Generating Multichannel Colored Noise For Underwater Acoustic Simulations

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***Abstract*—Underwater acoustic communication algorithms typically undergo numerical testing within simulated environments prior to real-world application. These simulations require noise samples that accurately reflect the ambient noise in the underwater environment. The common practice of employing a white Gaussian noise model for additive noise in simulations fails to capture the true nature of underwater noise. Given the wideband nature of most underwater communication systems and the frequency-dependence of ambient noise, the assumption of white noise becomes untenable. Furthermore, communication systems equipped with multiple hydrophones for spatial diversity experience correlated noise across hydrophones. This paper presents a simple technique for generating colored noise that is consistent with the spatial and temporal correlation characteristics of ambient noise measured at sea.**

I. Introduction

Underwater acoustic communication algorithms are often tested via numerical simulations prior to real-world deployment. In order to accurately estimate the performance of these algorithms, the simulations must accurately model the signal distortions introduced by the underwater channel. These distortions include time-varying frequency-selective fading due to multipath propagation, Doppler, and additive noise. While most channel modeling work has focussed on the former two [1]–[4], modeling of additive noise has received less attention. This paper is aimed at addressing this gap.

One may use recorded ambient noise from the ocean as additive noise samples for simulation. While this approach yields accurate results, it requires long uncontaminated noise recordings to be available. This limits the duration of transmission that can be simulated without repeating the noise samples. Another common practice is to employ a white Gaussian noise model to generate additive noise samples during simulation. While this approach is simple, it fails to capture the true nature of underwater noise. Ambient noise in the ocean is typically colored, with a frequency-dependent power spectral density [5]. The assumption of white noise is untenable for wideband underwater communication systems. Moreover, underwater ambient noise has directionality and therefore exhibits spatiotemporal correlation across hydrophones in a multichannel system. Even in the case of isotropic noise, the wideband nature of communication systems can lead to noise correlations across channels. A model where noise samples are independently generated for each receive hydrophone fails to capture this correlation. The generative model presented in this paper addresses both these limitations.

The Gaussian noise assumption holds in many underwater environments, especially in deeper or cooler waters. Warm shallow waters, however, exhibit a significant amount of non-Gaussian noise due to snapping shrimp and other biological sources [6]. Some polar regions also exhibit non-Gaussian noise due to ice cracking and bubbles from melting glaciers and icebergs [7]. We limit our discussion to the Gaussian noise model in this paper, but plan to extend the work to non-Gaussian noise in a follow-up paper. The Gaussian noise model is applicable to a wide range of underwater environments and is a good starting point for most underwater communication simulations.

II. Problem Statement

Consider an acoustic communication system with N receive hydrophones (channels) operating at a sampling rate of f_s samples per second. We wish to generate a simulated passband noise sequence $\{x_{it}\}$, where i is the channel number and t is a time index, with statistical properties similar to some measured Gaussian noise sample $\{\bar{x}_{it}\}$.

A Gaussian random process is fully characterized by its mean and covariance [8]. A hydrophone measures dynamic pressure variations (around a mean static pressure), and the measured Gaussian noise is therefore, by definition, zero-mean. We therefore have:

$$\{\bar{x}_{it}\} \sim \mathcal{N}(\mathbf{0}, \bar{\mathbf{R}}),$$

where $\bar{\mathbf{R}} = [\bar{R}_{ij\delta}]$ is a $N \times N \times (2L+1)$ covariance tensor, $i \in \{1 \cdots N\}$ and $j \in \{1 \cdots N\}$ are channel numbers, and $\delta \in \{-L \cdots L\}$ is the time lag. L is the maximum time lag, beyond which noise samples may be considered uncorrelated. Since our simulated noise must have similar statistical properties as the measured noise, we require:

$$\{x_{it}\} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}),$$

with $\mathbf{R} \approx \bar{\mathbf{R}}$. The problem then reduces to two sub-problems:

- Estimation of the covariance tensor $\bar{\mathbf{R}}$ from the measured noise samples $\{\bar{x}_{it}\}$, and
- Generation of a noise sequence $\{x_{it}\}$ with covariance tensor $\mathbf{R} \approx \bar{\mathbf{R}}$.

