$  are given by

$$\hat{\boldsymbol{\beta}}^{"}(t \mid \boldsymbol{\phi}) = \mathbf{U}^{"}(\boldsymbol{\phi})^{H} \, \tilde{\boldsymbol{x}}(t) \,, \tag{66}$$

$$\hat{\boldsymbol{s}}_{ASFD}(t \mid \boldsymbol{\phi}) = \mathbf{D}^{"}(\boldsymbol{\phi}) \tilde{\boldsymbol{x}}(t), \qquad (67)$$

$$\mathbf{D}^{"}(\phi) = \mathbf{U}^{"}(\phi)\mathbf{U}^{"}(\phi)^{H}.$$
(68)

Expressions for the estimate of  $\phi$  and the unconditional estimate of  $\tilde{s}(t)$  can be obtained using a procedure analogous to that for SD and TSD. Thus we get

$$\hat{\phi}_{ASFD} = \arg \max[\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}^{"}(\boldsymbol{\phi}) \tilde{\boldsymbol{x}}(t)], \qquad (69)$$

$$\hat{\boldsymbol{s}}_{ASFD}(t) = \mathbf{D}^{"}(\hat{\boldsymbol{\phi}}_{ASFD}) \tilde{\boldsymbol{x}}(t). \qquad (70)$$

The NMSE of the signal estimator defined in (70) is given by

$$\varepsilon_{ASFD}(\phi) \triangleq \frac{\sum_{t=1}^{T} E[\left(\hat{s}_{ASFD}(t) - \tilde{s}(t)\right)^{H} \left(\hat{s}_{ASFD}(t) - \tilde{s}(t)\right)]}{\sum_{t=1}^{T} E[\tilde{s}^{H}(t)\tilde{s}(t)]} = \varepsilon_{ASFD}^{(1)} + \varepsilon_{ASFD}^{(2)}(\phi),$$
(71)

where

$$\varepsilon_{ASFD}^{(1)} = \frac{NT}{\lambda},$$
(72)
$$\varepsilon_{T}^{T} \tilde{z}_{T}^{H}(\omega, b) \left(\mathbf{I}_{T} - \mathbf{D}_{T}^{H}(\hat{z}_{T})\right) \tilde{z}(\omega, b) \left[ \sum_{i=1}^{T} \tilde{z}_{T}^{H}(\omega) \mathbf{D}_{T}^{H}(\omega) \tilde{z}(\omega) \right]$$

$$\varepsilon_{ASFD}^{(2)}(\phi) = \frac{E\left[\sum_{t=1}^{\infty} \tilde{s}^{H}(t;\phi) \left(\mathbf{I}_{3N} - \mathbf{D}^{"}(\phi_{ASFD})\right) \tilde{s}(t;\phi)\right]}{E_{s}} \simeq 1 - \frac{E\left[\sum_{t=1}^{\infty} \tilde{s}^{H}(t) \mathbf{D}^{"}(\phi) \tilde{s}(t)\right]}{E_{s}} = 1 - \frac{\lambda^{"}}{\lambda},$$
(73)

$$\lambda'' = \frac{E_{s''}}{\sigma^2}, E_{s''} = \sum_{t=1}^{T} E[\tilde{s}''(t)^H \tilde{s}''(t)].$$
(74)

In (70),  $E_{s}$  is the total energy of the approximate signal vectors  $\tilde{s}''(t)$  over *T* snapshots and  $\lambda''$  is the ENR of the approximate signal. The NMSE  $\varepsilon_{ASFD}(\phi)$  also has two components  $\varepsilon_{ASFD}^{(1)}$  and  $\varepsilon_{ASFD}^{(2)}(\phi)$ . The first component  $\varepsilon_{ASFD}^{(1)}$  is the error due to noise, and it is analogous to the NMSE component  $\varepsilon_{ASFD}^{(1)}$  defined in (54). The second component  $\varepsilon_{ASFD}^{(2)}(\phi)$  arises from the signal modeling error due to the approximation (61). We have  $\varepsilon_{ASFD}^{(1)} = \varepsilon_{ED}/3$  while the other component  $\varepsilon_{ASFD}^{(2)}(\phi)$  is quite small in comparison to  $\varepsilon_{ASFD}^{(1)}(\phi)$  because the approximation in (61) introduces only a small error. It follows that  $\varepsilon_{ASFD} \simeq \varepsilon_{ED}/3$ . Hence the ASFD is expected to perform better than the ED. This prediction is confirmed by the asymptotic analysis in Section 4.6 and the simulation results presented in Section 5.

Expressions for the test statistics of ASFD, which are analogous to the corresponding expressions for SD and TSD, are given by

$$\gamma_{ASFD,Case(a)}(\boldsymbol{X}) = \sum_{t=1}^{I} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}^{"}(\hat{\phi}_{ASFD}) \tilde{\boldsymbol{x}}(t), \qquad (75)$$
$$\gamma_{ASFD,Case(b)}(\boldsymbol{X}) = \frac{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \mathbf{D}^{"}(\hat{\phi}_{ASFD}) \tilde{\boldsymbol{x}}(t)}{\sum_{t=1}^{T} \tilde{\boldsymbol{x}}^{H}(t) \left(\mathbf{I}_{3N} - \mathbf{D}^{"}(\hat{\phi}_{ASFD})\right) \tilde{\boldsymbol{x}}(t)}. \qquad (76)$$

### 4. Performance analysis

#### 4.1. Matched filter detector (MFD)

For the MFD, the probability of detection  $P_D$  and probability of false alarm  $P_{FA}$  are given by [42]

$$P_{FA} = Q\left(\eta / \sqrt{\frac{E_s \sigma^2}{2}}\right), P_D = Q\left(\left(Q^{-1}(P_{FA} - \sqrt{2\lambda})\right)\right),\tag{77}$$

where  $E_s$  is the energy of the pre-whitened signal summed over all snapshots and  $\lambda = E_s/\sigma^2$  is the signal energy to noise power ratio (ENR) defined in (34), Q (.) denotes the right-tail probability function of the standard normal distribution and  $Q^{-1}$ (.) denotes the inverse of this function.

