# Physics-aided Data-driven Modal Ocean Acoustic Propagation Modeling

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***Abstract*—Modeling acoustic propagation accurately is vital to numerous oceanic applications. However, physics-based acoustic propagation models require accurate prior environmental knowledge, which is often hard and expensive to acquire. Such a requirement can be relaxed by using data-driven machine learning techniques. Unfortunately, they are data-hungry and extrapolate poorly. We can potentially train machine learning algorithms with a lot less data and get them to extrapolate well by imposing constraints based on our knowledge of acoustic propagation. Our previous work proposed a physics-based data-aided high-frequency acoustic propagation modeling recipe based on the ray theory to do precisely this. The promising results obtained motivate us to further tailor the recipe for low-frequency applications. The theory of normal modes tends to be a more appropriate choice to model acoustic propagation at low frequencies. In this paper, we incorporate a modal acoustic propagation model in the structure of a neural network. We demonstrate the superiority of such an algorithm in estimating acoustic field, as compared with conventional data-driven machine learning techniques. We also show that this technique allows us to extract information about the environment, such as estimating the sound speed profile from acoustic observations.**

***Index Terms*—Normal mode, acoustic propagation model, data-aided acoustic modeling, SciML.**

## I. Introduction

Understanding acoustic propagation in the ocean is crucial for numerous applications, such as communication channel estimation [1], underwater source localization [2] and geo-acoustic inversion [3]. Oceans offer rich multipath environments that exhibit complicated constructive and destructive interference patterns. Conventional ocean acoustic propagation models solve the acoustic wave equation [4] using various mathematical techniques and simplifying approximations. While conventional models have matured over the past few decades, a key limitation for their effective use is the requirement of having full and accurate prior environmental knowledge. Accurately measuring required environmental parameters such as sound speed profile (SSP), seabed properties and sea surface properties, is often hard and expensive in practice.

Advances in data-driven machine learning (ML) resolve many problems that can not be addressed by conventional models. Modeling acoustic propagation through data-driven ML approaches does not need environmental knowledge. However, classical ML techniques are data-hungry and extrapolate poorly, and therefore poorly suited for most oceanic applications where data is difficult to obtain and consequently sparse. Recently, a synergetic strategy that embeds underlying domain knowledge into data-driven ML has emerged to handle such a dichotomy. This emerging technique is in the field of scientific machine learning (SciML) [5], [6]. Physics-informed neural networks (PINNs) [7], a popular strategy in SciML, encode scientific domain knowledge in the form of partial differential equations (PDEs). The PDEs are added as additional regularization terms in standard loss functions. There are a few recent works that preliminarily demonstrate the feasibilities of PINNs in learning solutions to the acoustic wave equation [8]–[11]. However, the use of SciML in the context of ocean acoustic modeling has not yet been extensively explored.

Augmenting the loss function is one of the possible strategies in SciML to inform a neural network (NN) by underlying domain knowledge, but is not the only effective one. Embedding the domain knowledge of acoustic propagation in structures of NNs, and using ML algorithms to train the NNs to solve the simplified wave equation also yields a promising strategy. Our previous work [12] extensively investigates this idea in high-frequency acoustic propagation modeling problems based on the ray theory [13]. It demonstrates the applicability and superiority of the proposed ray-based data-aided acoustic propagation modeling recipe in various case studies by benchmarking against classical ML techniques.

Ray theory applies a high-frequency approximation to solve the acoustic wave equation analytically [13], and is therefore not very accurate a low frequencies where the approximation is invalid. Normal mode models [14] are suitable and efficient alternatives for low-frequency oceanic applications. In this paper, we tailor the propagation modeling recipe based on the normal mode theory. We illustrate the proposed mode-based modeling framework in various scenarios and demonstrate its use in ocean acoustic field estimation problems and a SSP inversion problem. The proposed framework is flexible to incorporate any known domain knowledge, and can also embed smaller standard NNs to model unknown parameters. Moreover, it is data-efficient and generalizes well as compared to classical ML models.

## II. Method

### A. Normal mode theory

The acoustic wave equation is a second-order partial differential equation that describes acoustic propagation [13]:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p, \tag{1}$$

where $p$ denotes acoustic pressure, $t$ represents time and $c$ is sound speed.

We consider an acoustic propagation modeling problem due to a point source in a horizontally stratified (range-independent) two-dimensional (2D) ocean waveguide. A harmonic wave is a feasible solution to (1):

$$p(r,z,t)=\bar{p}(r,z)e^{i\omega t}, \tag{2}$$

where $\bar{p}(r,z)$ is complex pressure amplitude at a location with range $r$ and depth $z$, and $\omega=2\pi f$ denotes angular frequency. Substituting (2) back into (1) leads to the Helmholtz equation [4]:

$$k^2(z)\bar{p}(r,z)+\nabla^2\bar{p}(r,z)=0, \tag{3}$$

where $k(z)=\frac{\omega}{c(z)}$ represents wavenumber at depth $z$.

Classical normal mode models apply the separation of variables [15] to express acoustic field at location $(r,z)$ as a combination of a depth-dependent term and a range-dependent term:

$$\bar{p}(r,z)=\Psi(z)\Phi(r). \tag{4}$$

We substitute (4) into (3). After rearranging and simplifying, we obtain the modal equation [14]:

$$\rho(z)\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\rho(z)}\frac{\mathrm{d}\Psi(z)}{\mathrm{d}z}\right)+k_{\mathrm{z}}^2(z)\Psi(z)=0, \tag{5}$$

where

$$k_{\mathrm{z}}^2(z)=k^2(z)-k_{\mathrm{r}}^2, \tag{6}$$

and $\rho(z)$ is density and $k_{\mathrm{r}}$ represents horizontal wavenumber.