III. Covariance Estimation

Estimating the covariance tensor $\bar{\mathbf{R}}$ from the measured noise samples $\{\bar{x}_{it}\}$ is straightforward. Each of the components in the covariance tensor are estimated as unbiased sample covariances [9]:

$$\bar{R}_{ij\delta} = \frac{1}{T-2L-1} \sum_{t=1+L}^{T-L} \bar{x}_{it}\bar{x}_{j(t+\delta)}, \quad (1)$$

where T is the total number of time samples in the measured noise sequence.

IV. Noise Generation

Let $\{z_{it}\}$ be independent standard Gaussian random variates:

$$z_{it} \sim \mathcal{N}(0,1).$$

We generate the desired noise sequence $\{x_{it}\}$ from $\{z_{it}\}$ as follows:

$$x_{it} = \sum_{j,\tau} \alpha_{ij\tau} z_{j(t-\tau)}, \quad (2)$$

where $\alpha_{ij\tau}$ are the mixing coefficients and $\tau \in \{0 \cdots L\}$. We wish to determine mixing coefficients $\{\alpha_{ij\tau}\}$ such that the covariance tensor of $\{x_{it}\}$ matches the covariance tensor $\bar{\mathbf{R}}$ of the measured noise samples.

We compute the covariance tensor $\mathbf{R}$ of $\{x_{it}\}$:

$$\begin{aligned} R_{ij\delta} &= \mathbb{E}\left[x_{it}x_{j(t+\delta)}\right] \\ &= \mathbb{E}\left[\left(\sum_{k,\tau} \alpha_{ik\tau} z_{k(t-\tau)}\right)\left(\sum_{m,\nu} \alpha_{jm\nu} z_{m(t+\delta-\nu)}\right)\right] \\ &= \sum_{k,\tau} \alpha_{ik\tau}\alpha_{jk(\tau+\delta)}, \end{aligned}$$

since $\mathbb{E}[z_{k(t-\tau)}z_{m(t+\delta-\nu)}] = 1$ only if $k = m$, $t-\tau = t+\delta-\nu$ and 0 otherwise.

We determine mixing coefficients $\{\alpha_{ij\tau}\}$ such that the covariance tensor $\mathbf{R}$ of the generated noise matches the covariance tensor $\bar{\mathbf{R}}$ of the measured noise samples by solving an optimization problem:

$$\underset{\{\alpha_{ij\tau}\}}{\text{minimize}} \sum_{i,j,\delta} \left| \bar{R}_{ij\delta} - \sum_{k,\tau} \alpha_{ik\tau}\alpha_{jk(\tau+\delta)} \right|^2. \quad (3)$$

The problem is easily solved using gradient descent methods [10], with a good initial guess to start iterative optimization. We recommend an initial guess of $\alpha_{ii0} = 1 \;\forall\; i$ and $\alpha_{ij\tau} = 0$ otherwise.

Once the optimal mixing coefficients $\{\alpha_{ij\tau}\}$ are determined, the noise sequence $\{x_{it}\}$ is generated using the mixing coefficients and the standard Gaussian random variates $\{z_{it}\}$. The steps are summarized in Algorithm 1.

Algorithm 1: Generation of passband multichannel colored Gaussian noise, given passband noise sample or an estimated covariance tensor.

Input: Measured passband noise $\{\bar{x}_{it}\}$ or estimated covariance tensor $\bar{\mathbf{R}}$.
Output: Generated passband noise $\{x_{it}\}$.
Train the noise model:
1) If $\bar{\mathbf{R}}$ is not given as input, estimate it from $\{\bar{x}_{it}\}$ using (1).
2) Determine mixing coefficients $\{\alpha_{ij\tau}\}$ by solving the optimization problem (3).