#### 4.2. Energy detector (ED)

The test statistic  $\gamma_{ED}$  of the ED, defined in (32), is the total energy of the pre-whitened data vectors. It is the sum of squares of 6NT independent Gaussian random variables with variance  $\sigma^2/2$  under both hypotheses. Under  $H_0$ , the means of these random variables are zero; and under  $H_1$ , the sum of their means is equal to  $E_s = \sum_{t=1}^{T} E[\tilde{s}^H(t)\tilde{s}(t)]$ . It

follows that, under hypothesis  $H_0$ , we have  $\left(\frac{2}{\sigma^2}\gamma_{ED}\right) \sim \chi^2_{6NT}$ , i.e. the random variable  $\frac{2}{\sigma^2}\gamma_{ED}$  has chi-squared

distribution with 6*NT* degrees of freedom, and under hypothesis  $H_1$ , we have  $\left(\frac{2}{\sigma^2}\gamma_{ED}\right) \sim \chi'^2_{6NT}(2\lambda)$ , non-central chi-squared distribution with 6*NT* degrees of freedom and noncentrality parameter  $2\lambda$ . Thus we have

$$P_{FA,ED} = Q_{\chi^2_{6NT}} \left(\frac{2\eta}{\sigma^2}\right),\tag{78}$$

$$P_{D,ED} = Q_{\chi^2_{6NT}(2\lambda)} \left(\frac{2\eta}{\sigma^2}\right),\tag{79}$$

where the right-hand sides of (78) and (79) denote the right-tail probability functions of the respective distributions.

#### 4.3. Subspace detector (SD)

Derivation of expressions for  $P_D$  and  $P_{FA}$  requires determination of the probability density functions (PDFs) of the test statistic under both hypotheses. The test statistic is defined in (42) for case (a), noise variance known, and in (43) for case (b), noise variance unknown. It is difficult to determine the PDFs since the test statistics are highly nonlinear functions of the data. But we can determine approximate expressions for the PDFs by assuming that the estimate  $\hat{\phi}_{SD}$  is equal to the true value  $\phi$ . Validity of this approximation is discussed in Section 5. We note that  $\tilde{x}^H(t)\mathbf{D}(\phi)\tilde{x}(t)$  is a quadratic form involving a complex circular Gaussian random vector  $\tilde{x}(t)$ ,  $\mathbf{D}(\phi)$  is an idempotent matrix of rank M, and

$$\tilde{\boldsymbol{s}}^{H}(t)\mathbf{D}\left(\hat{\phi}_{SD}\right)\tilde{\boldsymbol{s}}(t) \cong \tilde{\boldsymbol{s}}^{H}(t)\mathbf{D}(\phi)\tilde{\boldsymbol{s}}(t) = \tilde{\boldsymbol{s}}^{H}(t)\tilde{\boldsymbol{s}}(t) = E_{S}.$$
(80)

We can apply Graybill's theorem [54,55] to find the probability distribution of the quadratic form  $\tilde{\boldsymbol{x}}^{H}(t)\mathbf{D}(\phi)\tilde{\boldsymbol{x}}(t)$ . Thus it can be readily shown that  $\left(\frac{2}{\sigma^{2}}\gamma_{SD,Case(a)}\right) \sim \chi^{2}_{2MT}$  under  $H_{0}$ , and  $\left(\frac{2}{\sigma^{2}}\gamma_{SD,Case(a)}\right) \sim \chi^{\prime^{2}}_{2MT}(2\lambda)$  under  $H_{1}$ . Therefore, the expressions for  $P_{FA}$  and  $P_{D}$  under case (a) can be written as

$$P_{FA,SD,Case(a)} = Q_{\chi^2_{2MT}} \left(\frac{2\eta}{\sigma^2}\right),\tag{81}$$

$$P_{D,SD,Case(a)} = Q_{\chi^2_{2MT}(2\lambda)} \left(\frac{2\eta}{\sigma^2}\right).$$
(82)

