NATURAL GRADIENT-BASED ADAPTIVE ALGORITHMS FOR SPARSE UNDERWATER ACOUSTIC CHANNEL IDENTIFICATION

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Abstract: *Natural-gradient (NG) adaptive algorithms are known to be superior to stochastic-gradient (SG) algorithms when the channel to be identified exhibits a known Riemannian structure. In sparse channel identification, for example, the improved-proportionate normalized least-mean-square (IPNLMS) is a well known NG algorithm which outperforms the classical SG normalized least-mean square (NLMS). Apart from Riemannian geometry, the L_0 norm is an alternative way to describe channel sparseness. In this work, a new algorithm is proposed for sparse underwater acoustic (UWA) channel estimation by incorporating the Riemannian metric of the IPNLMS and an appropriately defined L_0 norm of the channel taps into the same cost function. To cope with the high Doppler frequencies often encountered in UWA channels, carrier-phase estimation is incorporated into the algorithm. Based on data recorded over a sparse acoustic link, the superior performance of the proposed algorithm to existing NG and SG algorithms is validated.*

Keywords: *Underwater acoustic communications, channel estimation, natural gradient, sparse adaptive filters, NRLS, RLS, IPNLMS, L_0 norm.*

1. INTRODUCTION

In short- and medium-range shallow water acoustic links, the channel impulse responses are typically of hundreds of taps. Moreover, these responses are highly time-varying and exhibit a sparse structure [1-2]. Stochastic-gradient (SG) adaptive filters, such as the normalized least-mean square (NLMS) or the recursive least-squares (RLS), not only converge slowly to their steady-state values but also these steady-state values are far from the optimum ones. Parameter (filter tap) adaptation for this class of filters is performed by minimizing a cost function based on the gradient descent method [3]. In sparse channels, however, the negative of the ordinary (Euclidean) gradient does not represent the steepest descent direction of the cost function and therefore any gradient descent method will perform poorly [4].

The natural gradient (NG) method computes the steepest descent direction of the target cost function if knowledge of the Riemannian structure of the parameter space is available. For sparse channels, the optimum filter parameters are expected to lie close to the coordinate axes. Hence, the parameter space close to the axes should be regarded as warped in the following sense: any direction orthogonal to the axes should be larger than the ordinary Euclidean distance[5]. Two NG-based sparse adaptive algorithms that are reviewed and compared below are the improved proportionate normalized least-mean-square (IPNLMS) and the natural RLS (NRLS) [6].

Motivated by the capability of the L_0 norm to describe channel sparseness, the authors in [7] proposed the IPNLMS-l_0 algorithm by replacing the L_1 norm with the L_0 norm of the filter parameters. A novel NLMS was proposed for real-valued channels in [8] by incorporating the L_0 norm into the NLMS cost function. Based on our previous results [9], we propose a better way to enhance the performance of the IPNLMS algorithm by constraining its cost function with an appropriately defined L_0 norm of the filter parameters. In addition, carrier-phase estimation is included in the algorithm. The validity of the proposed algorithm is demonstrated by comparing its performance with two SG-based algorithms (NLMS and RLS) and three NG-based algorithms (NRLS, IPNLMS, and IPNLMS-l_0) using experimental data recorded over a sparse short-range acoustic link in a shallow water area.

Notation and definitions: Column vectors (matrices) are denoted by boldface lowercase (uppercase) letters. Superscripts $(\cdot)'$ and $(\cdot)^*$ stand for Hermitian transpose and conjugate, respectively. The $N\times N$ identity matrix is denoted by $\mathbf{I}_N$. For a complex number x, the L_1-norm is defined as $|x|_1=|\mathrm{Re}\{x\}|+|\mathrm{Im}\{x\}|$ and the complex sign function is denoted as $\mathrm{csgn}(x)=\mathrm{sgn}(\mathrm{Re}\{x\})+j\mathrm{sgn}(\mathrm{Im}\{x\})$, where $\mathrm{sgn}(\cdot)$ stands for the sign function of a real number. For a K-tap complex vector $\mathbf{x}$, the L_0 norm is denoted as $\|\mathbf{x}\|_0$ and equals to the number of the non-zero entries of $\mathbf{x}$ and the L_1 norm is denoted as $\|\mathbf{x}\|_1=\sum_{i=0}^{K-1}|x_i|$, where $|x_i|=\sqrt{x_i x_i^*}$.