Generate noise using the model:
3) Generate standard Gaussian random variates $\{z_{it}\}$.
4) Generate passband noise using (2).

V. Implementation

In order to aid optimization, it is advisable to scale the measured passband noise $\{\bar{x}_{it}\}$ to have approximately unit variance for each channel. Scaling does not change the correlation properties of the noise, but alters the absolute noise level. The scale factors can be recorded and the scaling can be undone after generating random noise samples, if the application requires generated noise levels to match the measured noise levels.

An implementation based on Algorithm 1 and scaling, as described above, is provided as an open-source Julia package at `https://github.com/org-arl/NoiseModels.jl`.

This implementation was used to generate all the results in the next section.

VI. Results

We demonstrate the method outlined in the previous sections by applying to a multichannel ambient noise dataset collected during the KAM11 experiment [11].

The KAM11 dataset used here is from a 12-channel vertical array with 20 cm spacing, deployed at 47 m depth in 100 m of water depth. A 55-second ambient noise sample (5-60 seconds in file `1860532F0083_C0_S4`), sampled at 39.062 kSa/s, was used for training. Since communication systems are bandlimited, we bandpass filtered the ambient noise data in the 5–15 kHz band to remove out-of-band noise before training. The net sensitivity of the receiver system in KAM11 has not been accounted for, and hence the recorded data should be considered uncalibrated for absolute acoustic pressure.

The power spectral density in Figure 1 shows a slow reduction in spectral level with frequency, as is typical of ambient noise in the ocean. The normal probability plot in Figure 2 shows that the noise data collected on all 12 channels is consistent with a Gaussian distribution. The training data was verified to be stationary by checking

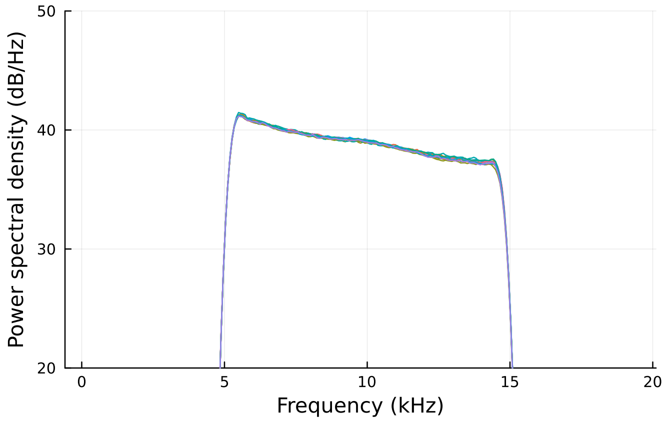

Figure 1: Power spectral density of each of the 12 channels in the KAM11 ambient noise dataset.

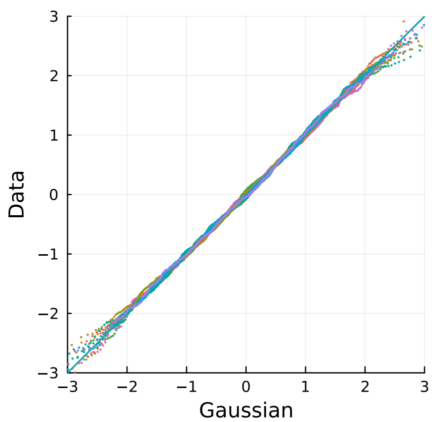

Figure 2: Normal probability plot of each of the 12 channels in the KAM11 ambient noise dataset. Data points along the diagonal of a normal probability plot indicate an agreement with Gaussian statistics. Significant deviations from the diagonal suggest non-Gaussianity.

the first and second order statistics in 1-second windows over the duration of the sample.