In case (b), the matrices  $\mathbf{D}(\hat{\phi}_{SD})$  and  $\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD})$  in the numerator and denominator of (43) are idempotent matrices of rank *M* and 3*N*-*M* respectively. Hence, on applying Graybill's theorem [54,55], we find that the numerator of (43),

scaled by the factor  $(2/\sigma^2)$ , has chi-squared distribution with 2MT degrees of freedom under hypothesis  $H_0$  and noncentral chi-squared distribution with 2MT degrees of freedom and noncentrality parameter  $\lambda$  under  $H_1$ . The denominator of (43), scaled by the factor  $(2/\sigma^2)$ , has chi-squared distribution with (6N-2M)T degrees of freedom under both hypotheses since  $\tilde{s}^H(t)(\mathbf{I}_{3N} - \mathbf{D}(\hat{\phi}_{SD}))\tilde{s}(t) = 0$  in view of (80). The numerator and denominator of (43) are statistically independent since they represent the squares of the norms of projections of  $\tilde{x}(t)$  onto mutually orthogonal subspaces. It follows that, under hypothesis  $H_0$ , the test statistic defined in (43) has F distribution, denoted by  $F_{2MT,(6N-2M)T}$ , and under hypothesis  $H_1$ , it has noncentral F distribution with noncentrality parameter  $2\lambda$ , denoted by  $F'_{2MT,(6N-2M)T}(2\lambda)$ . Hence the expressions for  $P_{FA}$  and  $P_D$  can be written as

$$P_{FA,SD,Case(b)} = Q_{F_{2MT,(6N-2M)T}}(\eta),$$
(83)

$$P_{D,SD,Case(b)} = Q_{F'_{2MT,(6N-2M)T}(2\lambda)}(\eta),$$
(84)

where  $Q_F(.)$  and  $Q_F(.)$  denote the right-tail probability functions of the respective distributions.

#### 4.4. *Truncated subspace detector (TSD)*

As in the case of SD, we can once again determine the approximate distributions of the test statistics, defined in (59) and (60), under the assumption that  $\hat{\phi}_{TSD} = \phi$ . Before proceeding further, we note some important differences between SD and TSD. It can be shown that

$$\tilde{\boldsymbol{s}}^{H}(t)\boldsymbol{\mathsf{D}}'(\boldsymbol{\phi})\tilde{\boldsymbol{s}}(t) = \tilde{\boldsymbol{s}}^{'H}(t)\tilde{\boldsymbol{s}}'(t), \qquad (85)$$

where  $\tilde{s}'(t) = \mathbf{U}'\boldsymbol{\beta}'(t)$  is the truncated signal vector defined in (46). It follows that  $\tilde{s}^{H}(t)\mathbf{D}'(\phi)\tilde{s}(t) \neq \tilde{s}^{H}(t)\tilde{s}(t)$ , whereas  $\tilde{s}^{H}(t)\mathbf{D}(\phi)\tilde{s}(t) = \tilde{s}^{H}(t)\tilde{s}(t)$ . Also, rank $(\mathbf{D}'(\phi)) = M'$ , whereas rank $(\mathbf{D}(\phi)) = M$ . Hence, we have

$$\left(\frac{2}{\sigma^2}\gamma_{TSD,Case(a)}\right) \sim \chi^2_{2M'T} \text{ under } H_0, \text{ and } \left(\frac{2}{\sigma^2}\gamma_{TSD,Case(a)}\right) \sim \chi'^2_{2M'T}(2\lambda') \text{ under } H_1, \text{ and}$$

$$P_{FA,TSD,Case(a)} = Q_{\chi^2_{2M'T}} \left(\frac{2\eta}{\sigma^2}\right),\tag{86}$$

$$P_{D,TSD,Case(a)} = Q_{\chi^2_{2M'T}(2\lambda')} \left(\frac{2\eta}{\sigma^2}\right).$$
(87)

where  $\lambda'$  is the ENR for the truncated signal defined in (56). For case (b), the numerator of (60), scaled by the factor  $(2/\sigma^2)$ , has chi-squared distribution with 2M'T degrees of freedom under hypothesis  $H_0$  and noncentral chi-squared distribution with 2M'T degrees of freedom and noncentrality parameter  $2\lambda'$  under  $H_1$ . The denominator of (60), scaled by the factor  $(2/\sigma^2)$ , has chi-squared distribution with (6N-2M')T degrees of freedom under hypothesis  $H_0$  and noncentral chi-squared distribution with (6N-2M')T degrees of freedom under hypothesis  $H_0$  and noncentral chi-squared distribution with (6N-2M')T degrees of freedom and noncentrality parameter  $2(\lambda - \lambda')$  under  $H_1$ . The numerator and denominator are statistically independent. It follows that, under hypothesis  $H_0$ , the test statistic defined in (60) has F distribution, denoted by  $F_{2MT,(6N-2M')T}$ , and under hypothesis  $H_1$ , it has a doubly noncentral F distribution denoted by  $F''_{2MT,(6N-2M')T}(2\lambda', 2\lambda-2\lambda')$ . Hence the expressions for  $P_{FA}$  and  $P_D$  can be written as

$$P_{FA,TSD,Case(b)} = Q_{F_{2M'T,(6N-2M')T}}(\eta),$$

$$P_{D,TSD,Case(b)} = Q_{F''_{2M'T,(6N-2M')T}(2\lambda',2\lambda-2\lambda')}(\eta),$$
(88)
(89)

where  $Q_{F}(.)$  and  $Q_{F''}(.)$  denote the right-tail probability functions of the respective distributions.