The modal equation derived in (5) is in the form of a classical Sturm-Liouville eigenvalue problem [16]. Theoretically, there are an infinite number of distinct mode solutions ($\Psi(z)$ and $k_{\mathrm{r}}$) to the modal equation (5). Normalized mode solutions form a complete set so that solutions to the wave equation can be represented as an infinite sum of the normal modes:

$$\bar{p}(r,z)=\sum_{m=1}^{\infty}\Psi_m(z)\Phi_m(r), \tag{7}$$

where $m$ denotes $m^{\text{th}}$ mode. The range-dependent term $\Phi_m(r)$ has a standard form in terms of the Hankel function [13]:

$$\Phi_m(r)=\frac{i}{4\rho(z_s)}\Psi_m(z_{\mathrm{s}})H_0^{(1,2)}(k_{\mathrm{r}m}r), \tag{8}$$

where $z_{\mathrm{s}}$ denotes source depth and $H_0^{(1,2)}$ refers to the Hankel function of first or second kind. The choice depends on radiation conditions. We adopt the Hankel function of the first kind since we assume energy is radiating outwards as $r$ approaches $\infty$. The asymptotic approximation to the Hankel function is often used in literature and (8) is approximated by:

$$\Phi_m(r)\approx\frac{i}{\rho(z_{\mathrm{s}})\sqrt{8\pi r}}e^{-i\frac{\pi}{4}}\Psi_m(z_{\mathrm{s}})\frac{e^{ik_{\mathrm{r}m}r}}{\sqrt{k_{\mathrm{r}m}}}. \tag{9}$$

Popular normal mode models, such as Kraken [17], seek all significant eigenfunction solutions $\Psi_m(z)$ and corresponding eigenvalues $k_{rm}$ to the modal equation (5) while satisfying boundary conditions and environment setup.

### B. Mode basis neural network

An imaginary $k_{\mathrm{r}m}$ makes $e^{ik_{\mathrm{r}m}r}$ an exponentially decaying term with respect to propagation range $r$. A real $k_{\mathrm{r}m}$ leads to a propagating mode that oscillates instead. The infinite sum in (7) can be approximated as a $n$-mode finite sum in far-field:

$$\bar{p}(r,z)\approx\frac{i}{\rho(z_{\mathrm{s}})\sqrt{8\pi r}}e^{-i\frac{\pi}{4}}\sum_{m=1}^{n}\Psi_m(z)\Psi_m(z_{\mathrm{s}})\frac{e^{ik_{\mathrm{r}m}r}}{\sqrt{k_{\mathrm{r}m}}}. \tag{10}$$

Although analytical solutions to (5) are not always available, general field solutions based on the normal mode theory approximately follow [14]:

$$\bar{p}(r,z)\approx\sum_{m=1}^{n}\left(A_m e^{ik_{\mathrm{z}m}z}+B_m e^{-ik_{\mathrm{z}m}z}\right)\Phi_m(r), \tag{11}$$

where $A_m$ and $B_m$ are scaling factors to make sure boundary conditions and environment setup are satisfied.

Even though the approximated field expression is provided in (11), the mode parameters $A_m$, $B_m$, $k_{\mathrm{r}m}$ and $k_{\mathrm{z}m}$ associated with each mode are calculable only if boundary conditions and all required environmental parameters are accurately known. Any missing environmental parameter prevents the application of the conventional normal mode model, or requires the modeler to estimate the parameter through other means (e.g. matched field processing). Such a requirement greatly limits practical uses of normal mode models as operating environments may not always be well understood.

We propose a *mode basis neural network* (MBNN) framework to enable the use of normal mode modeling in situations where accurate environmental knowledge is unavailable. The idea of encoding the physics of acoustic propagation into structures of data-driven ML has been extensively explored in our previous work [12], where we detail a hybrid modeling framework based on the ray theory for high-frequency underwater acoustic propagation modeling. Following a similar approach, in the MBNN framework, we encode the domain knowledge of acoustic propagation based on the normal mode theory into the structure of a standard NN to model low-frequency acoustic propagation in oceans. The MBNN model is differentiable so that well-developed automatic differentiation techniques [18] can be utilized to find optimal unknown mode parameters, providing acoustic measurements and corresponding measurement locations as training data.

The MBNN framework allows numerical propagation models based on the normal mode theory to be data-driven. At the same time, the structure of MBNN encodes essential physics and therefore improves the model's data efficiency and generalizability as compared to classical data-driven ML techniques. This leverages complementary strengths of data-driven ML and physics-based models to handle practical scenarios of partially known physics and limited data availability.