2. THE IPNLMS, IPNLMS-l_0, AND NRLS ALGORITHMS

The IPNLMS, NRLS and IPNLMS-l_0 algorithms have been derived for estimation of real-valued impulse responses encountered in network/acoustic echo cancelation applications. Here, we revisit these algorithms in the context of baseband (complex-valued) UWA channel estimation coupled with carrier-phase tracking. For the description of these algorithms, the following system model is used [10]:

$$y[n]=\sum_{k=0}^{K-1}h_k[n]x[n-k]e^{j\theta[n]}+w[n]=e^{j\theta[n]}\mathbf{h}[n]'\mathbf{x}[n]+w[n] \tag{1}$$

where $y[n]$ is the received baseband signal sampled at 2 samples/symbol, $\mathbf{h}[n]$ denotes the K-dimensional sparse impulse response at discrete time n, $\boldsymbol{x}[n]$ stands for the input signal, $\theta[n]$ is the residual carrier-phase occurring after imperfect Doppler compensation and/or mismatch between the transmitter and receiver sampling clocks, and $w[n]$ denotes the additive noise. Moreover, some important definitions that will be used below are the following:

$$\begin{aligned}
e[n]&=y[n]-\hat{\mathbf{h}}[n-1]'\mathbf{x}[n]e^{j\hat{\theta}[n-1]}=y[n]-\hat{\mathbf{h}}[n-1]'\mathbf{u}[n]\\
\mathbf{r}[n]&=\hat{\mathbf{h}}[n]-\hat{\mathbf{h}}[n-1]\\
\varepsilon[n]&=y[n]-\hat{\mathbf{h}}[n]'\mathbf{u}[n]=e[n]-\mathbf{r}[n]'\mathbf{u}[n]
\end{aligned} \tag{2}$$

where $\hat{\mathbf{h}}[n]$ and $\hat{\theta}[n]$ stand for the estimated impulse response (or equivalently the adaptive filter parameters) and carrier phase, respectively, $e[n]$ denotes the a priori prediction error of $y[n]$, $\mathbf{r}[n]$ is the channel update vector and $\varepsilon[n]$ stands for the a posterior prediction error of $y[n]$.

Before we describe the various algorithms, we first focus on $\hat{\theta}[n]$ estimation assuming knowledge of $\hat{\mathbf{h}}$[n]. This is accomplished via a second order phase locked loop (PLL) as follows[10]:

$$\hat{\theta}[n] = \hat{\theta}[n-1] + M_1\Phi[n] + M_2\sum_{m=0}^{n-1}\Phi[m], \quad \Phi[n] = -2\,\mathrm{Im}\left\{\hat{\mathbf{h}}[n]'\mathbf{u}[n]y[n]^*\right\} \tag{3}$$

where $\Phi[n]$ is the negative gradient of $|\varepsilon[n]|^2$ with respect to $\hat{\theta}[n-1]$ and M_1, M_2 are positive phase tracking parameters.

The IPNLMS algorithm [6] adjusts the filter parameters by using the Riemannian metric tensor $\mathbf{G}[n]$, a K×K diagonal positive definite matrix that describes the local curvature of the parameter space at $\hat{\mathbf{h}}[n]$. The channel update equation is given by

$$\hat{\mathbf{h}}[n] = \hat{\mathbf{h}}[n-1] + \mu\frac{e[n]^*\mathbf{G}[n-1]^{-1}\mathbf{u}[n]}{\mathbf{u}[n]'\mathbf{G}[n-1]^{-1}\mathbf{u}[n] + \delta\dfrac{1-\beta}{2K}}, \ \mu\in(0,1], \ \delta>0 \tag{4}$$

where μ is the step-size parameter and δ is a regularization parameter. The diagonal entries of $\mathbf{G}[n\text{-}1]^{-1}$ are given by

$$\frac{1-\beta}{2K} + (1+\beta)\frac{\left|\hat{h}_k[n-1]\right|}{2\left\|\hat{\mathbf{h}}[n-1]\right\|_1 + \varepsilon}, \ 0\le k\le K-1, \ \beta\in[-1,1] \tag{5}$$

where ε is a small positive constant (a typical value is 10^{-4}) to avoid division by zero when the algorithm is initialized and β is the parameter that controls the sparseness of the solution; larger β leads to more sparse solutions. Note that when β=-1, the IPNLMS reduces to the standard NLMS. In theory, $\mathbf{G}[n]$ does not assume a unique form provided that it remains positive definite (for instance, the L_1 norm could be replaced by other norms in Eq. (5)).

