Ocean Acoustic Propagation Modeling Using Scientific Machine Learning

Li Kexin and Mandar Chitre

ARL, Tropical Marine Science Institute and Department of Electrical and Computer Engineering, National University of Singapore.

***Abstract*—Ocean acoustic propagation models support a wide range of oceanic applications. Conventional approaches to modeling of acoustic propagation in an ocean environment usually require full environmental knowledge. Unfortunately, such knowledge is very difficult to accurately acquire. Measuring an adequate amount of accurate acoustic data required by conventional data-driven techniques is also difficult and expensive. We propose an ocean acoustic propagation modeling approach that requires very limited environmental knowledge, and small amount of acoustic measurements. By applying the concept of scientific machine learning, we can embed known scientific domain knowledge into data-driven machine learning techniques. One can efficiently learn to predict acoustic fields from much lesser training data compared with conventional data-driven techniques.**

I. Introduction

Acoustics has been extensively used in various oceanic applications, including but not limited to acoustic source localization [1], acoustic communication channel characterization [2], acoustic monitoring of marine biota [3], etc. A large number of such applications heavily rely on ocean acoustic propagation models. For example, an acoustic propagation model helps to provide a good estimate of underwater acoustic communication system performance within area of interest (AOI). An ocean acoustic propagation model is also an effective tool to predict and analyze acoustic signal propagation in an ocean environment. Ocean environments are geographically complicated and exhibit significant spatial and temporal variability. Accurate modeling acoustic propagation in an ocean environment is hard but essential.

Acoustic propagation is governed by the acoustic wave equation. Closed-form solutions to the wave equation are analytically intractable in general. Most commonly used ocean acoustic propagation models are derived from the wave equation with approximations. Ray models [4] and normal mode models [5] are two common examples. A good understanding of environmental knowledge is the key to modeling acoustic propagation accurately. Some of the common environmental parameters required in propagation models include sound speed profile, water depth, bathymetry, sea surface and seabed properties, etc. Unfortunately, such environmental knowledge is hard to accurately obtain.

Machine learning (ML) allows computers to automatically learn from data [6]. Neural networks (NN), a specific type of machine learning technique, are able to approximate arbitrary mathematical functions, according to the universal approximation theorem [7]. NN thus have the ability to predict the acoustic pressure field at any location, assuming sufficient amount of training data are available. Although researchers in oceanic engineering have successfully used conventional ML approaches to solve various underwater problems, they are severely limited by the availability of adequate quantity of high quality data, as the ocean environment tends to be noisy, and expensive to operate in.

How to accurately model acoustic propagation with limited environmental knowledge and a limited amount of data is the problem we address in this paper. Scientific machine learning (SciML) techniques could help alleviate this problem, as SciML leverages our understanding of physics, enabling the model to learn with much lesser data than conventional ML. SciML is an emerging technique which embeds scientific domain knowledge in data-driven ML [8]. It allows ML algorithms learn from very little data, or from noisy data, by using scientific domain knowledge to guide the learning. It also brings interpretability to trained model parameters. In this paper, we propose an ocean acoustic propagation modeling approach using SciML and validate its feasibility through simulation studies.

The rest of the paper is organized as follows. The problem formulation and proposed method are described in Section II. Section III presents simulation studies of proposed method with two conventional data-driven techniques and discusses our observations. Finally, Section IV concludes the paper. Table I lists symbols used in this paper.

II. Problem Statement and Method

To model ocean acoustic propagation using SciML, we start with the acoustic wave equation as our underlying scientific domain knowledge. The wave equation is a second-order linear partial differential equation which describes propagation of waves [9]. The acoustic wave equation is expressed as [4]:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p, \tag{1}$$

where p denotes acoustic pressure, t represents time, and c is sound speed. A feasible solution to (1) can be written as:

$$p(\boldsymbol{r}, t) = \bar{p}(\boldsymbol{r}) e^{i\omega t}, \tag{2}$$

TABLE I
SYMBOLS USED IN THE PAPER

Symbol	Description
t	Time
c	Sound speed
ω	Angular velocity
$\boldsymbol{r}$	Spatial coordinate of receiver
p	Acoustic pressure
$p(\boldsymbol{r}, t)$	Acoustic signal at location $\boldsymbol{r}$ and time t
$\bar{p}(\boldsymbol{r})$	Time-invariant acoustic signal at location $\boldsymbol{r}$
k	Wave number
$\boldsymbol{k}$	Wave propagation vector
A_j	Amplitude of j^{th} incoming wave
ϕ_j	Phase of j^{th} incoming wave
n	Number of incoming waves
$\boldsymbol{A}$	Collection of amplitude of n incoming waves
$\boldsymbol{\phi}$	Collection of phase of n incoming waves
$\hat{\boldsymbol{k}}$	Collection of wave propagation vector of n incoming waves
$\boldsymbol{y}_{\text{train}}$	Collection of measured acoustic fields in training data
$\boldsymbol{r}_{\text{train}}$	Collection of measurement locations in training data
α	Coefficient of amplitude penalty term
$\boldsymbol{\Theta}$	Unknown scientific model parameters
$\boldsymbol{\Psi}$	Collection of training data

where $\boldsymbol{r}$ refers to spatial coordinate, $\bar{p}(\boldsymbol{r})$ is time-invariant acoustic signal at location $\boldsymbol{r}$, and ω denotes angular frequency. We substitute (2) back into (1):

$$
\begin{aligned}
-\omega^2 \bar{p}(\boldsymbol{r}) e^{i\omega t} &= c^2 e^{i\omega t} \nabla^2 \bar{p} \\
\therefore \ k^2 \bar{p}(\boldsymbol{r}) + \nabla^2 \bar{p} &= 0,
\end{aligned}
\tag{3}
$$

where $k = \frac{\omega}{c}$ is called the wave number. A solution to (3) can be written as:

$$\bar{p}(\boldsymbol{r}) = A e^{i\phi} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{4}$$

where A and ϕ refer to amplitude and phase of a wave, and $\boldsymbol{k}$ denotes wave propagation vector. $\boldsymbol{k}$ can be an arbitrary vector satisfying:

$$\|\boldsymbol{k}\|_2 = k. \tag{5}$$

Any function of the form (4) solves the wave equation. The sum of n feasible solutions must also solve the wave equation due to linearity of the wave equation. Therefore, we can model the acoustic field $\bar{p}$ at receiver location $\boldsymbol{r}$ as the superposition of (4):

$$\bar{p}(\boldsymbol{r}) = \sum_{j=1}^{n} A_j e^{i\phi_j} e^{i\boldsymbol{k}_j\cdot\boldsymbol{r}}. \tag{6}$$

The intuition behind (6) is that we can visualize the acoustic field at any receiver location $\boldsymbol{r}$ as the superposition of multiple incoming waves with different amplitudes, phases and propagation directions (Fig. 1).

The exact values of A_j, ϕ_j and $\boldsymbol{k}_j$ for individual incoming waves are unknown. Fortunately, we can make use of ML tools, such as NN, as a powerful aid to find the unknown parameters. If we treat each of the terms in (6) as a neuron in a NN (Fig. 2), we can guarantee that the NN obeys the

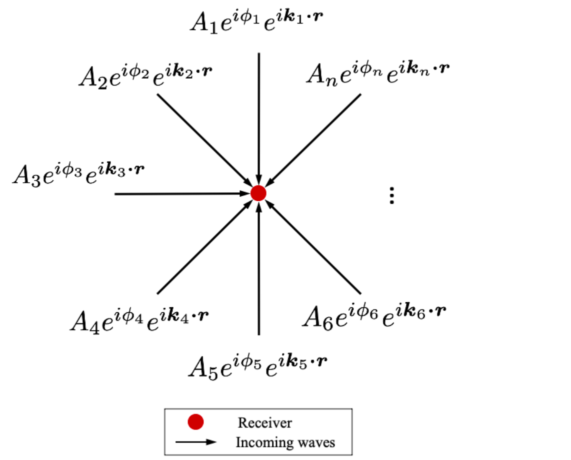

Fig. 1: Superposition of multiple incoming waves at a receiver location.

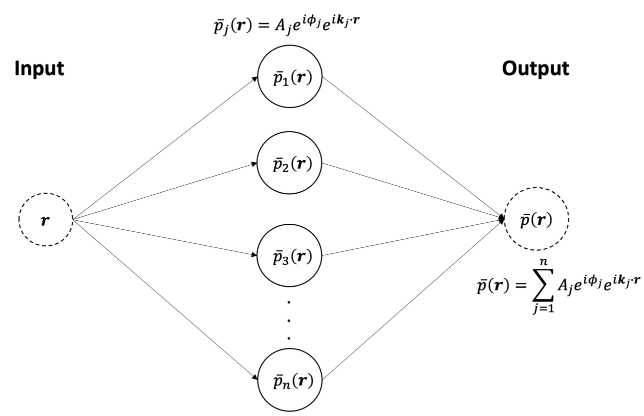

Fig. 2: Illustration of the RBNN model.

wave equation. Therefore, by design, our NN solves the wave equation. This is the beauty of SciML. Here, we name our ocean acoustic propagation modeling approach as *ray basis NN* (RBNN).