We illustrate our MBNN model formulation through two examples of 2D ocean waveguides with isovelocity SSP (Section II-B1) and non-isovelocity SSP (Section II-B2) respectively:

*1) Isovelocity ocean waveguides:* We consider an isovelocity ocean waveguide that has a constant sound speed $c$ and density $\rho$ with a water depth $D$. A general eigenfunction solution to (5) in this isovelocity ocean waveguide follows [14]:

$$\Psi_m(z) = A_m \sin(k_{zm} z) + B_m \cos(k_{zm} z). \tag{12}$$

We assume a pressure-release surface:

$$\Psi(0) = 0, \tag{13}$$

and a rigid bottom:

$$\left.\frac{\mathrm{d}\Psi}{\mathrm{d}z}\right|_{z=D} = 0. \tag{14}$$

Such boundary conditions further simplify (12) to:

$$\Psi_m(z) = \sqrt{\frac{2\rho}{D}} \sin(k_{zm} z). \tag{15}$$

The corresponding eigenvalue $k_{\mathrm{r}m}$ is derived as:

$$k_{\mathrm{r}m} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left((m+0.5)\frac{\pi}{D}\right)^2}, m = 1, 2, \cdots, n. \tag{16}$$

We assume that we do not know the exact values of $c$, $\rho$ and $D$. Due to missing environmental knowledge, conventional normal mode models cannot estimate acoustic fields. Fortunately, our proposed MBNN model can automatically learn the best-fitted values of the unknown mode parameters from acoustic data. We train a minimal set of unknown mode parameters and numerically calculate other unknowns using underlying physics to make sure our method generalizes well.

We denote the minimal unknown mode parameters whose values are yet to learn from acoustic observations as MBNN model trainable parameters:

$$\mathcal{T} = \{c, \rho, D\}. \tag{17}$$

We minimize the square difference between the estimated pressure amplitude $\bar{p}(r, z; \mathcal{T})$ and the acoustic field measurement $\hat{p}$ at a measurement location $(r, z)$ by tuning $\mathcal{T}$. The loss function is defined as:

$$L(r, z, \hat{p}; \mathcal{T}) = |\bar{p}(r, z; \mathcal{T}) - \hat{p}|^2. \tag{18}$$

Equation (18) is normally summed over a batch of training data in each iteration as per ML standards [19]. With the optimal trainable parameters $\mathcal{T}^*$ learnt from acoustic observations, we can readily estimate acoustic fields using (6), (10), (15) and (16).

*2) Non-isovelocity ocean waveguides:*

*a) Known SSP:* The MBNN framework is capable of modeling non-isovelocity ocean waveguides as well. As the modal equation cannot be analytically solved for ocean waveguides with non-isovelocity SSPs, approximate solutions are necessary. We use the WKB approximation [20] – one of the most widely used approximation techniques in normal mode literature, to approximate the depth-dependent term:

$$\Psi_m(z) \approx A_m \frac{e^{i\int_0^z k_{zm}(s)ds}}{\sqrt{k_{zm}(z)}} + B_m \frac{e^{-i\int_0^z k_{zm}(s)ds}}{\sqrt{k_{zm}(z)}}, \tag{19}$$

where

$$k_{zm}(z) = \sqrt{\left(\frac{\omega}{c(z)}\right)^2 - k_{\mathrm{r}m}^2}. \tag{20}$$

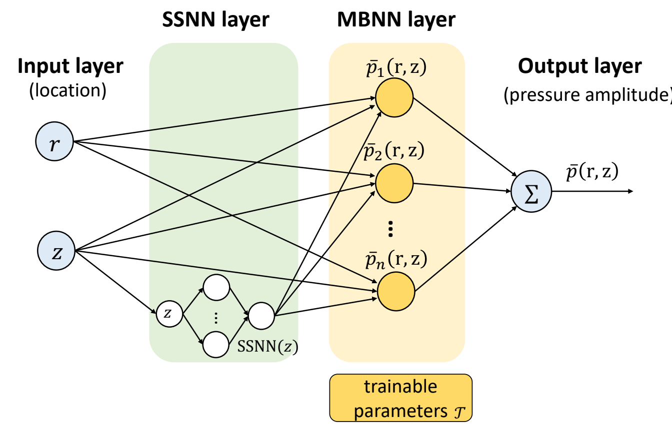



Fig. 1: The computational graph for (23) and (24) to estimate acoustic fields with unknown SSP.

We assume boundary conditions are unknown. This missing information introduces more unknown model parameters as compared to the isovelocity waveguide case in Section II-B1. When SSP is provided, we can use acoustic observations to find the optimal value of the trainable parameters:

$$\mathcal{T} = \{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{k}_{\mathrm{r}}\}, \tag{21}$$

where $\boldsymbol{A} = (A_1, A_2, \ldots, A_n)$, $\boldsymbol{B} = (B_1, B_2, \ldots, B_n)$ and $\boldsymbol{k}_{\mathrm{r}} = (k_{\mathrm{r}1}, k_{\mathrm{r}2}, \ldots, k_{\mathrm{r}n})$.

The missing environmental knowledge makes it hard to estimate the number of contributing modes $n$ precisely. When $\boldsymbol{A}$ and $\boldsymbol{B}$ are parts of the trainable parameters $\mathcal{T}$, conservatively setting $n$ to an upper bound of its possible range and adding $L_1$-norm regularization terms of $\boldsymbol{A}$ and $\boldsymbol{B}$ to encourage sparse solutions can help in model convergence. The loss function is updated to:

$$L(r, z, \hat{p}; \mathcal{T}) = |\bar{p}(r, z; \mathcal{T}) - \hat{p}|^2 + \alpha \|\boldsymbol{A}\|_1 + \beta \|\boldsymbol{B}\|_1, \tag{22}$$

where $\alpha$ and $\beta$ control the regularizations. With the trained optimal model parameters $\mathcal{T}^*$, acoustic fields in a non-isovelocity ocean waveguide with known SSP can be estimated using (10), (19) and (20).