#### 4.5. Approximate signal form detector (ASFD)

The test statistics of ASFD for cases (a) and (b), given by (75) and (76), are analogous to the corresponding test statistics for the TSD, with  $\mathbf{D}'(\phi)$  replaced by  $\mathbf{D}''(\phi)$ . Since  $\mathbf{D}''(\phi)$  is an idempotent matrix of rank *N*, the expressions for  $P_{FA}$  and  $P_D$  are analogous to those for the TSD. Thus we have

$$P_{FA,ASFD,Case(a)} = Q_{\chi^2_{2NT}} \left(\frac{2\eta}{\sigma^2}\right),\tag{90}$$

$$P_{D,ASFD,Case(a)} = Q_{\chi^2_{2NT}(2\lambda'')} \left(\frac{2\eta}{\sigma^2}\right),\tag{91}$$

$$P_{FA,ASFD,Case(b)} = Q_{F_{2NT,4NT}}(\eta),$$
(92)

$$P_{D,ASFD,Case(b)} = Q_{F_{2NT,ANT}(2\lambda^{"},2\lambda-2\lambda^{"})}(\eta), \qquad (93)$$

where  $\lambda''$  is defined in (74).

#### 4.6. Asymptotic performance analysis

The performance analysis in the preceding subsections is based on 3NT data samples provided by an AVS array of N sensors over T snapshots. It is of interest to also consider an asymptotic  $(3NT \rightarrow \infty)$  performance analysis since such an analysis facilitates an easy comparison of the performance of different detectors and provides a better insight into their relative merits. For case (a) and a given  $\phi$ , the test statistics of the detectors, defined in (32), (42), (59), and (75) are asymptotically  $(3NT \rightarrow \infty)$  Gaussian. Hence, if the noise variance is known, the asymptotic expression for the receiver operating characteristic (ROC) of each detector can be written as [42]

$$P_{D,Case(b)} \sim Q\left(\frac{\nu_0}{\nu_1}Q^{-1}(P_{FA}) - \frac{\mu_1 - \mu_0}{\nu_1}\right) = Q\left(\frac{Q^{-1}(P_{FA}) - \delta}{\tau}\right),\tag{94}$$

where ~ denotes asymptotic equality in the above expression, and

$$\tau = \frac{V_1}{V_0}, \delta = \frac{\mu_1 - \mu_0}{V_0}, \mu_j = E[\gamma; H_j], v_j^2 = \operatorname{var}(\gamma; H_j), j = 0, 1.$$
(95)

In (94),  $\mu_j$  and  $v_j$  are respectively the mean and standard deviation of  $\gamma$  under hypothesis  $H_j$ ,  $\tau$  is the ratio of standard deviations, and  $\delta$  is the parameter known as the deflection coefficient. It is seen from (94) that, for a given  $P_{FA}$ , the probability of detection  $P_D$  increases when  $\delta$  and / or  $\tau$  increases. Hence the relative magnitudes of  $\delta$  and the relative magnitudes of  $\tau$  provide measures for comparing the performance of the different detectors under consideration.

For the case of known noise variance, each detector has a test statistic of the form defined in (26) which is reproduced below:

$$\gamma_{Case(a)}(\boldsymbol{X}) = \sum_{t=1}^{T} \left[ 2\operatorname{Re}\left(\tilde{\boldsymbol{x}}^{H}(t)\hat{\boldsymbol{s}}(t)\right) - \hat{\boldsymbol{s}}^{H}(t)\hat{\boldsymbol{s}}(t) \right].$$

Using the expressions for  $\hat{s}(t)$  for different detectors defined in (31), (41), (52) and (70), and noting that  $\tilde{x}(t) = \tilde{w}(t)$ under  $H_0$  and  $\tilde{x}(t) = \tilde{s}(t) + \tilde{w}(t)$  under  $H_1$ , we can obtain the following results



Fig. 6: Variation of (a) standard deviation ratio  $\tau$  and (b) deflection coefficient  $\delta$  of TSD with number of retained modes *M*' for different values of SNR, for HLA (solid lines) and VLA (dashed lines), f = 350 Hz, array length N = 6, source bearing  $\phi = 20^{\circ}$ 

$$\mu_{0,ED} = 3NT\sigma^2, \mu_{1,ED} = E_s + 3NT\sigma^2, \nu_{0,ED}^2 = 3NT\sigma^4, \nu_{0,ED}^2 = 2\sigma^2 E_s + 3NT\sigma^4,$$
(96)

$$\tau_{ED} = \sqrt{1 + \frac{2\lambda}{3NT}}, \delta_{ED} = \frac{\lambda}{\sqrt{3NT}}, \qquad (97)$$

$$\mu_{0,SD} = MT\sigma^2, \mu_{1,SD} = E_S + MT\sigma^2, \nu_{0,SD}^2 = MT\sigma^4, \nu_{0,SD}^2 = 2\sigma^2 E_S + MT\sigma^4,$$
(98)

$$\tau_{SD} = \sqrt{1 + \frac{2\lambda}{MT}}, \delta_{SD} = \frac{\lambda}{\sqrt{MT}},$$
(99)

$$\mu_{0,TSD} = M'T\sigma^2, \mu_{1,TSD} = E_{S'} + M'T\sigma^2, \nu_{0,TSD}^2 = M'T\sigma^4, \nu_{0,TSD}^2 = 2\sigma^2 E_{S'} + M'T\sigma^4,$$
(100)

$$\tau_{TSD} = \sqrt{1 + \frac{2\lambda'}{M'T}}, \delta_{TSD} = \frac{\lambda'}{\sqrt{M'T}},$$
(101)

$$\mu_{0,ASFD} = NT\sigma^2, \mu_{1,ASFD} = E_{S"} + NT\sigma^2, \nu_{0,ASFD}^2 = NT\sigma^4, \nu_{0,ASFD}^2 = 2\sigma^2 E_{S"} + NT\sigma^4,$$
(102)

$$\tau_{ASFD} = \sqrt{1 + \frac{2\lambda''}{NT}}, \delta_{ASFD} = \frac{\lambda''}{\sqrt{NT}}.$$
(103)