Reducing the number and search space of unknown parameters in scientific model can greatly relax the amount of training data required. We assume sound speed in the AOI and source frequency to be known, so we can calculate the wave number k in advance. The unknown RBNN model parameters to train are denoted as a tuple $\boldsymbol{\Theta} = \left(\hat{\boldsymbol{k}}, \boldsymbol{A}, \boldsymbol{\phi}\right)$, where $\hat{\boldsymbol{k}} = (\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3 \ldots \boldsymbol{k}_n)$, $\boldsymbol{A} = (A_1, A_2, A_3 \ldots A_n)$ and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3 \ldots \phi_n)$.

In order to find the best values of the unknown A_j, ϕ_j and $\boldsymbol{k}_j$ in each neuron, we iteratively train our RBNN model given training data $\boldsymbol{\Psi} = (\boldsymbol{y}_{\text{train}}, \boldsymbol{r}_{\text{train}})$ through back-propagation to minimize following loss function:

$$\mathcal{L}(\boldsymbol{\Psi}, \boldsymbol{\Theta}) = \|\text{RBNN}(\boldsymbol{r}_{\text{train}}, \boldsymbol{\Theta}) - \boldsymbol{y}_{\text{train}}\|_2^2 + \alpha \|\boldsymbol{A}\|_1^2, \tag{7}$$

where $\boldsymbol{y}_{\text{train}}$ contains acoustic measurements, and $\boldsymbol{r}_{\text{train}}$ represents corresponding measurement locations in training data, α

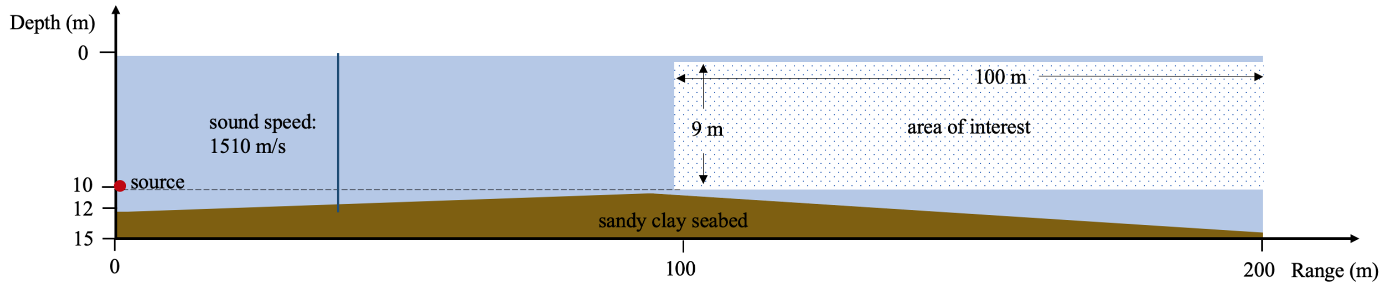

Fig. 3: Schematic showing 2D environment used in simulation study.

is a hyper-parameter that controls the weight of the penalty term. The L1-norm penalty term for amplitude of $\boldsymbol{A}$ is to encourage sparse solutions, incorporating the prior that we typically have a small number of discrete arrivals in a typical shallow water environment. This gives the model freedom to decide on the proper number of significant contributing arrivals during the training process, as the choice of n may be arbitrary.

III. Results and Discussion

A. Simulation studies

We demonstrate our idea in a simulated 2D AOI of 100 m $\times$ 9 m, in an environment with constant sound speed and range-dependent bathymetry as shown in Fig. 3. A 5 kHz continuous wave acoustic source is located at 10 m depth and 100 m from the AOI.

To validate the effectiveness of our proposed approach, we compare acoustic field patterns reconstructed by RBNN with a conventional data-driven ML approach and a Gaussian process regression (GPR) method. Conventional data-driven ML and GPR methods are commonly used to perform regression, interpolation and extrapolation based on training data. When conventional propagation models can not be used due to lack of necessary environmental knowledge, conventional NN and GPR models can be used as alternatives to predict acoustic field patterns.