Similarly to IPNLMS, the IPNLMS-l_0 [7] tries to better exploit the channel sparseness by using $\|\hat{\mathbf{h}}[n]\|_0$ instead of $\|\hat{\mathbf{h}}[n]\|_1$. For complex channel parameters, $\|\hat{\mathbf{h}}[n]\|_0$ may be approximated by the differentiable function

$$\left\|\hat{\mathbf{h}}[n]\right\|_0 \cong \sum_{k=0}^{K-1} 1 - e^{-\eta\left|\hat{h}_k[n]\right|_1}, \ \eta>0 \tag{6}$$

where the parameter η depends on the range of values of the channel taps. In this case, the diagonal entries of $\mathbf{G}[n\text{-}1]^{-1}$ are expressed as

$$\frac{1-\beta}{2K}+(1+\beta)\frac{1-e^{-\eta\left|\hat{h}_k[n-1]\right|_1}}{2\sum_{k=0}^{K-1}1-e^{-\eta\left|\hat{h}_k[n-1]\right|_1}+\varepsilon},\ 0\le k\le K-1,\ \beta\in[-1,1] \tag{7}$$

Using the above Riemannian metric tensor, the NRLS algorithm [6] is summarized by

$$\begin{aligned}
&\hat{\mathbf{h}}[n]=\hat{\mathbf{h}}[n-1]+\mathbf{G}[n-1]^{-1/2}\mathbf{R}[n]^{-1}\mathbf{G}[n-1]^{-1/2}\mathbf{u}[n]e[n]^*\\
&\mathbf{s}[n]=\mathbf{G}[n-1]^{-1/2}\mathbf{u}[n],\\
&\mathbf{R}[n]^{-1}=\lambda^{-1}\mathbf{R}[n-1]^{-1}+\frac{\lambda^{-2}\mathbf{R}[n-1]^{-1}\mathbf{s}[n]\mathbf{s}[n]'\mathbf{R}[n-1]^{-1}}{1+\lambda^{-1}\mathbf{s}[n]'\mathbf{R}[n-1]^{-1}\mathbf{s}[n]},\quad \mathbf{R}[0]=\left(\frac{1-\beta}{2K}\delta\right)\mathbf{I}_K
\end{aligned} \tag{8}$$

where λ is the forgetting factor and $\mathbf{R}[n]$ is an estimate of the input signal autocorrelation matrix. The update of NRLS can be explained from the fact that $\mathbf{G}[n]^{-1/2}\mathbf{R}[n]^{-1}\mathbf{G}[n]^{-1/2}$ is a positive definite matrix and so the vector $-\mathbf{G}[n]^{-1/2}\mathbf{R}[n]^{-1}\mathbf{G}[n]^{-1/2}\mathbf{u}[\mathrm{n}]\mathrm{e}[\mathrm{n}]^*$ becomes a gradient descent direction of the NRLS cost function. For a sampling rate of 2 samples/symbol, NRLS performance becomes very sensitive to the condition number of $\mathbf{R}[n]$. To circumvent this issue, we run two independent NRLS for the even and odd filter parameters and combine the results. Note again that when β=-1, NRLS reduces to the standard RLS.