We can draw some useful conclusions from equations (96)-(103). We recall that 3N > M > M'. It follows from (97) and (99) that  $\delta_{SD} > \delta_{ED}$  and  $\tau_{SD} > \tau_{ED}$ . It can be readily verified that the signal energy-to-noise power ratios (ENRs)  $\lambda$  (for the signal vector  $\tilde{s}$ ) and  $\lambda''$  (for the approximate signal vector  $\tilde{s}$ ") are very close to each other. It can also be verified that the value of  $\lambda'$  (ENR of the truncated signal vector  $\tilde{s}$ ") decreases very slowly as M' is reduced, and the rate of reduction of  $\lambda'$  becomes large only at very small values of M'. Therefore, it follows from (101) that the values

Detectors	Deflection coefficient $\delta$	Standard deviation ratio $\tau$	NMSE $\varepsilon$	Probability of detection $(P_D)$
TSD (HLA)	7.698	2.108	0. 561	0.9974
ASFD (HLA)	3.196	1.258	3.366	0.5383
SD (HLA)	2.022	1.111	8.57	0.1565
ED (HLA)	1.846	1.093	10.28	0.1178
TSD (VLA)	2.486	1.224	3.401	0.2963
ASFD (VLA)	2.644	1.218	3.811	0.3453
SD (VLA)	1.675	1.093	10.34	0.0886
ED (VLA)	1.529	1.078	12.41	0.0668

Table 1: Values of deflection coefficient  $\delta$ , standard deviation ratio  $\tau$ , NMSE  $\varepsilon$  and probability of detection  $P_D$  for different detectors, at SNR = -10 dB, T = 20 snapshots, for a 6-sensor AVS array



Fig. 7: Variation of (a) standard deviation ratio  $\tau$  and (b) deflection coefficient  $\delta$  vs. SNR for different detectors, f = 350 Hz, array length N = 6, source bearing  $\phi = 20^{\circ}$ 

of  $\delta_{TSD}$  and  $\tau_{TSD}$  keep increasing as M' is reduced until an optimal value of M' is reached, as shown in Fig. 6. (We recall that a different criterion of optimality, viz. minimization of the normalized mean square signal estimation error  $\varepsilon_{TSD}$ , was considered in Section 3. The two criteria may lead to slightly different optimal values of M'.) It is seen from Fig. 6 that  $\delta_{TSD} > \delta_{SD}$  and  $\tau_{TSD} > \tau_{SD}$  as long as the truncation of the signal vector in TSD is not too severe. We may therefore make the following predictions: (1) TSD performs better than SD, which in turn performs better than ED, and (2) the performance of TSD keeps improving as M' is reduced till it reaches its optimal value. In the case of ASFD, it is obvious from (97) and (103) that  $\delta_{ASFD} > \delta_{ED}$  and  $\tau_{ASFD} > \tau_{ED}$ , and therefore we can predict that ASFD performs better than ED. The performance of ASFD relative to that of the SD and TSD depends on the relative magnitudes of N, M and M'. It is seen from (99) and (103) that  $\delta_{ASFD} > \delta_{SD}$  and  $\tau_{ASFD} > \tau_{SD}$  if M > N. Consequently ASFD performs better than SD if 3N > M > N, and worse than SD if M < N. It is seen from (101) and (103) that  $\delta_{TSD} > \delta_{SD}$  and  $\tau_{TSD} > \tau_{SD}$  if M' < N. For HLA, optimal value of M' is very close to 1 and therefore optimal M' is almost always less than N. Hence, for HLA, TSD with optimal M' performs better than ASFD almost always. For VLA, it

turns out that optimal  $M' \cong N$  and consequently, for optimal M',  $\delta_{TSD} \cong \delta_{ASFD}$  and  $\tau_{TSD} \cong \tau_{ASFD}$ . Therefore, for VLA, the performance of ASFD is very close to that of TSD with optimal M'.

In order to illustrate the conclusions and predictions of the preceding paragraph, we shall consider an example with N = 6, T = 20, f = 350 Hz, M = 15, and SNR = -10 dB. SNR (in dB) is defined as

$$SNR = 10\log_{10}\frac{1}{N}\sum_{n=1}^{N}\frac{|p_n|^2}{\sigma_n^2},$$
(104)