*b) Unknown SSP:* The detailed SSP across the water column is often unknown. The unknown SSP makes the calculation of $k_{zm}(z)$ infeasible, even though the eigenvalue $k_{\mathrm{r}m}$ is provided. The MBNN is flexible to incorporate with standard NNs to model unknown physics. For example, we can implement a 1-input 1-output NN to model the SSP. We name the NN which models SSP as *sound speed neural network* (SSNN). Fig. 1 illustrates the overall structure of the MBNN framework that incorporates SSNN.

In order to train the SSNN, the trainable parameter $\mathcal{T}$ defined in (21) is modified to:

$$\mathcal{T} = \{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{k}_{\mathrm{r}}, \boldsymbol{S}\}, \tag{23}$$

where $\boldsymbol{S}$ contains all parameters (weights and bias) in the SSNN model. Calculating $k_{zm}(z)$ is feasible now using the trained SSNN:

$$k_{zm}(z) = \sqrt{\left(\frac{\omega}{\mathrm{SSNN}(z)}\right)^2 - k_{\mathrm{r}m}^2}, \tag{24}$$

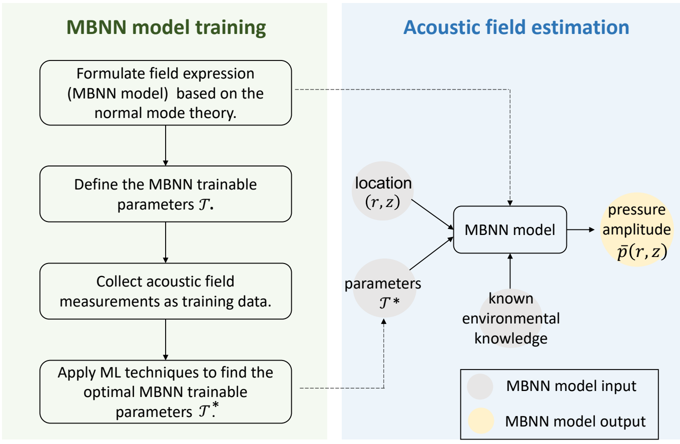



Fig. 2: Key steps in the proposed MBNN framework in model training stage and field estimation stage.

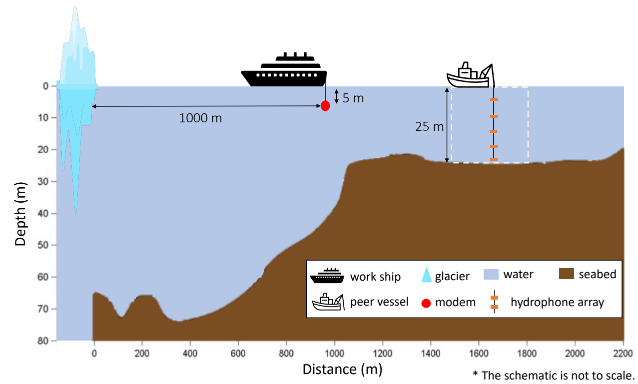



Fig. 3: Schematic of the simulated environment.

where $\text{SSNN}(z)$ is the estimated sound speed at depth $z$.

We employ the same loss function defined in (22) to learn optimal $\mathcal{T}^*$. We then use (10) and (19) with (24) to estimate acoustic fields in a non-isovelocity ocean waveguide with unknown SSP.

*3) Generalization to other mode models:* We have illustrated a few MBNN formulations in range-independent ocean waveguides and demonstrated how flexible the proposed modeling framework is in different scenarios. It is worth noting that formulations of normal mode models are application and environment specific. The idea of our MBNN modeling framework generalizes well to tackle different scenarios and can be applied to any variant of classical normal mode models. For example, the MBNN framework can incorporate adiabatic mode methods or coupled mode methods to model range-dependent environments [21]–[23].

Fig. 2 describes the steps involved in the MBNN model training stage and field estimation stage. For any ocean environment, the key is to have an analytical field solution or an approximated field solution based on the normal mode theory, and use a small number of acoustic measurements as training data to find the optimal MBNN trainable parameters $\mathcal{T}^*$. We can calculate other necessary physical quantities based on the trained MBNN parameters so as to estimate acoustic fields at locations of interest.

## III. Simulation studies

In [24], the authors study acoustic propagation and hydrological conditions at the Hans glacier front in Svalbard. This paper lays a foundation for several follow-up studies at the glacier [25]–[27]. We loosely use the environment described in this paper as the basis for our simulation studies to illustrate how one might model the acoustic propagation in a location where full environmental knowledge may be unavailable.

We assume that a work boat anchors at a location 1 km from the glacier. It carries an acoustic modem that emits 500 Hz continuous wave signals for acoustic communication and environmental monitoring. A peer survey boat executes exploration tasks in a region that is further away from the glacier. The bathymetry is approximately flat with a constant water depth of 25 m in the region in which the peer vessel operates. Fig. 3 depicts a schematic of the simulated environment. We use the Kraken normal mode model [28] to generate synthetic acoustic measurements in the simulated environment (with full environmental knowledge).