3. IPNLMS WITH L_0 NORM PENALTY

An instructive way to derive the IPNLMS algorithm is by minimizing the cost function

$$J_1[n]=\mathbf{r}[n]'\mathbf{G}[n-1]\mathbf{r}[n]+\mu\left|\varepsilon[n]\right|^2,\ \mu>0 \tag{9}$$

with respect to $\mathbf{r}[n]^*$ [5]. Since the L_0 norm represents the standard description of a sparse channel, constraining $J_1[n]$ with $\|\hat{\mathbf{h}}[n]\|_0$ would accelerate the convergence rate and decrease the misadjustment of the channel estimator. Towards this end, we minimize the following cost function:

$$J_2[n]=\mathbf{r}[n]'\mathbf{G}[n-1]\mathbf{r}[n]+\mu\left|\varepsilon[n]\right|^2+\alpha\left(\sum_{k=0}^{K-1}1-e^{-\eta\left|\hat{h}_k[n]\right|_1}\right),\ \mu,\alpha>0\,. \tag{10}$$

Note that $J_2[n]$ is not a convex function and thus, the algorithm could stall at a local minimum. However, based on experimental data processing from various channels, if α is chosen small enough then the algorithm always converges to meaningful results (as we will show below).

To derive the new algorithm, we employ the complex gradient operator defined in Appendix B of [3]. Thus, taking complex gradients with respect to $\mathbf{r}[n]^*$ we have:

$$\begin{aligned}
&\nabla\big(\mathbf{r}[n]'\mathbf{G}[n-1]\mathbf{r}[n]\big)=\mathbf{G}[n-1]\mathbf{r}[n]\\
&\nabla\left(\mu\left|\varepsilon[n]\right|^2\right)=\mu\nabla\big(\varepsilon[n]\big)\varepsilon[n]^*+\mu\underbrace{\nabla\big(\varepsilon[n]^*\big)}_{=0}\varepsilon[n]=-\mu\mathbf{u}[n]\big(e[n]^*-\mathbf{u}[n]'\mathbf{r}[n]\big)
\end{aligned} \tag{11}$$

Moreover, taking the derivative of the L_0 penalty term in Eq. (10) with respect to $r_k[n]^*$ we have

$$v_k[n] = \frac{\partial\left(\alpha \sum_{k=0}^{K-1} 1 - e^{-\eta \left|\hat{h}_k[n]\right|_1}\right)}{\partial r_k[n]^*} = \frac{1}{2}\alpha \cdot \eta \cdot \mathrm{csgn}\left(\hat{h}_k[n]\right) \cdot e^{-\eta \left|\hat{h}_k[n]\right|_1},\ k = 0, \ldots, K-1 \tag{12}$$

Setting the gradient of $J_2[n]$ equal to zero, we finally have the vector-form equation

$$\left(\mathbf{G}[n-1] + \mu\, \mathbf{u}[n]\mathbf{u}[n]'\right)\mathbf{r}[n] + \mathbf{v}[n] = \mu\, \mathbf{u}[n] e[n]^* \tag{13}$$

where the components of the vector $\mathbf{v}[n]$ are given in Eq. (12). From Eq. (13), we note that is rather tedious to solve for $\mathbf{r}[n]$ since $\mathbf{v}[n]$ depends on $\hat{\mathbf{h}}[n]$ in a non-linear fashion. It is plausible, however, to assume that $\mathbf{v}[n] \approx \mathbf{v}[n\text{-}1]$ at steady-state and so by using the matrix inversion lemma, the channel update equations of the proposed algorithm are:

$$\begin{aligned} &\mathbf{b}[n] = \mathbf{G}[n-1]^{-1}\mathbf{u}[n] \\ &\hat{\mathbf{h}}[n] = \hat{\mathbf{h}}[n-1] + \frac{\mu\, e[n]^* \mathbf{b}[n]}{1 + \mu\, \mathbf{u}[n]'\mathbf{b}[n]} - \underbrace{\left(\mathbf{G}[n-1]^{-1} - \frac{\mu\, \mathbf{b}[n]\mathbf{b}[n]'}{1 + \mu\, \mathbf{u}[n]'\mathbf{b}[n]}\right)\mathbf{v}[n-1]}_{\mathbf{d}[n]} \end{aligned} \tag{14}$$

The algorithm described by Eq. (3) and (14) will be called L_0-IPNLMS hereafter. From Eq.(14) note that the $K \times 1$ vector $\mathbf{d}$[n] is the analog of the *zero attractor* (forces inactive channel taps to remain close to zero value) as mentioned in [8]. A good rule of thumb for the choice of η is $5/q$, where q is the smallest non-zero channel tap amplitude. The algorithm is initialized for $\hat{h}_k[0]=0$, $k=0,\ldots,K$-1 and $\hat{\theta}[0] = 0$.