where  $p_n$  and  $\sigma_n^2$  are respectively the signal component of acoustic pressure and the variance of noise at the n<sup>th</sup> sensor. Only the pressure components of signal and noise are considered in the definition of SNR as per normal convention [20], to facilitate a fair comparison between the performance of AVS and APS (acoustic pressure sensor) arrays. Values of the channel parameters and source coordinates used in this example are the same as those listed in Section 5. For TSD, we have chosen the optimal values of M', which turn out to be M' = 1 for the HLA and M' = 5 for the VLA as shown in Fig. 6. For this example, we have the following values of ENRs:  $\lambda = 35.03$ ,  $\lambda' = 34.44$  (for M' = 1), and  $\lambda'' = 35.02$  for the HLA, and  $\lambda = 29.02$ ,  $\lambda' = 24.86$  (for M' = 5), and  $\lambda'' = 28.96$  for the VLA. Values of  $\delta$ ,  $\tau$ , the normalized mean square signal estimation error  $\varepsilon$ , and the asymptotic probability of detection  $P_D$  (for false alarm probability  $P_{FA} = 0.001$ ) for different detectors are tabulated in Table 1. The values of  $\delta$ ,  $\tau$ , and  $P_D$  confirm the predictions of the preceding paragraph. We also note that a reduction in the value of  $\varepsilon$  is almost always accompanied by an increase in the value of  $P_D$ , as expected. For VLA, we have  $\delta_{TSD} = 2.486 < \delta_{ASFD} = 2.644$  and  $\tau_{TSD} = 1.224 > 1.224$  $\tau_{ASFD} = 1.218$  in the present example. On substituting these values of  $\delta$  and  $\tau$  in (94), we see that  $P_D$  of ASFD is larger than  $P_D$  of TSD if  $\{Q^{-1}(P_{FA}) - 2.486\}/1.224 > Q^{-1}(P_{FA}) - 2.644\}/1.218$ , i.e. if  $P_{FA} > Q(34.72)$ . Since Q(34.72) <<1, it follows that  $P_D$  of ASFD is larger than  $P_D$  of TSD for almost all values of  $P_{FA}$ . It is however emphasized that, under different conditions,  $P_D$  of TSD-VLA with optimal M' may be higher than that of ASFD-VLA, but the difference in performance is always small.

Additional results based on the same example are provided in Figs. 6 and 7. Figure 6 shows the variation of  $\delta_{TSD}$  and  $\tau_{TSD}$  with M' for different values of SNR. For an HLA, the values of  $\delta_{TSD}$  and  $\tau_{TSD}$  are maximized at M' = 1, whereas for a VLA, they are maximized at M' = 5. It is also seen from Fig. 6 that the maximum values of  $\delta_{TSD}$  and  $\tau_{TSD}$  for the VLA are consistently smaller than those for the HLA at all values of SNR. Hence, for the TSD, the performance of an HLA may be expected to be consistently better than that of a VLA. The variation of  $\delta$  and  $\tau$  with SNR for different detectors is shown in Fig. 7. In this figure, plots for TSD are shown for the optimal values of M', viz.  $M'_{opt} = 1$  for the HLA and  $M'_{opt} = 5$  for the VLA. Figure 7 confirms once again that, if N < M < 3N and M' is optimal, we have  $\delta_{TSD} > \delta_{ASFD} > \delta_{SD} > \delta_{ED}$  and  $\tau_{TSD} > \tau_{ASFD} > \tau_{ED}$  for HLA, and  $\delta_{ASFD} \cong \delta_{TSD} > \delta_{ED}$  and  $\tau_{ASFD} \cong \tau_{TSD} > \tau_{SD} > \tau_{ED}$  for VLA.

It is known that the number of modes *M* increases as the frequency is increased. It follows from (99) that both  $\delta_{SD}$  and  $\tau_{SD}$  reduce with increasing frequency. We may therefore expect the performance of the SD to degrade as the frequency is increased till *M* reaches the threshold 3*N*. For all the other detectors, the performance is expected to be independent of frequency. All the predictions in the preceding paragraphs will be verified through simulation results presented in Section 5.

If noise variance is not known (case (b)), the test statistics defined in (43), (60) and (76) converge asymptotically to ratios of independent Gaussian random variables with non-zero means. The asymptotic PDFs of the test statistics have complicated expressions which do not provide any useful insights. Therefore, we shall not pursue the asymptotic analysis of case (b).

### 5. Simulation results

This section presents a detailed study of the performance of all the detectors. The ocean is modeled as a Pekeris channel [53] comprising a homogeneous water layer of constant depth over a fluid half-space. The following values



Fig. 8: Comparison of non-asymptotic and asymptotic theoretical results for the case of known noise variance.  $P_D$  vs. SNR at  $P_{FA}$ = 0.001. (a) HLA, (b) VLA



Fig. 9: Comparison of theoretical (non-asymptotic) and simulation results for the case of known noise variance.  $P_D$  vs. SNR at  $P_{FA} = 0.001$ . (a) HLA, (b) VLA

of channel, source and array parameters have been assumed unless otherwise stated: ocean depth h = 70 m, sound speed in water  $c_w = 1500$  m/s, sound speed in ocean bottom  $c_b = 1700$  m/s, density of water  $\rho_w = 1000$  kg/m<sup>3</sup>, density of ocean bottom  $\rho_b = 1500$  kg/m<sup>3</sup>, attenuation in ocean bottom  $\zeta = 0.2$  dB/ $\lambda_b$ , where  $\lambda_b = c_b/f$  is the wavelength in the ocean bottom, number of sensors N = 6 for both HLA and VLA, array depth  $z_1 = 40$  m for HLA and  $z_1 = 15$  m for topmost sensor in the VLA, inter-sensor spacing  $d = \lambda_w/2 = 15/7$  m, where  $\lambda_w = c_w/f$  is the wavelength in water, source at range r = 5000 m, depth  $z_s = 40$  m, bearing  $\phi = 20^0$  with respect to the endfire direction of the HLA, and frequency f = 350 Hz. At this frequency, the number of normal modes in the channel is M = 15. It is assumed that the signal does not vary from one snapshot to another. Detection is done using T = 20 snapshots of data, and the probability of false alarm is fixed at  $P_{FA} = 0.001$ .