We then consider a scenario where the bottom properties and boundary conditions are unknown (and so unavailable to the model). We aim to model acoustic propagation from the work boat to a nearby region around the peer vessel in Section III-A and infer the SSP using acoustic measurements collected at a constant depth in Section III-B.

In both cases, we need to train our MBNN using the simulated data. We randomly split the acoustic measurements[1] into a training dataset and a validation dataset based on a $70\% : 30\%$ ratio. The training dataset trains the MBNN trainable parameters $\mathcal{T}$, while the validation data is to implement early stopping [29] to avoid over-fitting during the training. We use 30 modes in the MBNN formulation.

### A. Acoustic field estimation

We consider two acoustic field estimation problems, one with known SSP and one with unknown SSP. We assume a 24-element vertical hydrophone array with a 1 m inter-element spacing to the peer vessel to collect acoustic field measurements in the measurement region. We wish to accurately estimate acoustic field in an area, without having to make measurements at all points in that area. To illustrate how this can be done, we define a *measurement region* where we make several measurements, and two 100 m regions on both sides of the measurement region as *extended region* to demonstrate field extrapolation. The measurement region and extended region together form the *area of interest* (AOI).

In [30], the authors find that the use of PINN does not benefit acoustic field estimation performance as compared to standard ML models. Our preliminary evaluation of PINN for this application agrees with this finding. We thus benchmark the field estimation performance of the proposed MBNN framework against two classical data-driven ML techniques

[1]The synthetic acoustic measurements used in simulation studies are peak-to-peak values from hydrophone output in millivolts.

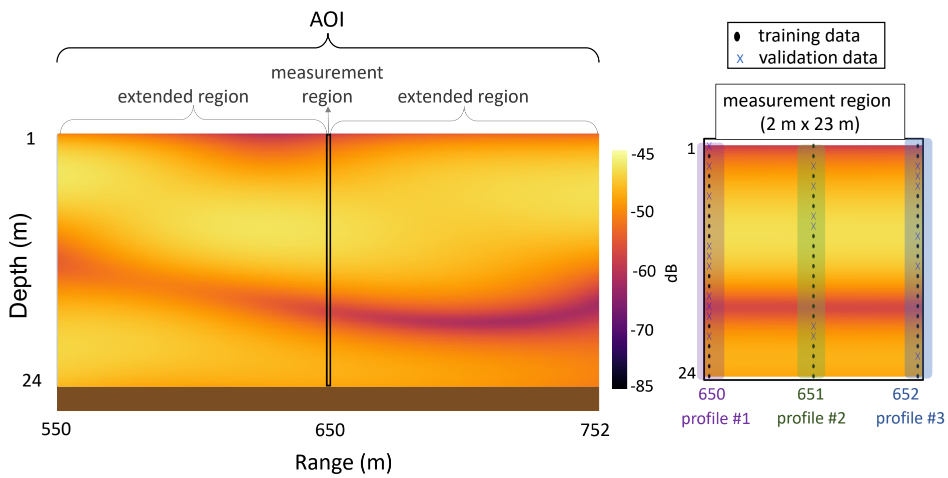
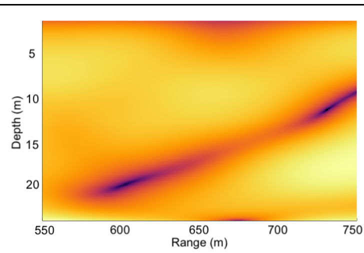
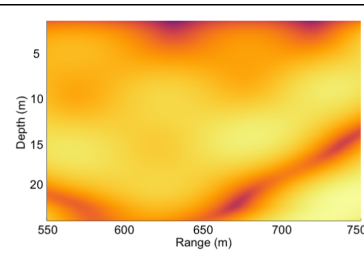
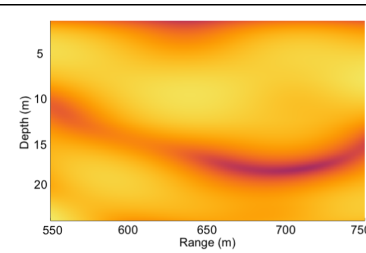
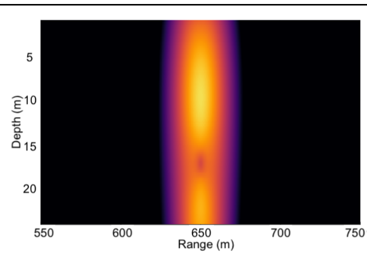
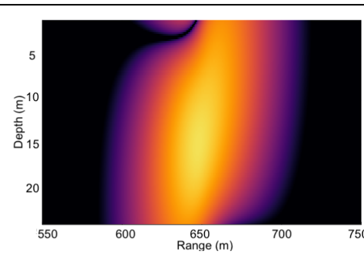
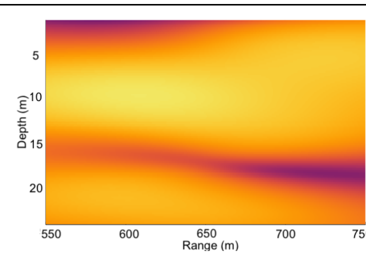
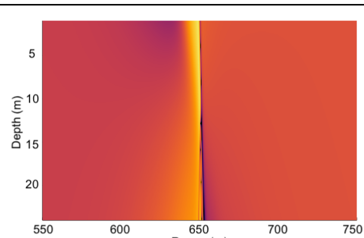
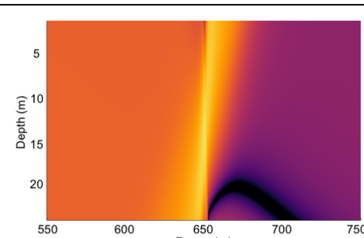
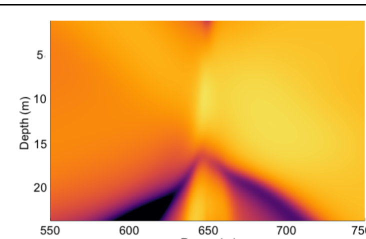



Fig. 4: Ground truth field pattern in the AOI for field estimation problem with known SSP. The training data and validation data used in the model training are labelled respectively.