4. EXPERIMENTAL RESULTS

In this section, a comparison between L_0-IPNLMS, IPNLMS, IPNLMS-l_0, NRLS, RLS, and NLMS in terms of steady-state misadjustment and channel tracking is reported. The channel to be estimated was recorded during the Focused Acoustic Fields (FAF'05) experiment off the coast of Pianosa, Italy, in July 22nd, 2005. The transmitter was attached on the hull of the research vessel Leonardo 4.5m below the sea level while the receiver was a moving autonomous underwater vehicle (AUV). The ocean depth was 85m and the range of the link was approximately 700m. The transmitted signal was a 6250bps-rate, QPSK-modulated m-sequence with 12kHz carrier frequency. Snapshots of the estimated channel impulse response amplitude are given in Fig.1(a). Note that the channel delay spread is approximately 36ms, corresponding to a channel length of 454 taps when the received signal is sampled at 2samples/symbol.

All the algorithm parameters, used for the results below, are given in Table 1. To ensure a fair comparison between L_0-IPNLMS, IPNLMS, and IPNLMS-l_0, we chose their parameters such that all algorithms show the same initial convergence rate. The parameters of NLMS, RLS, and NRLS were chosen for optimum convergence and steady-state performance.

Fig.1(b)-(e) show the learning curves of each algorithm for various values of the sparseness parameter β. Clearly, all the sparse algorithms outperform the standard NLMS and RLS corroborating the fact that the channel parameter space is better described by a Riemannian metric tensor. Moreover, all the IPNLMS-type of algorithms exhibit superior performance than NRLS for all β. Finally, note that L_0-IPNLMS outperforms both IPNLMS and IPNLMS-l_0 when β=*-0.5, 0, 0.5* while for β=*0.9* all three algorithms have the same performance. Thus, L_0-IPNLMS is a better choice since it is more robust to any mismatch in the parameter β.

Fig.1(f) illustrates the carrier-phase estimation for each algorithm for $\beta=0.5$. As expected, different algorithms give different carrier-phase estimates but the carrier-phase of the L_0-IPNLMS is the most reliable since it corresponds to the best channel estimate.