Before presenting the results, we recall that the ED and the ASFD do not require any prior information about the channel. The TSD requires the knowledge of wavenumbers of M' lowest order modes and the SD requires the knowledge of all the modal wavenumbers if an HLA is deployed. If a VLA is used, the SD and TSD require the knowledge of the modal eigenfunctions also, in addition to the modal wavenumbers.



Fig. 10: Bearing estimation errors of SD, TSD and ASFD. (a) Bias vs. SNR. (b) Root-mean-square error vs. SNR.



Fig. 11: Comparison of theoretical (non-asymptotic) and simulation results for the case of unknown noise variance.  $P_D$  vs. SNR at  $P_{FA} = 0.001$ . (a) HLA, (b) VLA

Figure 8 shows the theoretical plots of  $P_D$  versus SNR at  $P_{FA} = 0.001$  for different detectors for the case of known noise variance (case (a)). The non-asymptotic results ((77)-(79), (81)-(82), (86)-(87), (90)-(91)) are shown as dashed lines and the asymptotic results ((94) in conjunction with (96)-(103)) are shown as solid lines. The TSD results have been obtained using values of M' that minimize the NMSE  $\varepsilon_{TSD}$ . Performance of the unrealizable MFD is also shown to indicate the upper bound on  $P_D$  for any realizable detector. It is seen that the asymptotic and non-asymptotic results exhibit the same trend, even though there are some quantitative differences between the two sets of results. Among the realizable detectors, the ranking of HLA-based detectors in decreasing order of performance is TSD, followed by ASFD, SD, and ED. For VLA-based detectors, the order of TSD and ASFD is reversed. These results are consistent with the values of the deflection coefficient  $\delta$  shown in Table 1 and Fig. 6, and are in conformity with the predictions in Section 4.6. It is noteworthy that the ASFD, that does not require any channel information, compares favorably with the TSD that requires the knowledge of wavenumbers/eigen functions of the lowest M' modes of the channel. The ASFD compensates for the lack of channel information by exploiting the knowledge of an approximate relationship among different components of the signal measured by an AVS. The performance of the SD is poor even though it uses the modal wavenumber/eigenfunction information of all the M modes. This is so because the number of modes M is large and hence the NMSE of the SD is high, which leads to a degradation in performance. The performance of the ED is the poorest because it does not use any prior information.

In Fig. 9, the theoretical (non-asymptotic) results (dashed lines) are compared with simulation results (solid lines). For, SD, TSD and ASFD, two types of simulation results are considered, viz. simulations of realizable detectors that

do not assume prior knowledge of the bearing  $\phi$  (solid lines), and simulations of unrealizable detectors which assume that  $\phi$  is known (dotted lines). The variance of noise is assumed to be known (case (a)). The theoretical results are based on the expressions for  $P_{FA}$  and  $P_D$  given in Sections 4.1 to 4.5. The following observations can be made from



Fig. 12: Variation of  $P_D$  (at  $P_{FA} = 0.001$ ) with frequency for HLA with 6 sensors. SNR = -9 dB

Fig. 9. For MFD and ED, the simulation results match the theoretical predictions very closely. In the case of SD, TSD and ASFD, there is very good agreement between theoretical results and the simulation results that are based on the assumption that  $\phi$  is known. But the  $P_D$  of the realizable versions of these detectors are significantly lower than the theoretical predictions at low SNR. The difference between theoretical predictions and actual performance can be explained as follows. The theoretical results are based on the assumption that the bearing estimates  $\hat{\phi}_{SD}$ ,  $\hat{\phi}_{TSD}$  and  $\hat{\phi}_{ASFD}$  may be approximated by the true value  $\phi$ . This assumption has been made to simplify the theoretical analysis.

But the means and standard deviations of the bearing estimation errors keep increasing as SNR is reduced, as illustrated in Fig. 10. Consequently, for SD, TSD and ASFD, the gap between theoretical predictions and actual performance also keeps increasing as SNR is reduced. Even though the theoretical results tend to overestimate the actual performance, the former provide a useful insight into the comparative performance of different detectors.

It is also of interest to compare the performance of HLA-based detectors (Figs. 8 (a) and 9 (a)) and VLA-based detectors (Figs. 8 (b) and 9 (b)). It is seen that the performance of an HLA-based detector is better than that of a similar VLA-based detector. One reason for this difference is that noise at the sensors of the HLA is i.i.d., whereas noise at the sensors of the VLA is correlated and it has spatially varying variance. The whitening transformation leads to a reduction of the energy of the VLA signal vector and a consequent reduction in the performance of all VLA-based detectors. In the case of the TSD, there is an additional reason for the better performance of HLA. The optimal value of M' for the HLA is lower than that for the VLA due to the higher degree of correlation among the modal steering vectors of the HLA. Therefore the NMSE  $\varepsilon_{TSD}$  for the VLA is higher than that for the HLA, and the higher NMSE translates into a lower detector performance. Finally, it is seen from Fig. 10 that (1) for SD and TSD, the bearing estimation errors of ALA and VLA are almost the same. Therefore, for SD and TSD, the *actual* differences between the performance of HLA-based and VLA-based detectors (shown in Figs. 9(a) and 9(b)) are larger than the *theoretically predicted* differences (shown in Figs. 8(a) and 8(b)).

Figure 11 shows a comparison of theoretical (finite data) and simulation performances of TSD, ASFD and SD for the case of unknown noise variance  $\sigma^2$  (case (b)). A comparison of Figs. 9 and 11 shows that lack of knowledge of  $\sigma^2$  causes degradation in performance of all the detectors. But the degradation is less severe in the case of TSD. It is well known that this degradation can be arrested by employing secondary data vectors which are statistically identical to the noise-only data vectors [47].