TABLE I: Acoustic field estimation performance of the three models using a different number of profile measurements in the field estimation problem with known SSP.

| Model | Estimated field pattern [RMS test error (mV$_{pp}$)] | | |
|---|---|---|---|
| | measurements with profile #1 | measurements with profiles #1 & #2 | measurements with profiles #1–#3 |
| MBNN |  [4.67] |  [4.80] |  [1.19] |
| GPR |  [8.12] |  [7.50] |  [2.10] |
| DNN |  [8.89] |  [6.51] |  [4.59] |

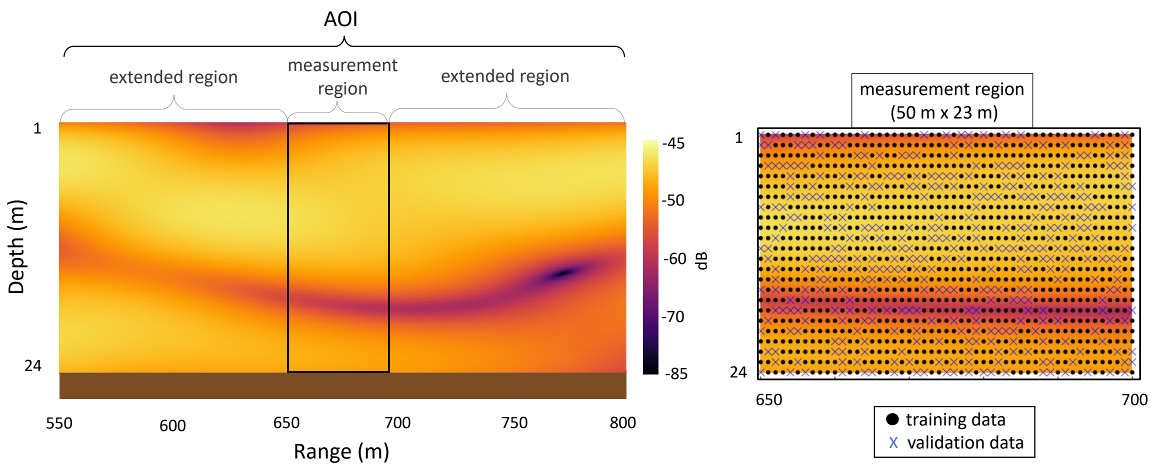



Fig. 5: Ground truth field pattern in the AOI for field estimation problem with unknown SSP. The collected training data and validation data used in the model training stage are labelled respectively.

– Gaussian process regression(GPR) and deep neural network (DNN). We design a composite kernel of a squared exponential isotropic kernel and a Matérn 5/2 ARD kernel for GPR, and implement a 2-input 1-output DNN with 3 hidden layers and ReLU activation function. We randomly initialize the MBNN and the DNN model parameters in each run. The hyper-parameters of the GPR model are fine-tuned by minimizing validation error. We carry out 10 Monte Carlo simulations for MBNN and DNN models, and present the field estimation results with the smallest validation error.

*1) Field estimation with knowledge of SSP:* We consider an ocean waveguide with a known non-isovelocity SSP, unknown seabed properties and unknown boundary conditions. We use the WKB approximation to formulate the MBNN model based on (10), (19) and (20). We use acoustic measurements to find optimal values of the trainable parameters $\mathcal{T}^*$ defined in (21).

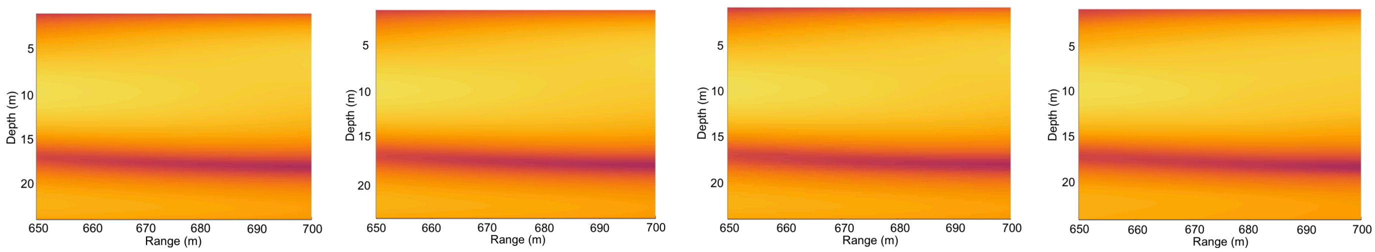
We deploy the hydrophone array to collect acoustic measurements in a 2 m $\times$ 23 m measurement region at three profiles, each spaced by a 1 m range in between. Fig. 4 shows ground truth field pattern in the AOI and the three profiles where we collect the measurements. In order to investigate the field estimation performance, we use the Kraken model to generate 464,600 test data with a resolution of 0.1 m in range and depth within the AOI. We investigate data efficiency of the three models by estimating the field patterns in the AOI using measurements collected at one profile (24 measurements), two profiles (48 measurements) or three profiles (72 measurements) in the measurement region.