We use a ray acoustic propagation model to generate 300 acoustic measurements at random locations within the AOI. 270 data out of 300 data are used as training data to train the three models. For RBNN and conventional NN models, we further provide 30 data as validation data to facilitate the training process. Validation data allow us to halt the training process at the right time to avoid overfitting (early stopping). Fig. 4a shows the locations of training data and validation data on top of ground-truth field pattern within the AOI. We set the number of neurons in our RBNN model as 100, i.e., we assume that the received signal comprises of no more than 100 incoming arrivals. We set hyper-parameter α in (7) as 0.01. For conventional ML approach, we build a 3 layer NN with total 3291 neurons and use Rectified Linear Unit (ReLU) as activation function. The choice of the number of neurons in the conventional NN is discussed in Section III-B. Input data to the conventional NN are normalized within the range of -1 to 1. It is important to note that choices of input normalization, model structure and activation function need to be carefully handled to allow conventional NN achieve good performance. For GPR model, we use a composite kernel which is the sum of a Matérn $\frac{5}{2}$ automatic relevance determination (ARD) kernel and a squared exponential isotropic kernel.

Simulation allows us to statistically evaluate the performance of the reconstructed field pattern by each of the three methods. We generate 90 000 test data using the ray model with a resolution of 0.1 m in range and depth within the AOI in order to carry out a rigorous evaluation.

When environmental parameters required in scientific models are unknown, it is impossible to reconstruct the entire field using forward propagation models given training data only. Even though inverse methods can be used to estimate environmental parameters from data in some cases, they are generally computationally infeasible [10], [11].

TABLE II
Model Complexity and RMS Test Error of Estimated Acoustic Field by the Three Methods

Method	Number of model parameters	RMS test error (dB)
RBNN	100	2.509
Conventional NN	3291	3.489
GPR	540	3.080

B. Discussion

The reconstructed fields by the three approaches are shown in Fig. 4. We observe that all of the three approaches are able to recover the rough field pattern. However, the fields reconstructed by conventional NN and GPR have very limited fidelity. The left side of the field reconstructed by the conventional NN is distorted. Although the RBNN model we implemented is much simpler than the other two models, our RBNN approach is able to learn the unknown model parameters from limited data and reconstruct the field pattern that is close to the ground-truth field. The same observations can be made from the number of model parameters and root mean square (RMS) error of the acoustic fields estimated by the three approaches as shown in Table II.