After training the model as described in Algorithm 1 with $L = 64$, we use the model to generate a 10 second multichannel noise sample. The power spectral density of the generated data is shown in Figure 3. We see that it agrees well with the power spectral density of KAM11 noise as seen in Figure 1. In Figure 4 and Figure 5, we see that the timeseries and spectrograms from the KAM11 noise dataset and the generated noise data are also very similar.

In order to demonstrate that the spatial correlation properties of the generated noise agree with the KAM11 data, we show frequency-wavenumber plots for both the KAM11 data and the generated noise in Figure 6. We can clearly see that most of the energy in the water column is propagating in the horizontal direction (small vertical wavenumber) in both the KAM11 data and generated noise. Some additional ambient noise directionality structure is also visible in the KAM11 data, and the same structure can be seen in the generated noise. The equivalent normalized noise directionality plots at 5 kHz and 10 kHz are shown in Figure 7.

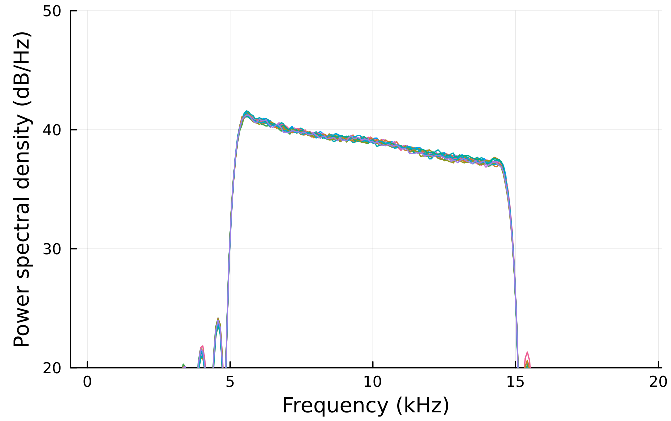

Figure 3: Power spectral density of each of the 12 channels of the noise generated using the trained model.

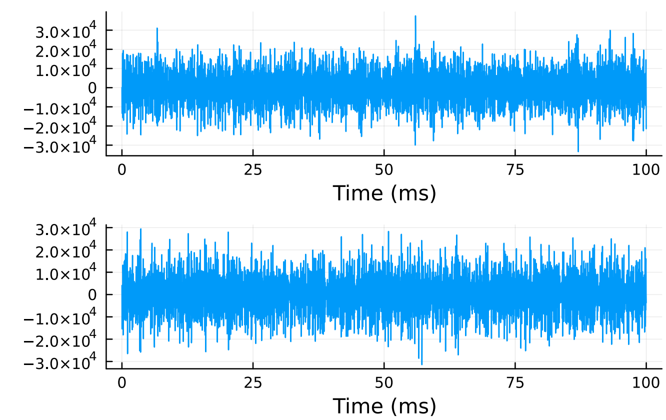

Figure 4: A comparison of time series for a 0.1-second sample from one channel for KAM11 ambient noise dataset (top) and noise generated using the trained model (bottom). The y-scale is proportional to the acoustic pressure.

VII. Discussion and Conclusion

We presented a simple method that can be used to generate realistic additive multichannel Gaussian noise for use in underwater acoustic simulations. The noise is statistically indistinguishable from noise recorded during experiments in areas where the ambient noise is stationary (over the timescales of interest) and Gaussian. The generated noise retains the temporal and spatial correlations arising from frequency-dependence and noise directionality.

In order to train the generative noise model, we require a sample of ambient noise from the environment of interest or a covariance tensor summarizing the noise statistics.