Table I shows the estimated field patterns and the corresponding root-mean-square (RMS) test errors in the AOI when different amounts of acoustic measurements are given. When acoustic measurements made at profile #1 & #2 are provided, the GPR and DNN models fail to extrapolate field patterns due to insufficient training data. The field estimated by the MBNN shows a rough field pattern with low fidelity. When the measurements made at profile #1–#3 are provided, the field estimated by the MBNN model aligns well with the ground truth field pattern. The GPR can extrapolate more details in the extended region. The DNN still performs poorly in extrapolation. The corresponding RMS test errors presented also justify our observations that the MBNN model outperforms the GPR and DNN models in terms of data efficiency and extrapolation performance.

*2) Field estimation without knowledge of SSP:* Sound speed in an ocean waveguide is measured by sending a conductivity, temperature and depth (CTD) sensor at various depths. When either the CTD sensor or equipment to survey sound speed at various depths is lacking, we do not know the exact SSP. In this case, we assume that no sound speed measurement is available due to the lack of a CTD sensor. Instead, we only have a rough understanding of a reasonable range that the SSP may fall in. Our conservative initial guess of the SSP is that it falls in a 100 m/s range between 1,400 m/s and 1,500 m/s and SSP variation should not exceed 35 m/s over the 25 m water depth.

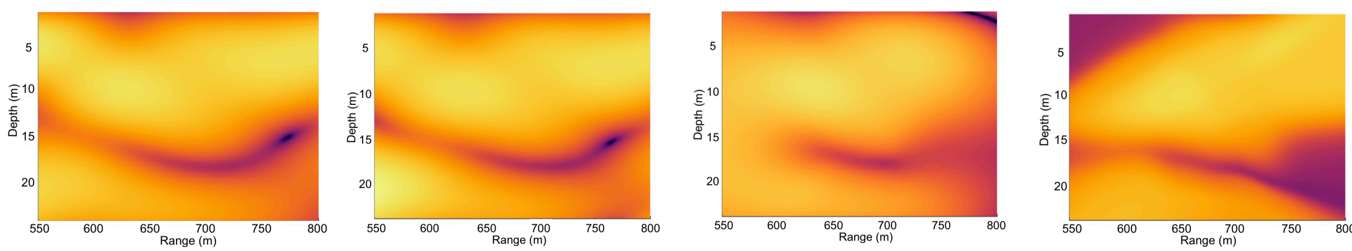
One may expect that more acoustic data is required to train the MBNN framework as the size of trainable parameter $\mathcal{T}$ increases as compared to the previous scenario. As shown in Fig. 5, we uniformly collect 1,224 acoustic measurements (24 elements/profile $\times$ 51 profiles) within a 50 m $\times$ 23 m measurement region. We assume the detailed SSP and seabed properties are unknown. We use a simple 1-input 1-output NN (SSNN) with 1 hidden layer and ReLU activation function to learn the SSP in the water column. We formulate the MBNN model based on the WKB approximation as (10), (19) and (24). We aim to estimate the acoustic field pattern in the AOI by learning the optimal trainable parameters $\mathcal{T}^*$ defined in (23) through acoustic measurements. We generate 575,000 acoustic measurements, with a resolution of 0.1 m in range and depth, as the test dataset over the AOI to rigorously quantify the field estimation performance.

As shown in Fig. 6, all of the three models can interpolate acoustic fields in the measurement region well. However, field patterns extrapolated in the extended region shown in Fig. 7 highlight the superiority of our proposed MBNN framework over the GPR and DNN models. The test errors shown in




(a) Ground truth (b) Estimation by MBNN (c) Estimation by GPR (d) Estimation by DNN

Fig. 6: The estimated field patterns in the measurement region when SSP is unknown. Panel (a) shows the ground truth field pattern. Panels (b)–(d) show the estimated fields by the MBNN, GPR and DNN models.




(a) Ground truth (b) Extrapolation by MBNN (c) Extrapolation by GPR (d) Extrapolation by DNN

Fig. 7: The estimated field patterns in the AOI when SSP is unknown. Panel (a) shows the ground truth field pattern. Panels (b)–(d) show the extrapolated fields by the MBNN, GPR and DNN models.

TABLE II: Acoustic field estimation performance of the three models in the field estimation problem with unknown SSP.

| Model | RMS test error ($\mathrm{mV_{pp}}$) | |
|---|---|---|
| | In measurement region | In AOI |
| MBNN | 0.039 | 0.87 |
| GPR | 0.011 | 2.10 |
| DNN | 0.10 | 2.88 |

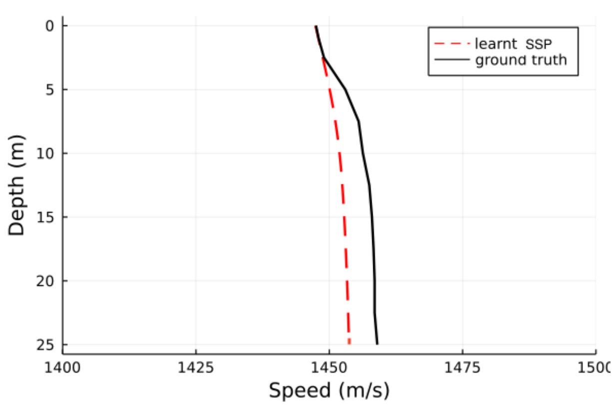



Fig. 8: The learnt SSP with the ground truth SSP in the field estimation problem. The SSNN is trained using acoustic measurements sampled across the water column in the measurement region.