We shall now study the variation of detector performance with frequency. Figure 12 shows simulation results for the variation of probability of detection  $P_D$  with frequency f, for  $P_{FA} = 0.001$  and SNR = -9 dB. All the other parameters have the values mentioned at the beginning of this section. When f = 30 Hz, the number of normal modes M is equal to 1, and thus the SD and the TSD are equivalent. As f is increased, M increases and consequently the performance of the SD suffers a progressive degradation. For f > 120 Hz, we have M > N = 6, and hence the performance of the SD dips below that of the ASFD. At f = 398 Hz, M = 3N = 18, and the performance of SD is the

same as that of ED. At still higher frequencies M is greater than 3N, and hence the SD cannot be used for detection. The performance of the other detectors does not vary with frequency. These results are in agreement with the predictions at the end of Section 4.6.



Fig. 13: Comparison of simulated performance of AVS and APS HLAs.  $P_D$  vs. SNR at  $P_{FA} = 0.001$ . (a) ED and (b) TSD

In the context of source localization applications, the superiority of AVS arrays over acoustic pressure sensor (APS) arrays has been well-established [3,7–23]. It is therefore interesting to compare the detection performance of AVS and APS arrays. Such a comparison is shown in Fig. 13. It is seen from Fig. 13 (a) that for the ED, the performance of the 6-sensor AVS HLA is equal to that of the 18-sensor APS HLA. This result can be explained by noting that the performance of the ED depends only on the ENR  $\lambda$ , and that the ENR at an N-sensor AVS array is equal to the ENR at a 3N-sensor APS array. However, in the case of the TSD, the performance of the 6-sensor AVS HLA is slightly better than that of the 18-sensor APS HLA. This difference is attributed to the superior bearing estimation capability of the AVS array due to additional directivity provided by the velocity measurements of the AVS [56]. Better bearing estimation translates into a better estimation of the signal vector and thus a better detection performance. These twin advantages of an AVS array, viz. higher SNR and directivity, over an APS array of the same size N and aperture (= (N-1)d for a uniform linear array) are recognized and documented in the literature on AVS array signal processing [3,56]. The ability of an N-sensor AVS array to attain the same level of performance as a 3N-sensor APS array is a result of great practical interest. In an array-based measurement system, a considerable portion of the cost is related to the construction, deployment and location calibration of sensor packages, and these costs depend on the number of sensor packages deployed [3]. Therefore, a reduction from 3N sensor packages in an APS array to N sensor packages in an AVS array will lead to a significant cost reduction. Moreover, in a towed array system, the drag on the towed array depends on the length of the array [57]. The drag can be reduced considerably if an APS array is replaced by a shorter AVS array.

### 6. Conclusion

A detailed investigation of four methods, viz. ED, SD, TSD and ASFD, for narrowband detection of an acoustic source in a range-independent shallow-ocean using an AVS array has been presented in this paper. Expressions for probability of false alarm and probability of detection were derived for the asymptotic case and the finite-data case, and the theoretical predictions were compared with simulation results. The signal vector at the sensor array is not known due to the unknown location of the source. Hence all detectors employ a generalized likelihood ratio test (GLRT) which involves a maximum likelihood estimation of the signal vector. The ED does not use any model for the array signal vector. The SD and TSD employ models based on the normal mode theory. The ASFD employs a model that exploits the relation between the acoustic pressure and particle velocity at each sensor. Different models lead to different signal-vector estimation errors. Expressions for the normalized mean square signal estimation error

(NMSE) were derived for each detector, and it was shown that there exists a strong negative correlation between the NMSE and the detector performance.

If an HLA is employed, the signal model used by the SD requires the knowledge of all the modal wavenumbers, while the TSD requires the knowledge of wavenumbers of a small number of the lowest order modes. If a VLA is employed, the knowledge of the corresponding mode functions is also required. No channel information is required by the ED and the ASFD. The ASFD exploits the knowledge of an approximate relationship among different components of the signal vector to achieve a significantly better performance than the ED.

Theoretical derivations as well as simulation results indicate that, in the case of an HLA, the best performance is achieved by TSD, followed by ASFD, SD, and ED in that order. These results are consistent with the NMSEs associated with these detectors. In the case of a VLA the performance of TSD and ASFD are very close to one another, and either of them may perform better depending on the values of various parameters. For all the detectors, the performance of the HLA is significantly better than that of the VLA because (1) the HLA provides a better estimate of the bearing of the source, and (2) noise at the VLA is spatially correlated whereas it can be assumed to be uncorrelated in the case of an HLA. TSD, SD, and ED are detection strategies which can be used by both APS and AVS arrays. For a given detector and array size, the performance of an AVS array is significantly better than that of the APS array. The ASFD is a detection strategy that is unique to an AVS array.

The analysis presented in this paper is based on the assumption that the environmental noise is Gaussian. It is known that the assumption of Gaussianity is not valid in all environments. Therefore, it would be of interest to enlarge the scope of the analysis to include non-Gaussian/impulsive noise.

Acknowledgments: The authors would like to thank the anonymous reviewers and Mr. P. V. Nagesha for useful comments and suggestions. This work was partly supported by a grant from National Institute of Ocean Technology, Chennai, India, under the Ocean Acoustics Programme.

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