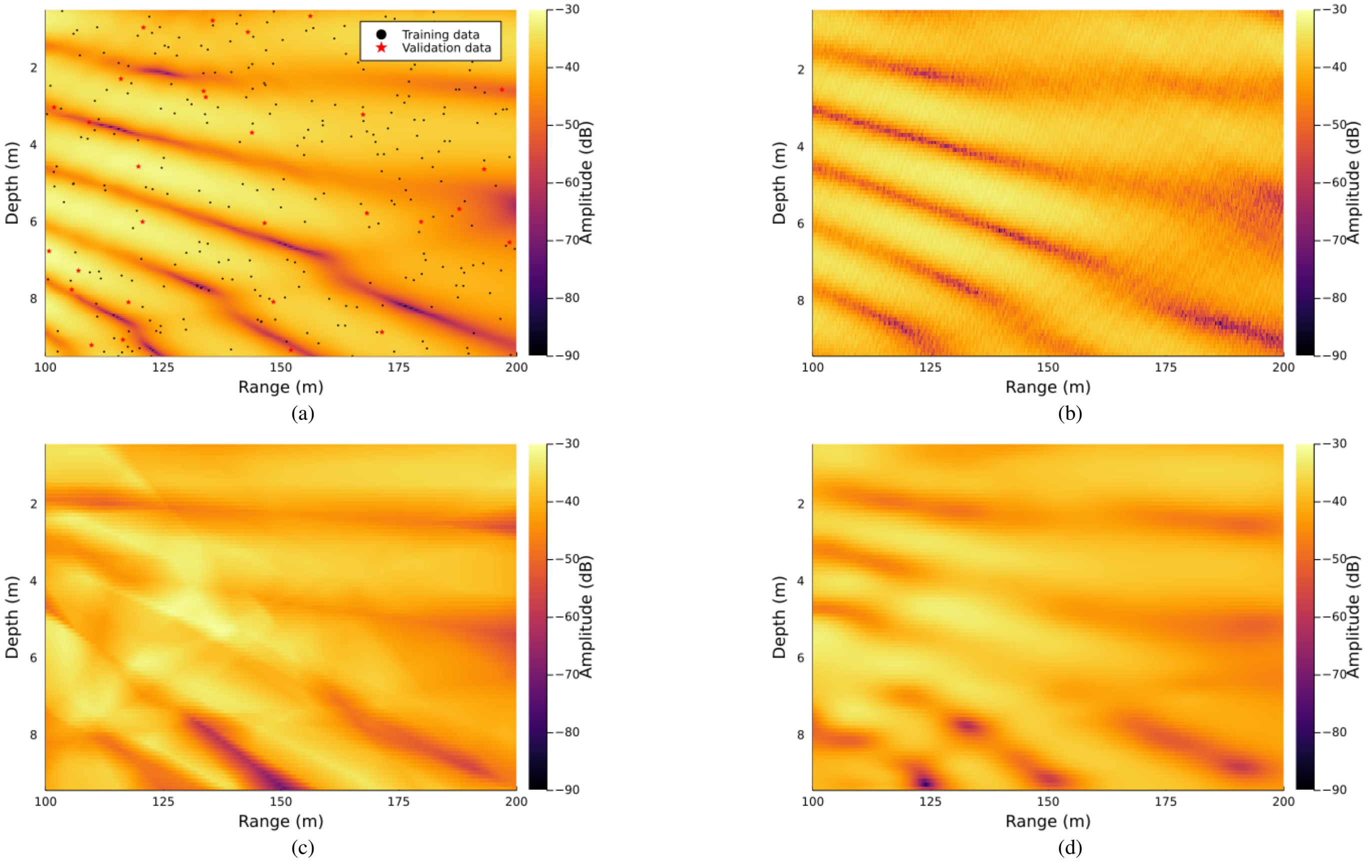

Fig. 4: Acoustic field of the AOI. (a) Ground-truth field by the ray acoustic propagation model. (b) Reconstructed field by the RBNN. (c) Reconstructed field by the conventional NN. (d) Reconstructed field by the GPR.

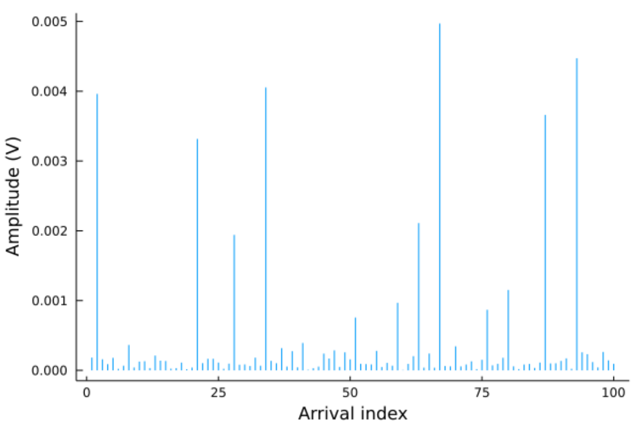

Fig. 5: Amplitude of 100 incoming arrivals in trained RBNN model.

Fig. 5 shows the amplitude of 100 incoming arrivals in the trained RBNN model. It clearly shows that there are only a few significant contributing arrivals whereas the remaining arrivals have relatively tiny amplitude. This result is in line with our expectation as arrivals with large number of bottom reflections experience strong attenuation.

The model complexity of the conventional NN used in the simulation studies is purposely chosen to be much higher than our RBNN model, as the conventional NN with lower model complexity fails to recover the main features of acoustic field. Although NN can be seen as a global function approximator, it normally requires a more complex model and a larger amount of accurate training data to efficiently eliminate ambiguities when the function it aims to estimate is complicated. By incorporating scientific domain knowledge, RBNN is able to reduce the number of unknowns and search space. It therefore has more advantages to learn from lesser data compared with conventional NN.

IV. Conclusion

SciML is a new concept that has been drawing increasing attention recently. In this paper, we applied this concept to solve the wave equation with limited data. While we formulated the solution to the wave equation, in this paper, in terms of a ray expansion, the same approach can be used with a normal mode expansion in case of environment with range-independent bathymetry but a non iso-velocity sound speed profile and low frequency.

Simulation studies show our proposed RBNN method outperforms the conventional NN and GPR methods in acoustic field reconstruction. Future research in propagation modeling through SciML will focus on dealing with more realistic scenarios, such as considering position error in acoustic measurements and benchmarking in 3D AOI.

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