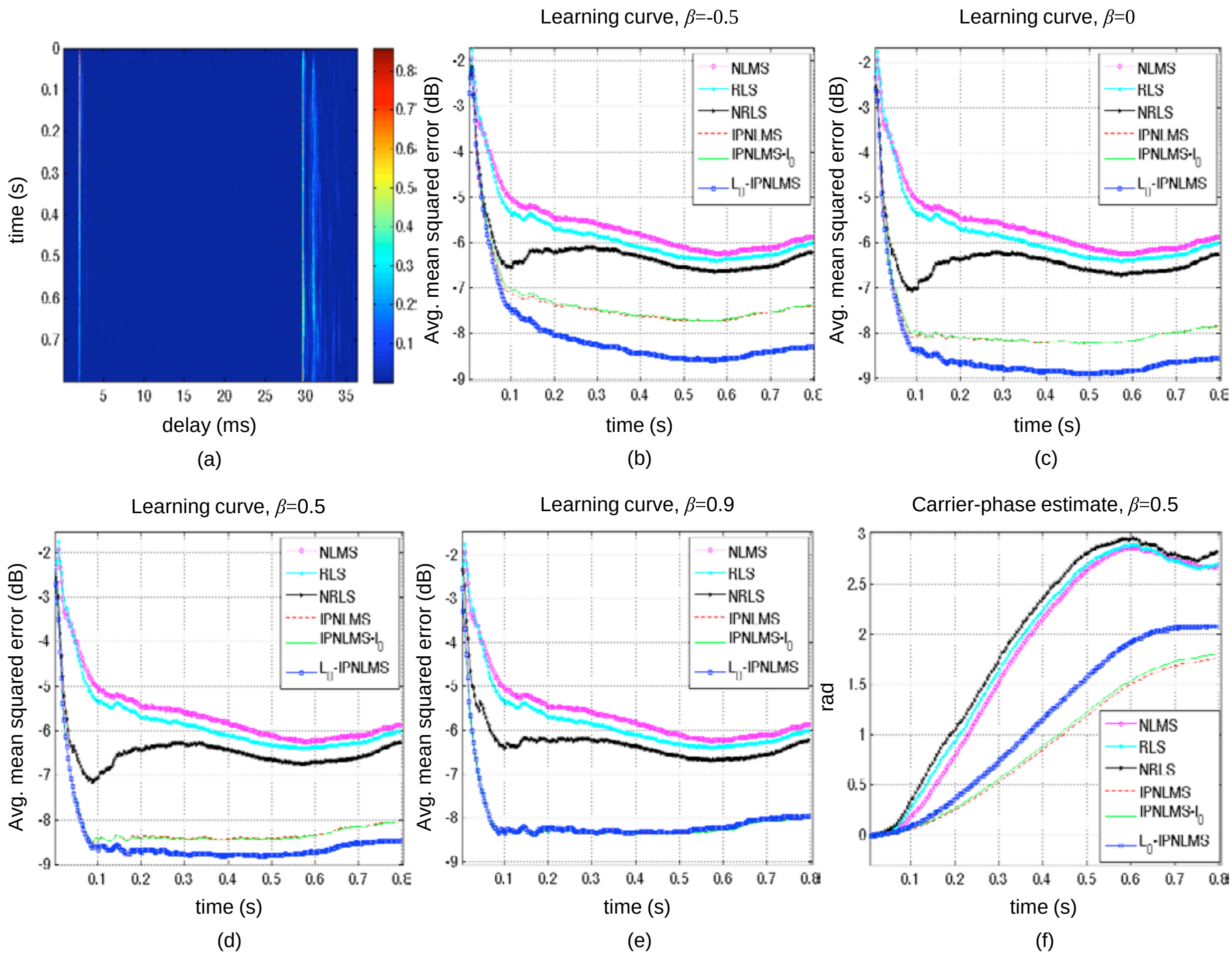

Fig.1: (a) snapshots of the time-varying channel impulse computed at a rate of 6250Hz. The horizontal axis represents multipath delay, the vertical axis represents absolute time and the colorbar represents the amplitude; (b)-(e) learning curves for all algorithms for various β; (f) carrier-phase estimates for all algorithms when $\beta=0.5$.

Algorithm parameters	λ	δ	μ	α	η	M_1	M_2
NLMS	-	0.5	0.8	-	-	$5{\cdot}10^{-3}$	$5{\cdot}10^{-5}$
RLS	0.993	0.02	-	-	-	$5{\cdot}10^{-3}$	$5{\cdot}10^{-5}$
NRLS	0.996	0.02	-	-	-	$5{\cdot}10^{-3}$	$5{\cdot}10^{-5}$
IPNLMS	-	0.5	0.8	-	-	$5{\cdot}10^{-3}$	$5{\cdot}10^{-5}$
IPNLMS-l_0	-	0.5	0.8	-	0.5	$5{\cdot}10^{-3}$	$5{\cdot}10^{-5}$
L_0-IPNLMS	-	-	3.5	10^{-3}	20	$5{\cdot}10^{-3}$	$5{\cdot}10^{-5}$

Table 1: Parameters for all algorithms.

5. CONCLUSIONS

We reviewed and investigated the performance of three NG-based (IPNLMS, IPNLMS-l_0, and NRLS) and two SG-based (RLS and NLMS) adaptive algorithms for sparse underwater acoustic (UWA) channel identification. The clear superiority of the NG-based algorithms was demonstrated by using signals recorded over a short-range shallow water channel. This result validates that sparse UWA channels exhibit an underlying Riemannian structure that can be exploited in adaptive filtering operations. Moreover, NRLS showed worse performance than IPNLMS and IPNLMS-l_0 for this particular channel. Finally, a new algorithm, the L_0-IPNLMS algorithm, was derived by penalizing the cost function of the IPNLMS with an appropriately defined L_0 norm of the filter parameters. Using the same experimental channel, the L_0-IPNLMS showed an improved performance compared to IPNLMS and IPNLMS-l_0 for different values of the sparseness parameter β.

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