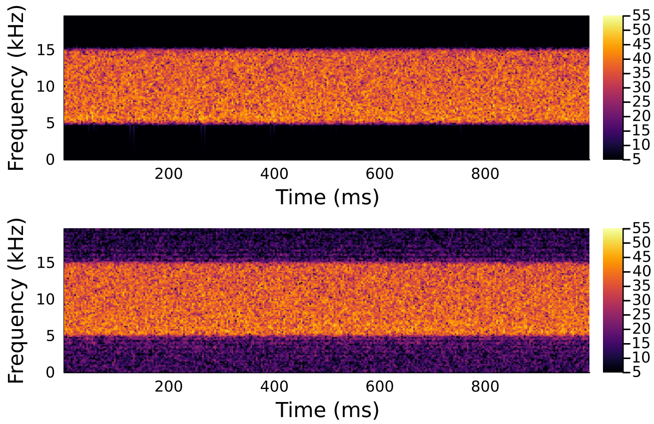

Figure 5: A comparison of spectrograms for a 1-second sample from one channel for KAM11 ambient noise dataset (top) and noise generated using the trained model (bottom). The color scale is in dB.

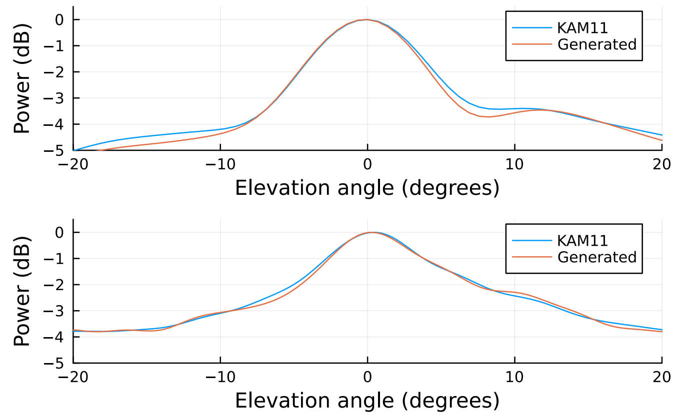

Figure 7: A comparison of normalized noise directionality plots for KAM11 ambient noise dataset and noise generated using the trained model at 5 kHz (top) and 10 kHz (bottom).

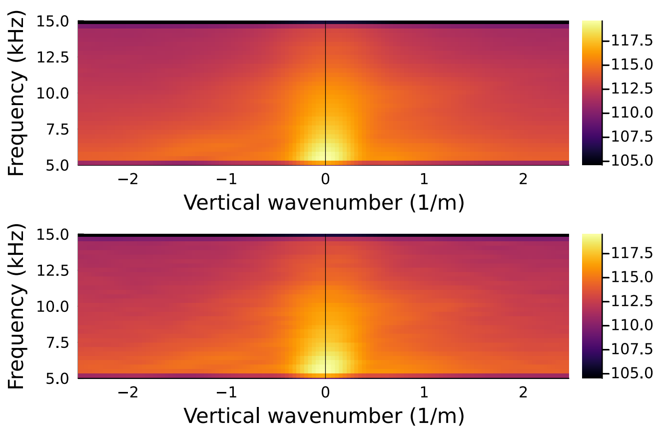

Figure 6: A comparison of frequency-wavenumber plots for for KAM11 ambient noise dataset (top) and noise generated using the trained model (bottom). The color scale is in dB.

While ambient noise data must be obtained by field measurements, only a small amount of training data is required to build the noise model. Once the model is trained, it can be used to generate an infinite amount of realistic noise samples. Benchmark channel replay datasets such as Watermark [4] provide a way for researchers to simulate acoustic propagation through a measured channel. The method described above may be used to publish channel noise covariance tensors to complements channel replay datasets, allowing researchers to not only simulate realistic acoustic propagation but also realistic ambient noise. While large acoustic recordings from the field could potentially serve the same purpose, the storage and distribution of channel noise covariance tensors is much easier and more cost-effective.

Acknowledgement

The author would like to thank James Preisig, Milica Stojanovic, Paul van Walree, Andrew Singer and Li Zhengnan for initial discussions that helped shape the work presented here, and for graciously providing the ambient noise data used to validate the work. The author would also like to thank the anonymous reviewers for their valuable suggestions that have helped improve the presentation of the work in this paper.

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