Table II support the observations we draw from Fig. 6 and Fig. 7. Although estimation of the SSP might not be of interest to the field estimation problem, the learnt SSP by the SSNN is close to the ground truth SSP in the 100 m/s span as shown in Fig. 8. It demonstrates the flexibility of the MBNN model to incorporate with standard NNs to model unknown physics.

### *B. Inversion for entire SSP*

In Section III-A2, we have demonstrated that our proposed MBNN model can learn the SSP reasonably well using the acoustic field measurements collected at 50 profiles when a CTD sensor is not provided. However, consider a scenario where measurements are not available through the entire water column, but rather at a few shallow depths only. Could one estimate the SSP in the entire water column with just a few shallow measurements? The MBNN framework can incorporate standard NNs to model unknown physics – in this case, the SSP. This makes the MBNN framework a useful tool for solving various inverse problems related to acoustic propagation modeling.

To illustrate the idea, we assume that an autonomous underwater vehicle (AUV) equipped with an acoustic sensor and CTD is deployed from the peer vessel and dives to an operating depth of 4 m. It then operates at a constant depth of 4 m, and therefore does not have access to profiles through the water column to make either CTD or acoustic measurements. Fig. 9 indicates the AUV's trajectory in the measurement region. The AUV uniformly makes 5 sound speed measurements at depths between the water surface and the operating depth of 4 m. We aim to learn the entire SSP using the acoustic field measurements collected at a nearly constant depth with the aid of just a few sound speed measurements made at shallow depths.

We assume boundary conditions and seabed properties are unknown. We use the same model formulation, trainable pa-

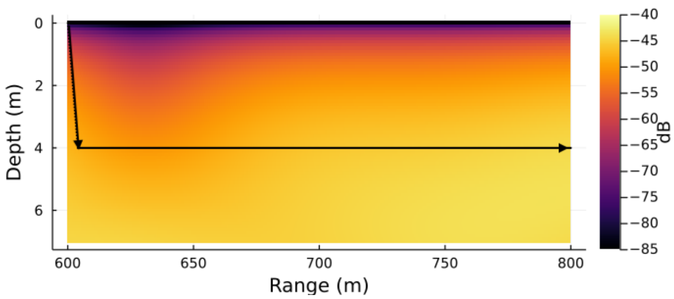



Fig. 9: The trajectory of AUV labelled as an arrow in the inversion of SSP application.

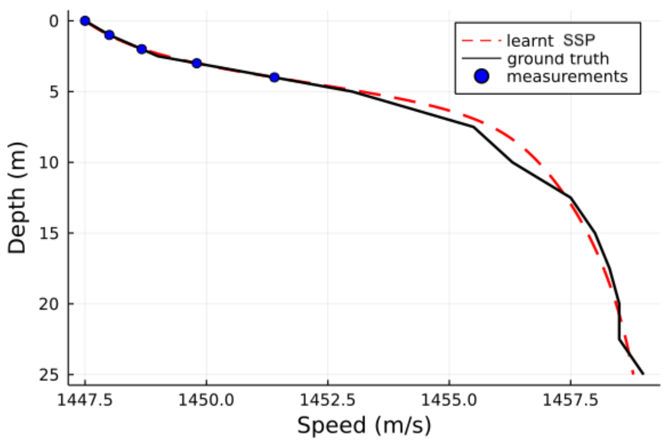



Fig. 10: The learnt SSP with the ground truth SSP in the SSP inversion application using acoustic measurements made at a nearly constant depth and a few sound speed measurements.

rameters and initial guess of the SSP defined in Section III-A2. The lack of strong spatial diversity of the collected field measurements makes this inversion problem particularly challenging.

The learnt SSP, benchmarked against the ground truth SSP, is shown in Fig. 10. We constrain the SSNN using the sound speed measurements made at the water surface, 1 m, 2 m, 3 m and 4 m in the loss function. The learnt SSP over 25 m depth is very close to the ground truth SSP, even for depths where no acoustic field measurement and sound speed measurement are provided. Training the MBNN model using acoustic measurements with stronger field variation can potentially reduce the training data size and improve the inversion accuracy.

## IV. Conclusion

Conventional normal mode propagation models, although are widely used, require accurate prior environmental knowledge. Environmental uncertainties thus significantly affect model estimation performance and thus limit its applicability in practice. We proposed a physics-based data-aided modal acoustic propagation modeling framework based on the normal mode theory. The proposed framework embeds the normal model theory of acoustic propagation into the structure of a NN, so as to enable a data-efficient propagation modeling framework. The proposed model is flexible to incorporate any known environmental knowledge and standard NNs to model unknown physics. At the same time, it extrapolates well and brings interpretability to the trained model parameters. We demonstrate the proposed modeling framework through several simulated application examples, and by benchmarking against classical ML techniques.

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